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Singular vectors in manifolds, countable intersections, and Dirichlet spectrum

A vector x = (x1, …, xd) in Rd is totally irrational if 1, x1, …, xd are linearly
independent over rationals, and singular if for any ε > 0, for all large enough T, there are solutions p in Zd and q in {1, …, T } to the inequality ‖qx − p‖ < εT −1/d. In previous work we showed that certain smooth manifolds of dimension at least two, and certain fractals, contain totally irrational singular vectors. The argument for proving this is a variation on an old argument employed by Khintchine and Jarn´ık. We now adapt this argument to show that for certain families of maps fi : Rd ⟶ Rni , certain manifolds contain points x such that fi(x) is a singular vector for all i. This countable intersection property is motivated by some problems in approximation of vectors by vectors with coefficients in a number field. I will review Khintchine's original argument and present additional consequences, among them that the Dirichlet spectrum is full, for arbitrary norms, in dimension d > 1, and improved rates of singularity on certain manifolds. Based on a joint work with Dmitry Kleinbock, Nikolaus Moshchevitin and Jacqueline Warren, and another joint work with Alon Agin.