

Some parabolic problems “forward-backward”

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The purpose of this talk is to review recent results on equations exhibiting a “forward-backward” structure. Our epitome in the linear case will be the so-called stationary Kolmogorov equation $yu_x - u_{yy} = f$ in the domain $(x_0, x_1) \times (-1, 1)$, which is forward parabolic in the upper half of the domain, and backward parabolic in the lower half. We exhibit explicit singular solutions for this equation (with infinitely smooth data). Hence, the solutions to the equation are regular if and only if the source term and lateral boundary data satisfy a finite number of orthogonality conditions. This is similar to well-known phenomena in elliptic problems in nonsmooth domains.

We then step on this linear analysis to address nonlinear equations such as $uu_x - u_{yy} = f$ in the vicinity of the linear shear flow, subject to perturbations of the source term and lateral boundary conditions. We construct smooth solutions thanks to an iterative scheme, taking care to satisfy the orthogonality conditions at every step of the process. We will also review related results by Sameer Iyer and Nader Masmoudi in the framework of the Prandtl system.

This is a joint work with Frédéric Marbach and Jean Rax.