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Stability and Shadowing of Non-invertible p-adic Dynamics

A continuous dynamical system is a couple (X, f) where (X, d) is a metric space and $f : X \rightarrow X$ is a continuous map (called dynamic). The notions of stability and shadowing, introduced in the second third of the 20th century in the works of Andronov, Pontrjagin, Bowen, and Sinai, play a fundamental role in several branches of dynamical systems. For a system (X, f) to be stable roughly means that its analytical properties (e.g. the behavior of its orbits) are not affected upon introducing sufficiently small noises in the system. A map f is said to be shadowing if every pseudo-orbit $\{x_n\}_{n \in \mathbb{N}}$ (i.e. $d(f(x_n), x_{n+1})$ is small for every $n \in \mathbb{N}$) is close, with respect to the supreme norm, to a real orbit $\{f^n(x)\}_{n \in \mathbb{N}}$.

The stability theory of compact topological manifolds with positive and finite dimensions is well-developed. The classical results describe how the two concepts of stability and shadowing are related, especially when the dynamic under consideration is a homeomorphism. However, the study of these dynamical concepts in zero-dimensional compact spaces has only recently started developing. A notable family of zero-dimensional compact spaces is the Cantor spaces. Examples of Cantor spaces include fractal sets such as the famous Cantor set and the metric space of the p-adic integers \mathbb{Z}_p .

This work concerns the dynamics of p-adic integers and, in particular, families of non-invertible maps that admit, however, right or left inverses. The results provide sufficient conditions for stability and shadowing properties to arise. As a consequence, the context developed unifies many of the known examples while, at the same time, providing new ones.