Diophantine Approximation, Fractal Geometry and Related Topic

Université Gustave Eiffel
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Laboratoire d'Analyse et
de Mathématiques Appliquées
(LAMA)

3rd — 7th June 2024

Program and Abstract Book

Schedule

Monday, June 3rd

08:50 - 09:20	**Registration**	
09:20 - 09:30	**Welcome**	
09:30 - 10:30	Victor Beresnevich	Khintchine's Theorem - one hundred years on! (4)
Coffee break		
11:00 - 11:30	Sam Chow	Multiplicative approximation on hypersurfaces (6)
11:40 - 12.10	Evgeniy Zorin	Exploring the Limits: Unbounded Diophantine Approximations and Matrix Transformations (31)
Lunch		
14:10 - 15:10	Nikolay Moshchevitin	Inhomogeneous approximation revisited (17)
15:20 - 15:50	Reynold Fregoli	Improvements to Dirichlet's Theorem in the multiplicative setup and equidistribution of averages along curves (11)
Coffee break		
16:20 - 16:50	Steven Robertson	A Combinatorial Approach to the $p(t)$-adic Littlewood Conjecture (22)
17:00 - 17:30	Niclas Technau	Counting Rational Points Near Manifolds (27)

Tuesday, June 4th

09:30 - 10:30	Barak Weiss	Singular vectors in manifolds, countable intersections, and Dirichlet spectrum (29)
Coffee break		
11:00 - 11:30	Noy Soffer Aranov	Hausdorff Dimension of the Set of Singular and Dirichlet Improvable Vectors in Function Fields. (26)
11:40 - 12.10	Erez Nesharim	The shifts of the Thue-Morse sequence have partial escape of mass (18)
Lunch		
14:10 - 15:10	Damien Roy	Parametric geometry of numbers and simultaneous approximation to geometric progressions (23)
15:20 - 15:50	Stéphane Fischler	Irrationality measures of values of E-functions (10)
Coffee break		
16:20 - 16:50	Taehyeong Kim	Infinitely badly approximable affine forms (15)
17:00 - 17:30	Agamemnon Zafeiropoulos	A variant of Kaufman's measures in two dimensions. (30)
19:00 Conference Diner		

Wednesday, June 5th

09:30 - 10:30	Dmitry Kleinbock	"Simultaneously dense and non-dense" orbits in homogeneous dynamics and Diophantine approximation (16)
Coffee break		
11:00 - 12:00	Frédéric Paulin	On Hausdorff dimension in inhomogeneous Diophantine approximation over global function fields (19)
12:10 - 12:40	René Pfitscher	Counting rational approximations in flag varieties (21)
Lunch		
Free afternoon		

Thursday, June 6th

09:30 - 10:30	Yann Bugeaud	On the b-ary expansion of e (5)
Coffee break		
11:00 - 11:30	Stéphane Seuret	An asymmetric version of the Littlewood conjecture (25)
11:40 - 12.10	Volodymyr Pavlenkov	Inhomogeneous Diophantine Approximation with restraint denominators on M_0-sets and some applications (20)
Lunch		
14:10 - 15:10	Cagri Sert	Projections of self-affine fractals (24)
15:20 - 15:50	Gaurav Aggarwal	Joint Equidistribution of Approximates (1)
Coffee break		
16:20 - 16:50	Shreyasi Datta	Rational points near manifolds and the Khintchine theorem (7)
17:00 - 17:30	Jiyoung Han	Quantitative Khintchine–Groshev theorem on S-arithmetic numbers (13)

Friday, June 7th

09:30 - 10:30	Anish Ghosh	Dynamics on the space of affine lattices and inhomogeneous Diophantine approximation. (12)
Coffee break		
11:00 - 11:30	Catalin Badea	Times-2 and Times-3 Invariant Measures and Exceptional Sets of Uniform Distribution (2)
11:40 - 12.10	Prasuna Bandi	Exact weighted approximation (3)
Lunch		
14:10 - 15:10	Stéphane Jaffard	Some interplays between multifractal analysis and Diophantine approximation (14)
15:20 - 15:50	Arnaud Durand	Capacities and (large) intersections for random sets in metric spaces, with applications in dynamical Diophantine approximation (9)
Coffee break		
16:20 - 16:50	Edouard Daviaud	A certain type of approximation by polynomials with algebraic coefficients (8)
17:00 - 17:30	Ioannis Tsokanos	Stability and Shadowing of Non-invertible p-adic Dynamics (28)

ABSTRACTS

(1)
- *Speaker*: Gaurav Aggarwal
- *Title*: Joint Equidistribution of Approximates
- *Abstract*: The distribution of integer points on varieties has occupied mathematicians for centuries. In the 1950's Linnik used an "'ergodic method" to prove the equidistribution of integer points on large spheres under a congruence condition. As shown by Maaß, this problem is closely related to modular forms. Subsequently, there were spectacular developments both from the analytic as well as ergodic side. I will discuss a more refined problem, namely the joint distribution of lattice points in conjunction with other arithmetic data. An example of such data is the "shape" of an associated lattice, or in number theoretic language, a Heegner point. In a completely different direction, a "Poincaré section" is a classical and useful tool in ergodic theory and dynamical systems. Recently, Shapira and Weiss, constructed a Poincaré section for the geodesic flow on the moduli space of lattices to study joint equidistribution properties. Their work in fact is very general but crucially uses the fact that the acting group has rank one. In joint work with Anish Ghosh, we develop a new method to deal with actions of higher rank groups. I will explain this and, if time permits, some corollaries in Diophantine analysis.

(2)
- *Speaker*: Catalin Badea
- *Title*: Times-2 and Times-3 Invariant Measures and Exceptional Sets of Uniform Distribution
- *Abstract*: We explore Furstenberg's times-2, times-3 conjecture, which poses the question of whether the normalized Lebesgue measure is the sole atom-free probability measure invariant under both times-2 and times-3 maps. Additionally, we analyze the size of exceptional sets associated with (almost) uniform distribution, which are linked to a sequence of positive integers and a measure on the circle. This is joint work with Sophie Grivaux.

(3)
- *Speaker*: Prasuna Bandi
- *Title*: Exact weighted approximation
- *Abstract*: I will discuss the results on the Exact ψ-approximable set in Diophantine approximation. Further, in the weighted setting, we will show that its Hausdorff dimension is equal to that of the ψ-well approximable set under certain conditions on ψ. This is a joint work with Reynold Fregoli.

(4)
- *Speaker*: Victor Beresnevich
- *Title*: Khintchine's Theorem - one hundred years on!
- *Abstract*: Khintchine's Theorem (1924) on rational approximations to real numbers is one of the most beautiful applications of the Borel-Cantelli lemma outside of probability theory. In this talk I will discuss problems and progress concerning inhomogeneous generalisations of this classical result, including the Duffin-Schaeffer Conjecture in the inhomogeneous setting. This is a joint work with Manuel Hauke and Sanju Velani.

(5)
- *Speaker*: Yann Bugeaud
- *Title*: On the b-ary expansion of e
- *Abstract*: Let $b \geq 2$ be an integer. The exponent v_b (resp., v'_b) and the uniform exponent $\widehat{v}_b$ (resp., $\widehat{v}'_b$) measure the quality of approximation to a real number by rational numbers whose denominator is a power of b (resp., is of the form $b^r(b^s - 1)$). Said differently and informally, we look at the lengths of the blocks of digit 0 (or of digit $(b-1)$) and at the lengths of repeated blocks in the base-b expansion of a

real number. We discuss several results on these four exponents and explain how an inequality between v_b' and $\widehat{v}_b'$ implies that the base-b expansion of any real number whose irrationality exponent is sufficiently close to 2 cannot be too 'simple', in the sense that it contains at least cn different blocks of digits of length n, for some $c > 1$ and every integer n sufficiently large. In particular, the b-ary expansion of e contains at least $10n/9$ different blocks of digits of length n, if n is large enough.

(6)
- *Speaker*: Sam Chow
- *Title*: Multiplicative approximation on manifolds
- *Abstract*: Diophantine approximation on manifolds has been a major theme for many decades and has seen remarkable recent progress. I discuss joint work in preparation with Han Yu on the multiplicative analogue.

(7)
- *Speaker*: Shreyasi Datta
- *Title*: Rational points near manifolds and the Khintchine theorem
- *Abstract*: We discuss a problem in Diophantine approximation which is related to counting rational points near a manifold. The proof uses tools from homogeneous dynamics and geometry of numbers. This is a joint work with Victor Beresnevich.

(8)
- *Speaker*: Édouard Daviaud,
- *Title*: A certain type of approximation by polynomials with algebraic coefficients
- *Abstract*: Let $N \in \mathbb{N}$ be an integer and $\mathcal{A} = \{q_1, ..., q_N\}$ be a set of algebraic numbers. Given $k \in \mathbb{N}$ call $\mathcal{P}_{\mathcal{A},k}$ the set of polynomials of degree smaller than k and coefficient in $\mathcal{A}$ by $\mathcal{P}_{\mathcal{A},k}$ and $\mathcal{P}_{\mathcal{A}}$ the collection of every polynomials with coefficient in $\mathcal{A}$, that is
$$\mathcal{P}_{\mathcal{A},k} = \left\{ P(X) = \sum_{i=0}^{k} a_i X^i, a_i \in \mathcal{A} \right\} \text{ and } \mathcal{P}_{\mathcal{A}} = \bigcup_{k \geq 0} \mathcal{P}_{\mathcal{A},k}.$$
Given $t \in (0,1)$ a natural question is to investigate the Hausdorff dimension of real numbers approximable at a given rate by elements $P(t)$ where $P \in \mathcal{P}_{\mathcal{A}}$, i.e., determining for every mapping $\phi : \mathbb{N} \to \mathbb{R}_+$,
$$\dim_H W_{\mathcal{A},t}(\phi) = \{x \in \mathbb{R} : \ |x - P(t)| \leq \phi(\deg(P)) \text{ for infinitely many } P \in \mathcal{P}_{\mathcal{A}}\}.$$
For instance, a consequence of the mass transference principle of Beresnevich and Velani shows that for $\mathcal{A}_1 = \left\{0, \frac{2}{3}\right\}$, $t_1 = \frac{1}{3}$, $\delta \geq 1$, $\phi(n) = t_1^{n\delta}$, one has
$$\dim_H W_{\mathcal{A}_1,t_1}(\phi) = \frac{\log 2}{\delta \log 3}.$$
In comparison, some quick calculation shows that for $\mathcal{A}_2 = \left\{\frac{1}{7}, \frac{3}{7}, \frac{5}{7}\right\}$, $t_2 = \frac{1}{7}$, $\delta > 1$, $\phi(n) = t_2^{n\delta}$, one has
$$\dim_H W_{\mathcal{A}_2,t_2}(\phi) = 0.$$
In this talk, we provide some general results regarding this problem and some partial results aiming at describing the possible behaviors one can encounter. We will more particularly show that the dichotomy between the two cases mentioned occurs when t is the inverse of a Pisot number and the coefficient are in $\mathbb{Z}[t]$. In particular, the results presented will feature new techniques regarding the mass transference principle when the reference measure is not Lebesgue and the sequence of balls we consider overlaps substantially.

(9)
- *Speaker*: Arnaud Durand
- *Title*: Capacities and (large) intersections for random sets in metric spaces, with applications in dynamical Diophantine approximation
- *Abstract*: We consider, in general metric spaces, the classes of random sets that are bound to intersect almost surely any deterministic set with positive capacity in a given gauge function. This property yields a lower bound on the size of the intersection of these random sets with arbitrary deterministic sets. It also implies Falconer's large intersection property. As an illustration, we study limsup sets based on random balls, and present a connection with Dvoretzky's covering problem and dynamical Diophantine approximation.

(10)
- *Speaker*: Stéphane Fischler
- *Title*: Irrationality measures of values of E-functions
- *Abstract*: E-functions are a class of special functions introduced by Siegel in 1929; they include the exponential and Bessel functions. Values of E-functions at algebraic numbers are never Liouville : they are never extremely well approximated by rationals. If an E-function with rational coefficients is evaluated at a rational number, a more precise result holds : if irrational, the value has exponent of irrationality 2, like a randomly chosen number. This is a joint work with Tanguy Rivoal, based mostly on results of Shidlovsky, Chudnovsky, André and Beukers.

(11)
- *Speaker*: Reynold Fregoli
- *Title*: Improvements to Dirichlet's Theorem in the multiplicative setup and equidistribution of averages along curves
- *Abstract*: In this talk, I will discuss uniform approximation by rationals vectors in the multiplicative set-up. Curiously enough, in this context, Dirichlet's Theorem is improvable, and, for $m \times n$ matrices the correct constant is bounded above by 2^{-m+1}. One can also show that almost all matrices are uniformly approximable by the function $x^{-1}(logx)^{-1+\varepsilon}$ for any $\varepsilon > 0$. This emerges from the study of certain measures defined by averaging along particular curves the action of the full diagonal group on the space of $(m+n)$-dimensional unimodular lattices. The talk is based on a joint work with P. Bandi and D. Kleinbock

(12)
- *Speaker*: Anish Ghosh
- *Title*: Dynamics on the space of affine lattices and inhomogeneous Diophantine approximation.
- *Abstract*: We establish a new dynamical result on the space of affine lattices. Using this, we uncover some new Diophantine properties of affine forms. Joint work with Gaurav Aggarwal.

(13)
- *Speaker*: Jiyoung Han
- *Title*: Quantitative Khintchine–Groshev theorem on S-arithmetic numbers
- *Abstract*: In this talk, I would like to introduce two analogs of an S-arithmetic generalization of Diophantine approximation problems. One way to obtain quantitative results for Diophantine approximation over the real field is by utilizing Schmidt's counting theorem on the family of expanding Borel sets. We will explore how this approach can be extended to S-arithmetic Diophantine approximation, taking into consideration certain limitations.

(14)
- *Speaker*: Stéphane Jaffard
- *Title*: Some interplays between multifractal analysis and Diophantine approximation
- *Abstract*: Multifractal analysis deals with the determination of the pointwise regularity of everywhere irregular functions. It is therefore not surprising that many functions which had been proposed as examples or counter-examples of "pathological" functions turned out to be multifractal. What is more remarkable is that they share a common property: their pointwise regularity exponent at a point x depends in a simple way on the Diophantine approximation exponent of x. This is the case of the famous examples that Riemann proposed in his "Habilitationshrift" of functions that are *Riemann integrable*, but not *Cauchy integrable*, and which have jumps at rational numbers. It is also the case for a trigonometric series which Riemann proposed as a tentative example of a continuous nowhere differentiable function. Other important examples were obtained as particular cases of *Davenport series* and, more recently, it was also the case for the *Brjuno function* which was introduced by Yoccoz because it encapsulates a key information concerning analytic small divisor problems in dimension 1, this function now being one example among an important family. We will show that direct and wavelet methods are in competition or can be combined in order to determine the pointwise regularity exponents of these functions and how these methods explain why their regularity exponents are related with the Diophantine approximation properties of the point considered. Finally, we will mention several open problems concerning the regularity of such functions.

(15)
- *Speaker*: Taehyeong Kim
- *Title*: Infinitely badly approximable affine forms
- *Abstract*: In this talk, we will consider infinitely badly approximable affine forms in the sense of inhomogeneous Diophantine approximation. We introduce a new concept of singularity for affine forms and characterize the infinitely badly approximable property by this singular property. We also discuss some applications of this characterization.

(16)
- *Speaker*: Dmitry Kleinbock
- *Title*: "Simultaneously dense and non-dense" orbits in homogeneous dynamics and Diophantine approximation
- *Abstract*: Consider a non-compact homogeneous space X with the action of a diagonal one-parameter subgroup. It is known that the set of points in X with bounded forward orbits has full Hausdorff dimension. Question: what about points with forward orbits both bounded and accumulating on a given $z \in X$? We prove that, barring a certain obvious obstruction, those points also form a set of large Hausdorff dimension. This is motivated by the subject of improving Dirichlet's Theorem in Diophantine approximation. Joint work with Manfred Einsiedler and Anurag Rao.

(17)
- *Speaker*: Nikolay Moshchevitin
- *Title*: Inhomogeneous approximation revisited
- *Abstract*: We will discuss classical and modern results related to linear inhomogeneous Diophantine approximation. We recall the history of the problem from Kronecker's approximation theorem, transference theory developed by Khintchine, Jarník and Cassels to recent results concerning grids of lattices. In particular we explain a new theory of k-divergence lattices which was introduced recently by U. Shapira and discuss the results of our recent paper N. Moshchevitin, A. Rao, U. Shapira "Badly approximable grids and k-divergent lattices", arXiv:2402.00196.

(18) • *Speaker*: Erez Nesharim
• *Title*: The Thue-Morse sequence has partial escape of mass over $\mathbb{F}_2((1/t))$
• *Abstract*: Every Laurent series in the field $\mathbb{F}_q((1/t))$ has a continued fraction expansion whose digits are polynomials. De-Mathan and Teulie proved that the degrees of the partial quotients of the left shifts of every quadratic Laurent series are unbounded. Shapira and Paulin improved this by showing that, in fact, a positive proportion of the degrees are bigger than any bound. We show that their result is best possible in the following sense: For the Laurent series over $_mathbbF_2((1/t))$ whose sequence of coefficients is the Thue-Morse sequence, this proportion is strictly less than 1. This talk is based on a work in progress with Uri Shapira and Noy Soffer-Aranov.

(19) • *Speaker*: Frédéric Paulin
• *Title*: On Hausdorff dimension in inhomogeneous Diophantine approximation over global function fields
• *Abstract*: We study inhomogeneous Diophantine approximation by elements of a global function field (over a finite field) in its completion for a discrete valuation. Given an (m, n) matrix A with coefficients in this completion and a small $r > 0$, we obtain an effective upper bound for the Hausdorff dimension of the set $\mathrm{Bad}_A(r)$ of r-badly approximable m-dimensional vectors, using an effective version of entropy rigidity in homogeneous dynamics for an appropriate diagonal action on the space of integral grids. We further characterize the matrices A for which $\mathrm{Bad}_A(r)$ has full Hausdorff dimension for some $r > 0$ by a Diophantine condition of singularity on average. This is a joint work with Taehyeong Kim and Seonhee Lim.

(20) • *Speaker*: Volodymyr Pavlenkov
• *Title*: Inhomogeneous Diophantine Approximation with restraint denominators on M_0-sets and some applications
• *Abstract*: This is a joint result with Evgeniy Zorin.
In this talk I will present a Schmidt-type theorem for Diophantine approximations with restraint denominators of sufficiently slow growth on M_0-sets. Basically, the balance condition between the growth rate of denominators and the decay rate of the Fourier transform of Rajchman measure, supported on M_0-sets, will be considered; this balance condition implies a Schmidt-type result.
I will also show some applications: Khintchine Theorem on the set of Liouville numbers; Hausdorff dimension of subsets of inhomogeneous ψ-well approximable real numbers; Schmidt-type theorem for Diophantine approximations with denominators of polynomial growth.

(21) • *Speaker*: René Pfitscher
• *Title*: Counting rational approximations in flag varieties
• *Abstract*: In the divergence case of Khintchine's theorem, Schmidt obtained an asymptotic formula for the number of rational approximations of bounded height to almost every real number. Using exponential mixing in the space of lattices, we prove versions of this theorem for intrinsic diophantine approximation on quadrics, grassmannians, and other examples of flag varieties.

(22) • *Speaker*: Steven Robertson
• *Title*: A Combinatorial Approach to the $p(t)$-adic Littlewood Conjecture
• *Abstract:* Let p be a prime and let $p(t)$ be an irreducible polynomial with coefficients in a field $\mathbb{K}$. In 2004, de Mathan and Teulié stated the p-adic Littlewood conjecture (p-*LC*) in analogy to the classical Littlewood conjecture. This talk focuses on the

analogue of p-LC over the field of formal Laurent series with coefficients in $\mathbb{K}$, known as the $p(t)$-adic Littlewood conjecture ($p(t)$-LC). Specifically, two metric results are provided on $p(t)$-LC with an additional growth function f. The first - a Khintchine-type theorem for $p(t)$-adic multiplicative approximation - enables one to determine the measure of the set of counterexamples to $p(t)$-LC for any choice of f. The second complements this by showing that the Hausdorff dimension of the same set is maximal when $p(t) = t$ in the critical case where $f = \log^2$.
These statements are proved by developing a dictionary between Diophantine approximation in function fields and the so-called number wall of a sequence - an infinite array containing the determinant of every finite Toeplitz matrix generated by that sequence. This unique methodology provides a complementary approach to the classical strategies used to attack a problem in Diophantine approximation: namely, Ergodic Theory and Number Theory.

(23)
- *Speaker*: Damien Roy
- *Title*: Parametric geometry of numbers and simultaneous approximation to geometric progressions
- *Abstract*: An important problem in Diophantine approximation is to determine, for a given positive integer n, the supremum $\widehat{\lambda}_n$ of the exponents $\widehat{\lambda}_n(\xi)$ of uniform simultaneous rational approximation to geometric progressions $(1, \xi, \xi^2, \ldots, \xi^n)$ whose ratio ξ is either a transcendental real number or an algebraic real number of degree $> n$. In 1969, Davenport and Schmidt provided an upper bound on $\widehat{\lambda}_n$ and, via geometry of numbers, they deduced a corresponding lower bound on the exponent of best approximation to such ξ by algebraic integers of degree at most $n+1$. The same general transference principle applies to other classes of numbers, like approximation to ξ by algebraic units of degree at most $n + 2$, as Teulié showed in 2001. Recall that Dirichlet's theorem on simultaneous rational approximation yields $\widehat{\lambda}_n \geq 1/n$. However, we still don't know, for any $n \geq 3$, if $\widehat{\lambda}_n$ is equal to $1/n$ or strictly greater.

  In this talk, we concentrate on the cases $n = 2$ and $n = 3$. For $n = 2$, I showed in 2003 that the upper bound of Davenport and Schmidt for $\widehat{\lambda}_2$ is best possible, namely that $\widehat{\lambda}_2 = 1/\gamma \cong 0.618$, where γ stands for the golden ratio. Then, for many years, I thought that $\widehat{\lambda}_3$ could be equal to the positive root $\lambda_3 \cong 0.4245$ of the polynomial $T^2 - \gamma^3 T + \gamma$, until I realized that it is strictly smaller. As the argument lead only to a very small improvement on the upper bound, I simply published, in 2008, the proof that $\widehat{\lambda}_3 \leq \lambda_3$.

  In the presentation, we take the point of view of parametric geometry of numbers. We first recall the basic facts that we need about n-systems and dual n-systems. For $n = 2$, we explain why a point $(1, \xi, \xi^2)$ with optimal exponent $\widehat{\lambda}_2(\xi) = 1/\gamma$ admits a very simple self-similar dual 3-system, we give generic algebraic relations between the points of $\mathbb{Z}^3$ that realize this map up to a bounded difference, and we show how these in turn determine the point $(1, \xi, \xi^2)$. One can hope that a similar phenomenon holds for each $n \geq 2$. For $n = 3$, assuming that $\widehat{\lambda}_3(\xi) = \lambda_3$, we find an interesting self-similar dual 4-system attached to the point $(1, \xi, \xi^2, \xi^3)$ and algebraic relations with similar properties between the points that realize it up to bounded difference. However, they eventually lead to a contradiction...

  In general, the theory attaches a dual n-system $\mathbf{P} = (P_1, \ldots, P_n)\colon [0, \infty) \to \mathbb{R}^n$ to any non-zero point $\mathbf{u}$ of $\mathbb{R}^n$, and $\mathbf{P}$ is unique up to bounded difference. This encodes most of the Diophantine approximation properties of $\mathbf{u}$. For a geometric progression $\mathbf{u} = (1, \xi, \xi^2, \xi^3)$ in $\mathbb{R}^4$ with $\widehat{\lambda}_3(\xi) > \sqrt{2} - 1 \cong 0.4142$, we can show that the behavior of $\mathbf{P}$ is qualitatively much simpler than that of a general dual 4-system. Moreover, the differences $P_3(q) - P_1(q)$ and $P_4(q) - P_2(q)$ both tend to infinity with q. Based on this, we deduce the existence of a sequence of integral bases of $\mathbb{R}^4$ which, in a simple way, realize $\mathbf{P}$ up to a bounded difference. We propose this as a tool to improve the present upper bound λ_3 on $\widehat{\lambda}_3(\xi)$. By contrast, the current way of studying $\widehat{\lambda}_n(\xi)$ for a general n is to form a sequence of so-called minimal points for $\mathbf{u} = (1, \xi, \dots, \xi^n)$, which can be loosely described as a sequence of points of $\mathbb{Z}^{n+1}$ that realize the first component P_1 of $\mathbf{P}$ up to bounded difference.

(24)
- *Speaker*: Cagri Sert
- *Title*: Projections of self-affine fractals
- *Abstract*: If a subset X of $\mathbb{R}^d$ is projected onto a linear subspace then the Hausdorff dimension of its image is bounded above by the rank of the projection and by the dimension of the set X itself. When the Hausdorff dimension of the image is smaller than both of these values the projection is called an exceptional projection for the set X. By the classical theorem of Marstrand, the set of exceptional projections of a Borel set always has Lebesgue measure zero when considered as a subset of the relevant Grassmannian. I will describe some results from an ongoing systematic study of the exceptional projections of self-affine sets describing, among others, a mechanism to create exceptional projections. As an application, we will discuss an example of a strongly irreducible self-affine set in $\mathbb{R}^4$ whose set of exceptional projections includes a nontrivial subvariety of the Grassmannian. Ongoing joint work with Ian Morris.

(25)
- *Speaker*: Stéphane Seuret
- *Title*: An asymmetric version of the Littlewood conjecture
- *Abstract*: In this talk, we study an asymmetric version of the Littlewood conjecture proposed by Y. Bugeaud. A parameter $\sigma \in [0,1]$ being fixed, we study the set $B(\sigma)$ of those pairs of real numbers (x, y) such that$\inf_{q \geq 1}(q \cdot \max(\|qx\| \, \|qy\|)^{1+\sigma} \min(\|qx\| \, \|qy\|)^{1-\sigma}) > 0$. Counter-examples to the Littlewood conjecture would belong to $B(0)$ and appear as an interpolation from the set $B(1)$ corresponding to the badly approximable vectors in dimension 2. We prove that for every $\sigma \in [0,1]$, $B(\sigma)$ has Hausdorff dimension 2, and propose some natural conjectures around such sets. Joint work in progress with F. Adiceam.

(26)
- *Speaker*: Noy Soffer Aranov
- *Title*: Hausdorff Dimension of the Set of Singular and Dirichlet Improvable Vectors in Function Fields.
- *Abstract*: We compute the Hausdorff dimension of the set of singular vectors in function fields and bound the Hausdorff dimension of the Dirichlet improvable vectors in this setting. Our results are a function field analog of the results of Cheung and Chevallier. This is part of joint work with Taehyeong Kim.

(27)
- *Speaker*: Niclas Technau
- *Title*: Counting Rational Points Near Manifolds
- *Abstract*: Choose your favourite, compact manifold M. How many rational points, with denominator of bounded size, are near M? We report on joint work with Damaris Schindler and Rajula Srivastava addressing this question. Our new method reveals an intriguing interplay between number theory, harmonic analysis, and homogeneous dynamics.

(28)

- *Speaker*: Ioannis Tsokanos
- *Title*: Stability and Shadowing of Non-invertible p-adic Dynamics
- *Abstract*: A continuous dynamical system is a couple (X, f) where (X, d) is a metric space and $f : X \mapsto X$ is a continuous map (called *dynamic*). The notions of stability and shadowing, introduced in the second third of the 20th century in the works of Andronov, Pontrjagin, Bowen, and Sinai, play a fundamental role in several branches of dynamical systems. For a system (X, f) to be *stable* roughly means that its analytical properties (e.g. the behavior of its orbits) are not affected upon introducing sufficiently small noises in the system. A map f is said to be shadowing if every pseudo-orbit $\{x_n\}_{n\in\mathbb{N}}$ (i.e. $d(f(x_n), x_{n+1})$ is small for every $n \in \mathbb{N}$) is close, with respect to the supreme norm, to a real orbit $\{f^n(x)\}_{n\in\mathbb{N}}$.
The stability theory of compact topological manifolds with positive and finite dimensions is well-developed. The classical results describe how the two concepts of stability and shadowing are related, especially when the dynamic under consideration is a homeomorphism. However, the study of these dynamical concepts in zero-dimensional compact spaces has only recently started developing. A notable family of zero-dimensional compact spaces is the *Cantor spaces*. Examples of Cantor spaces include fractal sets such as the famous Cantor set and the metric space of the p-adic integers $\mathbb{Z}_p$.
This work concerns the dynamics of p-adic integers and, in particular, families of non-invertible maps that admit, however, right or left inverses. The results provide sufficient conditions for stability and shadowing properties to arise. As a consequence, the context developed unifies many of the known examples while, at the same time, providing new ones.

(29)

- *Speaker*: Barak Weiss
- *Title*: Singular vectors in manifolds, countable intersections, and Dirichlet spectrum
- *Abstract*: A vector $x = (x_1, ..., x_d)$ in $\mathbb{R}^d$ is totally irrational if $1, x_1, ..., x_d$ are linearly independent over rationals, and singular if for any $\epsilon > 0$, for all large enough T, there are solutions p in $\mathbb{Z}^d$ and q in $\{1, ..., T\}$ to the inequality $\|qx - p\| < \epsilon T^{-1/d}$. In previous work we showed that certain smooth manifolds of dimension at least two, and certain fractals, contain totally irrational singular vectors. The argument for proving this is a variation on an old argument employed by Khintchine and Jarník. We now adapt this argument to show that for certain families of maps $f_i : \mathbb{R}^d \to \mathbb{R}^{n_i}$, certain manifolds contain points x such that $f_i(x)$ is a singular vector for all i. This countable intersection property is motivated by some problems in approximation of vectors by vectors with coefficients in a number field. I will review Khintchine's original argument and present additional consequences, among them that the Dirichlet spectrum is full, for arbitrary norms, in dimension $d > 1$, and improved rates of singularity on certain manifolds. Based on a joint work with Dmitry Kleinbock, Nikolaus Moshchevitin and Jacqueline Warren, and another joint work with Alon Agin.

(30)

- *Speaker*: Agamemnon Zafeiropoulos
- *Title*: A variant of Kaufman's measures in two dimensions.
- *Abstract*: An old result of Kaufman showed that the set **Bad** of badly approximable numbers supports a family of probability measures with polynomial decay rate on their Fourier transform. We show that the same phenomenon can be observed in a two-dimensional setup: we consider the set

$$\mathcal{B} = \{(\alpha, \gamma) \in [0, 1]^2 : \inf \|q\alpha - \gamma\| > 0\}$$

and we prove that it supports certain probability measures with Frostman dimension arbitrarily close to 2 and Fourier transform with polynomial decay rate. (Joint work with S. Chow and E. Zorin).

(31)
- *Speaker*: Evgeniy Zorin
- *Title*: Exploring the Limits: Unbounded Diophantine Approximations and Matrix Transformations
- *Abstract*: In this talk, I will present our recent advancements on the shrinking target problem of matrix transformation on tori and their subvarieties. For tori, we can provide sharp asymptotic results in a remarkably broad setting. This research has been naturally linked with expansion of the Mass Transference Principle to unbounded conditions, a tool which holds an independent interest on its own. I will present the progress in this direction as well.

  In a much more refined case of subvarieties of tori, our findings are (yet) less sharp. So far, we have established the Khintchine theorem for curves in a two-dimensional torus and a Jarník-type theorem for straight lines (the latter result is conditional under *abc*-conjecture for high exponents of approximations). The latter topic clearly leads to a very interesting, rich and fascinating theory. Joint project with W. Baowei, B. Li, L. Liao, and S. Velani.