The Gurtin–Pipkin heat equation: Old and new results

Vittorino PATA

Politecnico di Milano
`vittorino.pata@polimi.it`

We consider an abstract version of the integrodifferential equation

$$\partial_t u(t) - \int_0^\infty g(s)\Delta u(t-s)ds = 0,$$

modeling hereditary heat conduction of Gurtin–Pipkin type. Under suitable albeit quite general assumptions on the convolution kernel g, the equation generates a contraction semigroup $S(t) = e^{t\mathbb{A}}$ acting on a certain Hilbert space. Although the decay properties of $S(t)$ are nowadays well understood, several important issues related to the structure of the spectrum of the infinitesimal generator $\mathbb{A}$ have not yet been investigated.

In this talk, we provide some answers in that direction, demonstrating in particular the impossibility to have arbitrarily fast decays. For the most relevant physical case of the exponential kernel, we will also prove that the semigroup fulfills the so-called spectrum determined growth condition, telling that the decay type of $S(t)$ is fully dictated by the spectrum of $\mathbb{A}$. In some cases, the optimal decay rate turns out to be actually attained.