Applied Analysis and Modeling: a conference in honor of Olivier Goubet

Monday Nov 4th

9:00–10:00 **Welcome, registration and coffee**
Salle Kampé de Fériet

10:00–10:30 **Conference Opening**
Olivier Goubet: his life, his work

10:30–11:20 **Anne de Bouard (École polytechnique)**
From additive stochastic Zakharov system to multiplicative stochastic nonlinear Schrödinger equation

We study the convergence of a Zakharov system, which couples a Schrödinger equation for the electric field envelope with a wave equation for the variation in ion density. This system is a simplified model for Langmuir turbulence and the introduction of a noise describes the influence of external perturbations of the ion density. In the subsonic limit, the system formally converges to a stochastic nonlinear Schrödinger equation, but this limit becomes singular in the presence of noise. It will be shown that the system can nevertheless be rewritten in the approximation-diffusion regime and that this limit can thus be studied rigorously using predictor-corrector methods. This work is in collaboration with Grégoire Barrué, Arnaud Debussche and Rita Nader (ENS Rennes).

11:20–12:10 **Arnaud Debussche (ENS Rennes)**
From correlated to white transport noise in fluid models

Stochastic fluid model with transport noise are popular, the transport noise models unresolved small scales. The main assumption in these models is a very strong separation of scales allowing this representation of small scales by white - ie fully decorrelated - noise. It is therefore natural to investigate whether these models are limits of models with correlated noises. Also, an advantage of correlated noises is that they allow classical calculus. In particular, it allows to revisit the derivation of stochastic models from variational principle and allows to derive equation for the evolution of the noise components. The advantage of having such equations is that in most works, the noise components are considered as given and stationary with respect to time which is non realistic. Coupling stochastic fluid models with these gives a more realistic system.

12:10–12:40 **Brahim Alouini (Université de Monastir)**
Asymptotic dynamics of certain solutions for an extended nonlinear Gross-Pitaevskii equation with critical nonlinear damping in $\mathbb{R}^3$

Recent experiments have revealed the formation of stable droplets in a dipolar Bose-Einstein condensate (**BEC**). This surprising result has been explained, experimentally, by the stabilization given by the three body loss process appearing in the form of a critical damping term in an extended Gross-Pitaevskii equation (**eGPE**) modelling the formation of these dipolar quantum droplets. The purpose of the current paper is to study the dynamics of solutions to this equation (**eGPE**). In the first part, we present a new mathematical study that validates this prediction by proving that the nonlinear damping prevents the collapse and ensures the existence of global-in-time solutions. The asymptotic dynamics of these solutions will be discussed as well, especially when the system is free (without potential). We show that all global solutions behave as free waves asymptotically in time.

12:45–14:30 **Lunch at RU "La table d'Henry", (S)pace Pariselle**

14:30–15:20 **Anne-Laure Dalibard (Sorbonne Université)**
Some parabolic problems "forward-backward"

The purpose of this talk is to review recent results on equations exhibiting a "forward-backward" structure. Our epitome in the linear case will be the so-called stationary Kolmogorov equation $yu_x - u_{yy} = f$ in the domain $(x_0, x_1) \times (-1, 1)$, which is forward parabolic in the upper half of the domain, and backward parabolic in the lower half. We exhibit explicit singular solutions for this equation (with infinitely smooth data). Hence, the solutions to the equation are regular if and only if the source term and lateral boundary data satisfy a finite number of orthogonality conditions. This is similar to well-known phenomena in elliptic problems in nonsmooth domains.

We then step on this linear analysis to address nonlinear equations such as $uu_x - u_{yy} = f$ in the vicinity of the linear shear flow, subject to perturbations of the source term and lateral boundary conditions. We construct smooth solutions thanks to an iterative scheme, taking care to satisfy the orthogonality conditions at every step of the process. We will also review related results by Sameer Iyer and Nader Masmoudi in the framework of the Prandtl system.

This is a joint work with Frédéric Marbach and Jean Rax.

15:20–16:10 **Carole Rosier (Université du Littoral Côte d'Opale)**
Modeling of underground flows. Application to reactive transport

In this talk, we present models that are alternatives to the 3D-Richards equation for describing water flow in shallow aquifers. They exploit the low thickness of the aquifer and consist in capturing very different physical phenomena: the fast and essentially vertical leakage coming from the surface through an unsaturated soil and the slow and essentially horizontal displacement in the saturated part of the aquifer. These models are easier to manipulate numerically since the original 3D problem is replaced by the coupling of a 2D problem with several independent 1D-problems (which can be solved in parallel). This implies significant time savings in the numerical processing. An asymptotic analysis is used to prove that each model of the new class and the 3D-Richards equation are associated with the same effective problem for any time scale [C. Bourel, C. Choquet, C. Rosier and M. Tsegmid (2020)]. The mathematical study of this class of models is particularly delicate because of the nonlinearities, the free boundary between each area and the difficulty resulting from the coupling between the two zones which is expressed in terms of flux at the interface. We show how taking into account the low compressibility of the fluid eliminates the nonlinearity in the time derivative of the Richards equation. Then, we use the general framework of parabolic equations in non-cylindrical domains introduced by Lions in [J. L. Lions (1957)] to give a global in time existence result for this problem [S. Al Nazer, C. Rosier, M. Tsegmid (2022)]. This model is then coupled to the transport equations describing kinetic chemistry as well as to the algebraic system describing equilibrium chemistry [R. Awada, J. Carrayrou, C. Rosier (to appear)].

16:10–14:40 **Coffee break**

16:40–17:30 **Min Chen (Purdue University)**

Mathematical analysis of Bump to Bucket problem

In numerical simulations of surface water waves, when there is a deformation on the bottom, it is a common practice to transform form the boundary deformation data to the free surface. In this talk, we investigate this procedure, by comparing the waves generated by the moving bottom (Bump) and by the initial surface variation (Bucket), using linear and nonlinear Boussinesq-type models. This is a joint work with Olivier Goubet and Shenghao Li.

17:30–19:30 **Poster session & appetizers**
Salle Delattre, M3

Tuesday Nov 5th

9:00–9:50 **Samir Adly (Université de Limoges)**
The limit dynamic of the Su-Boyd-Candès accelerated gradient system when the asymptotic vanishing damping coefficient α becomes large: a singular perturbation approach

In a real Hilbert space setting, we concentrate on the continuous dynamical system introduced by Su, Boyd, and Candès as a low-resolution ordinary differential equation (ODE) version of Nesterov's accelerated gradient method (NAG). This inertial system, represented as $(\text{AVD})_\alpha$, is driven by the gradient of the function f that is subject to minimization and features a damping mechanism with an asymptotic vanishing coefficient of the form α/t, where $\alpha \geq 3$. Selecting a sufficiently large α is pivotal for ensuring the desirable asymptotic convergence characteristics of the trajectories. Specifically, for a general convex function f, choosing $\alpha > 3$ ensures an asymptotic convergence rate of the values at $o(1/t^2)$, in addition to the weak convergence of the trajectories towards the optimal solutions. In the case of strongly convex functions f, the convergence rate asymptotically achieves the order of $1/t^{\frac{2\alpha}{3}}$, improving with increasing α. To elucidate the influence of the parameter α on the convergence properties of $(\text{AVD})_\alpha$, our analysis reveals that an appropriate time scaling of $(\text{AVD})_\alpha$ yields trajectories that closely resemble those produced by the continuous steepest descent method associated with f (the gradient flow), particularly when α is substantially large. This approach highlights a singular perturbation phenomenon as the analysis transitions from a second-order evolution equation to a first-order one. Such a transition is instrumental in comprehending the shift in the convergence rate from $1/t$ to $1/t^2$, distinguishing the steepest descent method from NAG.

9:50–10:40 **Marion Darbas (Université Sorbonne Paris Nord)**
Multimodal analysis and inverse problems for brain imaging

In this presentation, I will present the coupling of two brain imaging modalities: electroencephalography (EEG) and diffuse optical tomography (DOT). In the first part, I will introduce a model for generating synthetic coupled data based on neurovascular coupling. Through dimensional analysis, we will validate an EEG model that incorporates time-dependent sources of electrical brain activity, alongside a time-harmonic DOT model with time-varying optical parameters of the brain. We will simulate synthetic EEG and DOT data derived from the same brain activity on a realistic 3D head model. In the second part, I will investigate two inverse problems: source localization in EEG and parameter identification in DOT. I will provide theoretical and numerical results.

This is a joint work with Stephanie Lohrengel and Benjamin Sulis, LMR URCA, Reims.

10:40–11:10 **Coffee break**

11:10–12:00 **Serge Dumont (Université de Perpignan)**
Stochastics Models of Interfaces with Damage: A Numerical Study

Interphases between solids play a crucial role in the behaviour of structures, whether natural or industrial. It is very important to predict their ageing under various stresses (mechanical, thermal, environmental, etc.). The difficulty in modelling is essentially due to the very small dimensions of the interfaces compared with those of the structures. A classical technique for overcoming this difficulty is to use asymptotic techniques, with the interphase modelled by an interface [C. Licht (2007), S. Dumont, F. Lebon and R. Rizzoni (2014)]. These techniques can be used to introduce a damage parameter, which can be interpreted as a microcrack density [E. Bonetti, G. Bonfanti, F. Lebon and R. Rizzoni (2017)].

Since the damage is not totally predictable, stochastics models are introduced into the damage evolution equation. However, these problems are difficult to analyse mathematically [C. Bauzet, E. Bonetti, G. Bonfanti, F. Lebon and G. Vallet (2017)] and solve numerically. Numerical simulations on academic examples are presented to illustrated the properties of the proposed models.

12:00–12:30 **Simone Nati Poltri (Centre Inria de l'Université de Bordeaux)**
Asymptotic Analysis of Electrocardiology Modeling after Pulsed Field Ablation

We focus on the mathematical study of pulsed electric field ablation (PFA), an innovative cardiac ablation technique for the treatment of cardiac arrhythmias. In particular, we would like to compare it with radio-frequency ablation (RFA), a thermal ablation that is currently the most commonly used technique. This work aims to modify the classical bidomain mode, which describes the propagation of intracellular and extracellular potentials in the heart, to introduce a region ablated by RFA or PFA. Both types of ablation involve isolation of a pathological area, but we describe them differently by using appropriate transmission conditions at the interface between the ablated and the not-ablated area. In the case of RFA, we assume that both intracellular and extracellular potentials are affected, resulting in Kedem-Katchalsky-type conditions at the interface. In contrast, in the case of PFA, we study the static bidomain model and we assume that the thickness of the electroporated (EP) region is small compared with the whole domain and proportional to a parameter ε. Moreover, we assume that within the EP region the intracellular conductivity scales with a factor ε^2. We provide a formal asymptotic analysis at any order by considering an asymptotic expansion of the intracellular and extracellular potentials both outside and inside the EP area. This allows us to derive transmission conditions at the interface for PFA at any order, that read as non-homogeneous boundary conditions for the jump of the extra-cellular potential and its normal derivative, and as Neumann conditions for the intracellular potentials. Moreover, we give a proof of the asymptotic expansion by deriving estimates of H^1- and L^2-norms of the errors. The asymptotic expansion was validated by numerical convergence tests.

12:45–14:30 **Lunch at RU "La table d'Henry", (S)pace Pariselle**

14:30–15:20 **Youcef Mammeri (Université Jean Monnet Saint-Étienne)**
Imaging and spatio-temporal modelling of plant-pathogen lesions

Due to the complexity of mechanisms driving epidemic progression, few models exist that accurately reproduce lesion growth in vivo. Mathematical models that incorporate morphological imaging data offer a promising multidisciplinary approach. After reviewing some preliminary models, we will explore how daily imaging data combined with machine learning and reaction-diffusion models can describe lesion dynamics. This is a joint effort with Melen Leclerc, Stéphane Jumel, Frédéric Hamelin, and Nicolas Parisey from INRAe UMR Igepp, 35650 Le Rheu, France.

We will conclude by discussing recent collaborative work with O. Goubet et al. at the forest scale.

15:20–16:10 **Alain Miranville (Université Le Havre Normandie)**
Some generalizations of the Cahn-Hilliard equation

In this talk, we discuss generalizations of the Cahn-Hilliard equation due to E. Fried et al. and to S. Forest. We also study the convergence of the solutions of these models to those of the original Cahn-Hilliard equation.

16:10–16:40 **Coffee break**

16:40–17:30 **Vittorino Pata (Politecnico di Milano)**
The Gurtin–Pipkin heat equation: Old and new results

We consider an abstract version of the integrodifferential equation

$$\partial_t u(t) - \int_0^\infty g(s)\Delta u(t-s)ds = 0,$$

modeling hereditary heat conduction of Gurtin–Pipkin type. Under suitable albeit quite general assumptions on the convolution kernel g, the equation generates a contraction semigroup $S(t) = e^{t\mathbb{A}}$ acting on a certain Hilbert space. Although the decay properties of $S(t)$ are nowadays well understood, several important issues related to the structure of the spectrum of the infinitesimal generator $\mathbb{A}$ have not yet been investigated. In this talk, we provide some answers in that direction, demonstrating in particular the impossibility to have arbitrarily fast decays. For the most relevant physical case of the exponential kernel, we will also prove that the semigroup fulfills the so-called spectrum determined growth condition, telling that the decay type of $S(t)$ is fully dictated by the spectrum of $\mathbb{A}$. In some cases, the optimal decay rate turns out to be actually attained.

17:30–18:00 **Jordan Berthoumieu (CY Cergy Paris Université)**
Orbital stability of a chain of dark solitons for general nonintegrable Schrödinger equations with non-zero condition at infinity

We consider travelling wave solutions for a general nonlinear Schrödinger equation

$$i\partial_t\Psi + \partial_x^2\Psi + \Psi f(|\Psi|^2) = 0 \quad \text{on } \mathbb{R}\times\mathbb{R}, \qquad (NLS)$$

when the condition at infinity is $|\Psi(t,x)| \to 1$, as $|x| \to \infty$. More precisely, we prove the orbital stability of a chain of travelling waves whose speeds are well ordered, taken close to the speed of sound c_s and such that the solitons are initially localized far away from each other. The proof relies on the arguments developed by F. Béthuel, P. Gravejat and D. Smets and first introduced by Y. Martel, F. Merle and T.-P. Tsai.

19:30–22:00 **Social diner**
La brasserie de La Paix, 25 Pl. Rihour, 59000 Lille.

Wednesday Nov 6th

9:00–9:50 **Amandine Véber (Université Paris Cité)**
Stochastic models for the growth of filamentous fungi

Filamentous fungi form a large family of species playing an important role in the functioning of many ecosystems. They develop in space thanks to the growth and multiplication of filaments (also called hyphae) which allow the absorption and sharing of nutrients and other molecules. In this talk, we shall first present a stochastic growth-fragmentation model for the development of a hyphal network, whose main aim is to identify a small number of key parameters describing the growth of the fungus in homogeneous conditions (in particular, in lab conditions) and to understand and quantify the impact of different forms of stress on this growth. One major limitation of this model is that it does not take the spatial structure of the network into account. In the second part of the talk, we shall discuss how to integrate this spatial dimension into a more complex measure-valued process, whose dynamics depends on its past states.

The results presented are joint work with Vincent Bansaye (Ecole Polytechnique), Lena Kuwata (Univ. Paris Cité) and Milica Tomasevic (CNRS, Ecole Polytechnique) on the maths side, and Cécilia Bobée, Florence Chapeland-Leclerc, Thibault Chassereau, Pascal David, Eric Herbert, Christophe Lalanne, Clara Ledoux, Gwenaël Ruprich-Robert, all at LIED (Univ. Paris Cité) on the biology and physics side.

9:50–10:40 **Guillaume Decocq (Université de Picardie Jules Verne)**
Natura insilico Goubet & Decocq 2001: when vegetation ecologists meet mathematicians

In 1992, the American writer John Gray was claiming that “Men are from Mars, Women are from Venus”. But surprisingly, he said nothing about ecologists and mathematicians. Yet essentialism may also at play between these two “species” rub each other. This is the aim of my talk: shedding light on differences and convergences between two disciplines that have a lot to do together, but often ignore it. As a case study, I will relate the story of my collaboration with Olivier Goubet, which started twenty-four years ago, in 2001. As a vegetation ecologist, I came from the wild, more precisely from woodlands where I was studying an invasive plant species named American black cherry or, as botanists say, Prunus serotina Ehrh. (botanists love insulting plants in Latin). As a mathematician, Olivier was living in silico, speaking an unknown language. The first challenge was thus to communicate, to try to found a common language. We succeeded. Then came the vocabulary, with new words such as canopy or ergodicity, prior to the concepts themselves. After one year, the experiment could start. For this, we recruited a Guinea pig called Emmanuelle, which rapidly became a hybrid species capable of transforming in natura data into in silico data. She built one population model, then one landscape model and finally an all-in-one model called PRUNUS, which reproduced the spatial spread of the invasive black cherry over the Compiègne forest in north France. The experiment was so successful that the vegetation ecologist and the mathematician never stopped collaborating. Still, colleagues from both civilizations joined them and now the hybrid population has well established in Amiens and is still growing. Long-distance dispersal sometimes happened, with diaspores that have recruited in Lille and Lyon… like a plant invasion dynamics!

10:40–11:10 **Coffee break**.

11:10–12:00 **Pascal Poullet (Université des Antilles)**
A numerical study of Parity-Time symmetric systems of two coupled nonlinear Schrödinger equations

The study of coupled systems of nonlinear Schrödinger equations is fascinating because of the diversity of the applications that can be considered. These systems can describe phenomena in nonlinear optics, condensed matter physics and plasma physics. But exploring the existence, uniqueness and stability of solutions are also areas that contribute to our understanding of physical phenomena. In this presentation, we'll revisit the properties that a coupled system verifies, in particular the Parity-Time symmetry property, which is one of the concepts that have led to a plethora of developments for physicists. We'll see how the numerical study of this coupled system illustrates theoretical results and allows us to make further additions, particularly in the critical case.

12:00–12:30 **Killian Verdure (Université de Reims Champagne-Ardenne)**
Real-valued nonlinear recursive dynamical system with a probabilistic perturbation

We are interested in a dynamical system satisfying a non-linear, partly deterministic recurrence relation of order k, with a stochastic noise term:

$$X_{t+k} = \Phi_0 \left(X_t, ..., X_{t+k-1}\right) + \varepsilon_t$$

We give sufficient conditions on Φ_0 ensuring that our system admits an ACIM (absolutely continuous invariant measure). To that end, we associate a transform on $\mathbb{R}^k$ with our system, and show that the associated transfer operator satisfies a Lasota-Yorke inequality.

12:45–14:30 **Lunch at RU "La table d'Henry", (S)pace Pariselle**.

14:30–15:20 **Vicentiu Radulescu (AGH University of Krakow)**
Normalized solutions of (p, q)-equations with mass supercritical growth

I report on some results included in a recent paper with Li Cai (J. Differential Equations, 2024). I discuss the qualitative properties of solutions with prescribed norm for a class of elliptic equations driven by the (p, q)-Laplace operator and with lack of compactness. The reaction term is assumed to be continuous and satisfying weak mass supercritical mass conditions. I shall discuss the existence of ground states, as well as the basic behavior of the ground state energy. The approach is based on the direct minimization of the energy functional on the linear combination of Nehari and Pohozaev constraints intersected with a closed ball of suitable radius.

15:20–16:10 **Louis Dupaigne (Université Claude Bernard Lyon 1)**
Constante optimale dans l'inégalité de Poincaré

La constante optimale dans l'inégalité de Poincaré est souvent difficile à calculer et on connaît encore peu d'exemples. Je présenterai de nouveaux exemples de variétés à poids, obtenus par transformation conforme d'un cône euclidien, pour lesquels la constante est calculable. Des phénomènes nouveaux apparaissent lorsqu'on fait varier la mesure de ce cône, son ouverture, que l'on suppose que sa base n'est pas circulaire ou qu'on le plonge dans une classe plus large de variétés, de type ""produit tordu". Travail avec I. Gentil, N. Simonov et S. Zugmeyer.

16:10–17:00 **Claudio Muñoz (Universidad de Chile)**
Asymptotic Stability of the Fourth Order ϕ^4 kink for general perturbations in the energy space

The fourth order ϕ^4 model extends the classical ϕ^4 model of quantum field theory to the fourth order case, but sharing the same kink solution. It is also the dispersive counterpart of the well-known parabolic Cahn-Hilliard equation. Mathematically speaking, the kink is characterized by a fourth order nonnegative linear operator with a simple kernel at the origin but no spectral gap. In this talk, I will consider the kink of this theory, and will sketch the orbital and asymptotic stability for any perturbation in the energy space.

Poster Session & Appetizers

Monday Nov 4th at 17:30, Salle Delattre

- **Diana Al Zareef** (Université de Technologie de Compiègne)
  A convergence result for a non-local eikonal equation modeling dislocation dynamics

  Dislocation dynamics are crucial for understanding the mechanical properties of materials, particularly in one-dimensional (1D) models. Dislocations are line defects in crystals, and their motion driven by external stress explains metallic plastic deformation. The Peach-Koehler force, derived from linear elasticity equations, governs their motion. In this study, we focus on a simplified 1D version of a model originally proposed in 2D by Rodney, Le Bouar, and Finel. Specifically, we consider dislocations as parallel lines moving within an elastic crystal plane, described by a non-local eikonal equation:

  $$\begin{cases} \partial_t v(x,t) = (\mathcal{K} \star v(\cdot,t))(x)\partial_x v(x,t) & \text{in } \mathbb{R} \times (0,T), \\ v(x,0) = v_0(x) & \text{in } \mathbb{R}, \end{cases}$$

  where v represents the scalar unknown function, and $\mathcal{K}$ is a kernel function dependent on the crystal's physical properties. This non-local and non-monotonic equation presents significant challenges for proving uniqueness via standard methods.

  The primary objective of this research is to establish the convergence of a periodic numerical scheme for the discretized non-local eikonal equation and to extend these results to the non-periodic case. Preliminary results include the existence, uniqueness, and monotonicity of the solution, as well as total variation decay and entropy estimates.

- **Bouchra Bensiali** (École Centrale Casablanca) *Boundary layer analysis of a d-dimensional penalization method for Neumann or Robin boundary conditions*

  In this talk, we present a d-dimensional extension of a fictitious domain penalization technique that we previously proposed for Neumann or Robin boundary conditions. We apply Droniou's approach for non-coercive linear elliptic problems to obtain the existence and uniqueness of the solution of the penalized problem, and we derive a boundary layer approach to establish the convergence of the penalization method. The developed boundary layer approach is adapted from the one used for Dirichlet boundary conditions, but in contrast to the latter where coercivity enables a straightforward estimate of the remainders, we reduce the convergence of the penalization method to the existence of suitable supersolutions of a dual problem. These supersolutions are then constructed as approximate solutions of the dual problem using an additional formal boundary layer approach. The proposed approach results in an advection-dominated problem, requiring the use of appropriate numerical methods suitable for singular perturbation problems. Numerical experiments, using upwind finite differences, validate both the convergence rate and the boundary layer thickness, illuminating the theoretical results. Finally, we investigate the applicability of the suggested method to problems raised on moving domains such as those associated with the simulation of population dynamics under climate change. This talk comprises joint work with Jacques Liandrat, Centrale Méditerranée, I2M.

- **Ahmad Safa** (Université de Picardie Jules Verne) *Asymptotic expansion of the solutions to a regularized Boussinesq system*

We consider the propagation of surface water waves described by the Boussinesq system. We introduce a regularized Boussinesq system obtained by adding a non-local pseudo-differential operator define by $\widehat{g_\lambda[\zeta]} = |k|^\lambda \hat{\zeta}_k$ with $\lambda \in]0,2]$. Now, we display a twofold approach: first, we study theoretically the existence of an asymptotic expansion for the solution to the Cauchy problem associated to this regularized Boussinesq system with respect to the regularizing parameter ϵ. Then, we compute numerically the function coefficients of the expansion (in ϵ) and verify numerically the validity of this expansion up to order 2. We also check the numerical L^2 stability of the numerical algorithm.

- **Madeline Chauvier** (Université Polytechnique Hauts-de-France) *An existence result and simulations of a space charges problem applied to HVDC transmission lines*

  The large-scale development of renewable energies induces new challenges for the transmission of electricity over high-voltage lines. In this context, it makes sense to use direct current instead of alternative current, as is the case nowadays. However, not only does direct current transmission increase air ionization, but the constant polarisation also generates space charges. Our aim is to study the system of non-linear partial differential equations modelling this phenomenon. In this work, we will present an existence result obtained by adding a viscosity term to the so-called continuity equation and passing to the limit in the viscosity coefficient. We will also propose a numerical method, based on a least-squares minimization algorithm. Numerical results will be validated by comparison with analytical solutions on a simple geometry. Results on more realistic geometries will also be presented.

- **Robin Colombier** (Université Polytechnique Hauts-de-France) *A finite volume scheme for the quantum Navier-Stokes system*

  In order to better understand how semiconductors work we aim to simulate the flow of electrons, which when there are many, can be modelized using fluid mechanics, by the system of quantum Navier-Stokes equations given for $t \in [0,T]$ and $\mathbf{x} \in \Omega$:

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p(\rho) - 2\epsilon^2 \rho \nabla \dfrac{\Delta \sqrt{\rho}}{\sqrt{\rho}} + r\rho \mathbf{u} = 2\nu \nabla \cdot (\rho D(\mathbf{u})), \\ \rho_{|t=0} = \rho_0, \quad \rho \mathbf{u}_{|t=0} + \rho_0 u_0, \end{cases} \tag{1}$$

  with $\quad p(\rho) = \rho^\gamma, \quad \gamma > 1, \quad \epsilon > 0, \nu > 0$ and $r > 0$. Here the unknowns are the particle density ρ and the particle velocity $\mathbf{u}$ and $\Omega = \mathbb{T}^d$ is the torus in dimension d ($1 \leq d \leq 2$).

  By noting $\mathbf{v} = \epsilon \nabla log(\rho)$ and $\mathbf{w} = \mathbf{u} + \nu \mathbf{v}/\epsilon$, we can rewrite (1) as an augmented system of order 2 in the new variables $\rho, \mathbf{w}, \mathbf{v}$. The new system allow for a maximum of order 2 term and is more suited for the numerical scheme, and BD-entropy can be defined. The objective of our work is to propose a finite volume scheme allowing to define a discrete BD-entropy. Some 1D benchmarks as the grey soliton and the dispersive Riemann problem, as well as a 2D benchmark on cartesian grids illustrate the performance and the accuracy of the numerical scheme.

- **Amélie Dupouy** (Centre Inria de l'Université de Lille) *Diffusion on a moving domain*

The main goal is to prove the existence of a solution to the problem, and its convergence to a travelling wave profile. We start by proving the existence of this travelling wave solution. Then, we compute a priori bounds, using the decrease of free energies. We write a numerical scheme to approximate the system. It allows us to represent the convergence towards the travelling wave. Using the monotonicity of the numerical scheme we then prove some a priori estimates on the numerical solution. Finally, these estimates enable us to prove the existence of a solution to our scheme.

- **Pierre Gervais** (Université de Lille) *On a self consistent Vlasov-Fokker-Planck equation with general interactions*

  The self consistent Vlasov-Fokker-Planck equation models a large system of particles which is subject to external confinement, long-range interactions between particles, and thermalization mechanisms. Motivated by applications in particle accelerator physics, we consider interaction potentials which may be singular and non-symmetric with respect to the relative position of the particles. Under suitable assumptions, including in particular Poisson (Coulomb) interactions, We prove existence, uniqueness and stability of steady states. The rate of convergence towards the steady state is quantitative and improves former results. This presentation is based on a joint work with Maxime Herda (Centre Inria de l'Université de Lille).

- **Jade Le Quentrec** (Université de Reims Champagne-Ardenne) *Étude théorique et numérique d'équations de Schrödinger avec terme de défaut*

  Dans le poster proposé, nous présentons l'étude de l'équation de Schrödinger faisant intervenir un terme de défaut, qui matérialise la présence d'une impureté le long d'une hypersurface Σ de $\mathbb{R}^n$ dans le milieu non linéaire. L'équation considérée s'écrit

  $$i\frac{\partial u}{\partial t} = \Delta u + |u|^{2\sigma}u + Zu\delta_\Sigma$$

  où δ_Σ désigne la mesure de Dirac sur Σ.

  Nous expliquerons comment la présence de ce terme de défaut influera l'étude du caractère localement ou globalement bien posé de l'équation, et l'existence de solutions qui explosent en temps fini. Nous montrerons ensuite sur les simulations sur machine du comportement de solutions particulières. Une attention particulière sera portée à la discrétisation du défaut en différences finies.

- **Erwan Le Quiniou** (Université de Lille) *Variational characterization of the quasilinear Gross–Pitaevskii dark solitons*

  We study a quasilinear Schrödinger equation with nonzero conditions at infinity in one dimension. We obtained a continuous branch of traveling-wave solutions, given by dark solitons indexed by their speed. Neglecting the quasilinear term, one recovers the Gross–Pitaevskii equation, for which the branch of dark solitons minimizes the energy at fixed momentum.

  In two submitted papers, we investigated how the quasilinear term affects the variational properties of the dark solitons. For weak quasilinear interactions, these dark solitons can be obtained as minimizers of the energy at fixed momentum via a concentration-compactness argument. While, for stronger quasilinear interactions, a cusp appears in the energy-momentum diagram of the solitons and we identified three behaviors depending of the speed of the wave. The fast waves remain minimizers, the waves of intermediate speeds, before the cusp, are local minimizers and the slow waves are saddle points under the constraint of fixed momentum. We obtained these local variational properties by performing the spectral analysis on the Hessian of a modified energy functional and proceeding as in the seminal work of Grillakis, Shatah and Strauss.

The aim of this poster is to present our results for the minimization problem and the local variational properties of the dark solitons of the quasilinear equation and to sketch some important steps of the analysis. This presentation is based on a joint work with André de Laire (Université de Lille).

- **Sebastián Tapia** (Centre Inria de l'Université de Lille) *Fractal Dynamics in Quintic NLS Soliton Collisions: A Review*

  In this work we explored the multi bouncing and fractal phenomena observed in the velocity-phase relationship during soliton collisions in the quintic nonlinear Schrödinger equation. We studied and used a classical finite difference scheme adapted for simulating these collisions, followed by the derivation and simulation of the governing system of ODEs for soliton trajectories and velocities. Additionally, we discussed potential invariants that may help clarify the origins of the fractal patterns, both within the original PDE and the reduced ODE system.

- **Céline Wang** (Université de Lille) *Matrix Population Models for Prunus Serotina*

  The American black cherry (*P. serotina* Ehrh.) is a tree species native to North America that was introduced into European forests in the 17th century. Over the last three decades or so, it has spread to the forests of Western and Central Europe, competing with local species. In order to model its dynamic, we constructed and studied (simplified) demographic matrix models to explore the local population dynamics of this species. The species population on a plot of land in year n is described by a state vector $s_n = M_n \cdots M_1 s_0$, where M_i is a Leslie (or Lefkovitch) matrix random variable attached to the year i. Next, the aim is to study the asymptotic behavior of the sequence $s_{n n \in \mathbb{N}^*}$, its saturation time behavior. We focused on the limit $\frac{1}{n} \log s_n$. In the first constructed model, the matrices M_i are i.i.d and take two possible values. In this case, the limit only depends on the probability p of the M_i distribution. To study the existence and the uniqueness of this limit, ergodic theory results from Furstenberg and Kingman are used. The limit can also be expressed as an integral with respect to an invariant measure. This expression allows us to prove the continuity of the limit with respect to the probability.