The limit dynamic of the Su-Boyd-Candès accelerated gradient system when the asymptotic vanishing damping coefficient α becomes large: a singular perturbation approach

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In a real Hilbert space setting, we concentrate on the continuous dynamical system introduced by Su, Boyd, and Candès as a low-resolution ordinary differential equation (ODE) version of Nesterov's accelerated gradient method (NAG). This inertial system, represented as $(\mathrm{AVD})_\alpha$, is driven by the gradient of the function f that is subject to minimization and features a damping mechanism with an asymptotic vanishing coefficient of the form α/t, where $\alpha \geq 3$. Selecting a sufficiently large α is pivotal for ensuring the desirable asymptotic convergence characteristics of the trajectories. Specifically, for a general convex function f, choosing $\alpha > 3$ ensures an asymptotic convergence rate of the values at $o(1/t^2)$, in addition to the weak convergence of the trajectories towards the optimal solutions. In the case of strongly convex functions f, the convergence rate asymptotically achieves the order of $1/t^{\frac{2\alpha}{3}}$, improving with increasing α. To elucidate the influence of the parameter α on the convergence properties of $(\mathrm{AVD})_\alpha$, our analysis reveals that an appropriate time scaling of $(\mathrm{AVD})_\alpha$ yields trajectories that closely resemble those produced by the continuous steepest descent method associated with f (the gradient flow), particularly when α is substantially large. This approach highlights a singular perturbation phenomenon as the analysis transitions from a second-order evolution equation to a first-order one. Such a transition is instrumental in comprehending the shift in the convergence rate from $1/t$ to $1/t^2$, distinguishing the steepest descent method from NAG.