(1) Compute the number of zero modes for a fermion in the n-dimensional irrep. of $SU(2)$ coupled to an instanton background of charge K, on some spin 4-manifold M.

(2) Consider $\mathcal{N}=1$ SYM in $d=8$ with algebra $so(2n+1)$. [This is Yang-Mills theory coupled to a Weyl fermion in the adjoint representation.] Choose a subalgebra

$$su(2) \oplus su(2)' \oplus so(2n-3)$$

and put the theory on $S^4 \times M^4$, such that there is a unit instanton charge on the S^4. Show that the effective theory on M^4 has a global anomaly.

(3) (a) Assuming that the standard model gauge group is $SU(3) \times SU(2) \times U(1)_Y$, and the $SU(3) \times SU(2)$ representation is fixed, show that (up to $U(1)_Y$ redefinitions) there are exactly <u>two</u> $U(1)_Y$ assignments compatible with anomaly cancellation. (Assume that $Q_Y(\nu_R) = 0$.)

(b) Show that if the standard model gauge group is $\frac{SU(3) \times SU(2) \times U(1)_Y}{\mathbb{Z}_6}$ only the assignment of hypercharge we see in nature is anomaly-free.

(c) If you imposed gauge-gravity mixed anomaly cancellation above, try to obtain the same results without that assumption, assuming only that $U(1)_Y$ is really $U(1)_Y$ and not $\mathbb{R}_Y$, so hypercharges are rational numbers.

[Difficult but fun! See 1907.00514 by Lohitsiri and Tong for the solution]

(4) Show that IIB sugra is the only solution to the local gravitational anomaly equations worked out in the lecture, assuming $n_{1/2} = 1$.