Boundary layer analysis of a d-dimensional penalization method for Neumann or Robin boundary conditions

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In this talk, we present a d-dimensional extension of a fictitious domain penalization technique that we previously proposed for Neumann or Robin boundary conditions. We apply Droniou's approach for non-coercive linear elliptic problems to obtain the existence and uniqueness of the solution of the penalized problem, and we derive a boundary layer approach to establish the convergence of the penalization method. The developed boundary layer approach is adapted from the one used for Dirichlet boundary conditions, but in contrast to the latter where coercivity enables a straightforward estimate of the remainders, we reduce the convergence of the penalization method to the existence of suitable supersolutions of a dual problem. These supersolutions are then constructed as approximate solutions of the dual problem using an additional formal boundary layer approach. The proposed approach results in an advection-dominated problem, requiring the use of appropriate numerical methods suitable for singular perturbation problems. Numerical experiments, using upwind finite differences, validate both the convergence rate and the boundary layer thickness, illuminating the theoretical results. Finally, we investigate the applicability of the suggested method to problems raised on moving domains such as those associated with the simulation of population dynamics under climate change. This talk comprises joint work with Jacques Liandrat, Centrale Méditerranée, I2M.