Excercises on nonrelativistic conformal field theory

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Bound state in 2D

Consider the quantum mechnics problem of a particle in an attractive δ-functional potential in 2D:

$$V(\mathbf{x}) = -c\delta(\mathbf{x}) \tag{1}$$

Regularize the potential in any way convenient for you and show that the potential has a bound state with energy that scales as $\sim \exp(-\#/c)$ in the limit $c \to 0$.

A descendant opertor that is primary?

Let $O(\mathbf{x})$ be a primary operator in a nonrelativistic CFT: $[K_i, O(\mathbf{x})] = [C, O(\mathbf{x})] = 0$. When there exists a linear combination of $[P_i, [P_i, O(\mathbf{x})]]$ and $[H, O(\mathbf{x})]$ that is also primary? (consult 07065.3746 for the Schrödinger algrebra).

Anyons

One example of nonrelativistic CFT in two spatial dimensions is the theory of nonrelativistic (abelian) anyon. These particles are characterized by the statistical angle θ that varies between 0 (bosons) and π (fermions). For example, the wave function of two anyons has to have the form

$$\psi(z_1, z_2) = (z_1 - z_2)^{\theta/\pi}\tilde{\psi}(z_1, z_2) \tag{2}$$

where z_i is the complex coorindates of the particles ($z_i = x_i + iy_i$) and $\tilde{\psi}$ is a single-valued function of the arguments.

Find the dimension of the lowest charge-two operator in this theory.