

Multiplicative middle convolution for KZ-type equations and construction of representations of braid groups

Haru Negami

12 December 2023

The Long-Moody construction is a method for obtaining a representation of the braid group from a given representation of the semidirect product of the free group F_n and the braid group B_n defined by Artin representation. The braid group is defined as the mapping class group of a closed disk with n holes, $D \setminus \{n\text{-points}\}$, and the fundamental group of $D \setminus \{n\text{-points}\}$ is isomorphic to F_n . Thus, Artin representation has a geometric realization such as the transformation of the loops on $D \setminus \{n\text{-points}\}$. Wada generalized the Artin representation and obtained the group invariants of links.

In this talk, we first introduce the extension of the method to obtain an infinite series of representations of the braid group by using the convolution of Dettweiler-Reiter[1], defined by an integral transform. In particular, this construction has a correspondence with the multiplicative middle convolution of KZ-type equations in the case of pure braid groups, giving an irreducible representation. Second, we further extend the method to one of the Wada types of semidirect product. We then discuss the connection between this construction and the unitarity of the representation.

References

- [1] M. Dettweiler and S. Reiter, *An Algorithm of Katz and its Application to the Inverse Galois Problem*, Journal of Symbolic Computation, 30(6), 761-798, (2000)
- [2] K. Hiroe and H. Negami, *Long-Moody construction of braid representations and Katz middle convolution*, <https://arxiv.org/pdf/2303.05770.pdf>