Bounds for the eigenvalues of a matrix polynomial by using Tropical algebra and application to the numerical computation of eigenvalues

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We study the problem of computing the eigenvalues of a matrix polynomial. We introduce a general scaling technique, based on tropical algebra, which applies in particular to the companion form. This scaling is based on the construction of an auxiliary tropical polynomial function, depending only on the norms of the matrix coefficients. The tropical roots of this tropical polynomial which provide a priori estimates of the modulus of the eigenvalues, are the non-differentiability points or equivalently, the opposites of the slopes of a certain Newton polygon. We show that the sequence of absolute values of the eigenvalues of a matrix polynomial is majorized by a sequence of these tropical roots. This extends to the case of matrix polynomials some bounds obtained by Hadamard, Ostrowski and Polyá for the roots of scalar polynomials.

We also provide a new location result, showing that under assumption involving condition numbers, there is one group of "large" eigenvalues, which have a maximal order of magnitude, given by the largest root of the auxiliary tropical polynomial. A similar result holds for a group of small eigenvalues.

We show experimentally that this scaling improves the backward stability of the computations, particularly in situations when the data have various orders of magnitude.

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