Variational characterization of the quasilinear Gross–Pitaevskii dark solitons

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We study a quasilinear Schrödinger equation with nonzero conditions at infinity in one dimension. We obtained a continuous branch of traveling-wave solutions, given by dark solitons indexed by their speed. Neglecting the quasilinear term, one recovers the Gross–Pitaevskii equation, for which the branch of dark solitons minimizes the energy at fixed momentum.

In two submitted papers, we investigated how the quasilinear term affects the variational properties of the dark solitons. For weak quasilinear interactions, these dark solitons can be obtained as minimizers of the energy at fixed momentum via a concentration-compactness argument. While, for stronger quasilinear interactions, a cusp appears in the energy-momentum diagram of the solitons and we identified three behaviors depending of the speed of the wave. The fast waves remain minimizers, the waves of intermediate speeds, before the cusp, are local minimizers and the slow waves are saddle points under the constraint of fixed momentum. We obtained these local variational properties by performing the spectral analysis on the Hessian of a modified energy functional and proceeding as in the seminal work of Grillakis, Shatah and Strauss.

The aim of this poster is to present our results for the minimization problem and the local variational properties of the dark solitons of the quasilinear equation and to sketch some important steps of the analysis. This presentation is based on a joint work with André de Laire (Université de Lille).