A finite volume scheme for the quantum Navier-Stokes system

Robin COLOMBIER

Université Polytechnique Hauts-de-France
robin.colombier@uphf.fr

In order to better understand how semiconductors work we aim to simulate the flow of electrons, which when there are many, can be modelized using fluid mechanics, by the system of quantum Navier-Stokes equations given for $t \in [0, T]$ and $\mathbf{x} \in \Omega$:

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p(\rho) - 2\epsilon^2 \rho \nabla \dfrac{\Delta \sqrt{\rho}}{\sqrt{\rho}} + r\rho \mathbf{u} = 2\nu \nabla \cdot (\rho D(\mathbf{u})), \\ \rho_{|t=0} = \rho_0, \quad \rho \mathbf{u}_{|t=0} + \rho_0 u_0, \end{cases} \tag{1}$$

with $\quad p(\rho) = \rho^\gamma, \quad \gamma > 1, \quad \epsilon > 0, \nu > 0$ and $r > 0$. Here the unknowns are the particle density ρ and the particle velocity $\mathbf{u}$ and $\Omega = \mathbb{T}^d$ is the torus in dimension d $(1 \leq d \leq 2)$.

By noting $\mathbf{v} = \epsilon \nabla log(\rho)$ and $\mathbf{w} = \mathbf{u} + \nu \mathbf{v}/\epsilon$, we can rewrite (1) as an augmented system of order 2 in the new variables $\rho, \mathbf{w}, \mathbf{v}$. The new system allow for a maximum of order 2 term and is more suited for the numerical scheme, and BD-entropy can be defined. The objective of our work is to propose a finite volume scheme allowing to define a discrete BD-entropy. Some 1D benchmarks as the grey soliton and the dispersive Riemann problem, as well as a 2D benchmark on cartesian grids illustrate the performance and the accuracy of the numerical scheme.