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A certain type of approximation by polynomials with algebraic coefficients

Let $N \in \mathbb{N}$ be an integer and $\mathcal{A} = \{q_1, ..., q_N\}$ be a set of algebraic numbers. Given $k \in \mathbb{N}$ call $\mathcal{P}_{\mathcal{A},k}$ the set of polynomials of degree smaller than k and coefficient in $\mathcal{A}$ by $\mathcal{P}_{\mathcal{A},k}$ and $\mathcal{P}_{\mathcal{A}}$ the collection of every polynomials with coefficient in $\mathcal{A}$, that is \begin{align}
\mathcal{P}{\mathcal{A},k}=\left\{P(X)=\sum{i=0}^k a_i X^i, a_i \in\mathcal{A}\right\}\text{ and }\mathcal{P}{\mathcal{A}}=\bigcup{k\geq 0}\mathcal{P}_{\mathcal{A},k}.
\end{align}
Given $t \in (0, 1)$ a natural question is to investigate the Hausdorff dimension of real numbers approximable at a given rate by elements $P(t)$ where $P \in \mathcal{P}_{\mathcal{A}}$, i.e., determining for every mapping $\phi : \mathbb{N} \to \mathbb{R}_+$, $\dim_H W_{\mathcal{A},t}(\phi) =$ $\{x \in \mathbb{R} : |x - P(t)| \leq \phi(\deg(P))$ for infinitely many $P \in \mathcal{P}_{\mathcal{A}}\}.For instance, a consequence of the mass transference principle of B$
$\{0, \frac{2}{3}\}, t_1 = \frac{1}{3}, \delta \geq 1, \phi(n) = t_1^{n\delta}$, one has $\dim_H W_{\mathcal{A}_1,t_1}(\phi) = \frac{\log 2}{\delta \log 3}. In comparison, some quick calculation shows that for \mathrm{A}_2 =$
$\{\frac{1}{7}, \frac{3}{7}, \frac{5}{7}\}, t_2 = \frac{1}{7}, \delta > 1, \phi(n) = t_2^{n\delta}$, one has $\dim_H W_{\mathcal{A}_2,t_2}(\phi) = 0. In this talk, we provide some general results regarding this proble$