A convergence result for a non-local eikonal equation modeling dislocation dynamics

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Dislocation dynamics are crucial for understanding the mechanical properties of materials, particularly in one-dimensional (1D) models. Dislocations are line defects in crystals, and their motion driven by external stress explains metallic plastic deformation. The Peach-Koehler force, derived from linear elasticity equations, governs their motion. In this study, we focus on a simplified 1D version of a model originally proposed in 2D by Rodney, Le Bouar, and Finel. Specifically, we consider dislocations as parallel lines moving within an elastic crystal plane, described by a non-local eikonal equation:

$$\begin{cases} \partial_t v(x,t) = (\mathcal{K} \star v(\cdot,t))(x) \partial_x v(x,t) & \text{in } \mathbb{R} \times (0,T), \\ v(x,0) = v_0(x) & \text{in } \mathbb{R}, \end{cases}$$

where v represents the scalar unknown function, and $\mathcal{K}$ is a kernel function dependent on the crystal's physical properties. This non-local and non-monotonic equation presents significant challenges for proving uniqueness via standard methods.

The primary objective of this research is to establish the convergence of a periodic numerical scheme for the discretized non-local eikonal equation and to extend these results to the non-periodic case. Preliminary results include the existence, uniqueness, and monotonicity of the solution, as well as total variation decay and entropy estimates.