Using dileptons to estimate the initial temperature of QCD matter[1,2]

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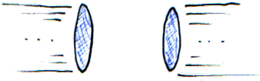
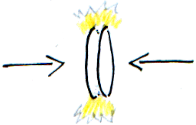
1 based on collaboration w/ J. Churchill, L. Du, C. Gale and S. Jeon

2 supported by the ANR under grant No. 22-CE31-0018

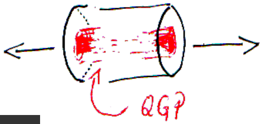


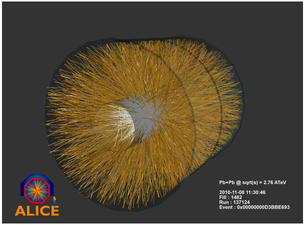


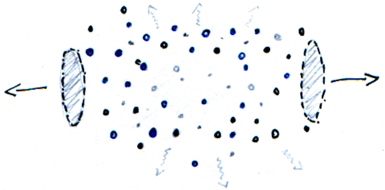
$$\mathcal{L} = -\frac{1}{4} F^2 + \sum_f \bar{\psi} \left(i \not{D} - m_f \right) \psi$$

w/ non-Abelian fields

$$F_{\mu\nu} = \frac{1}{ig} \left[D_\mu, D_\nu \right]$$

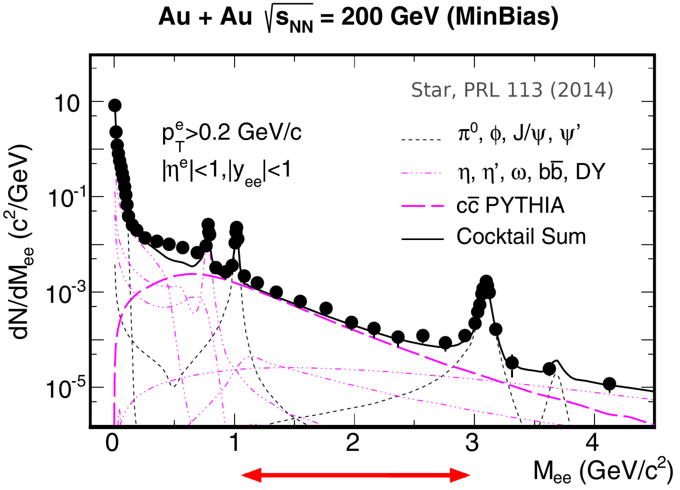




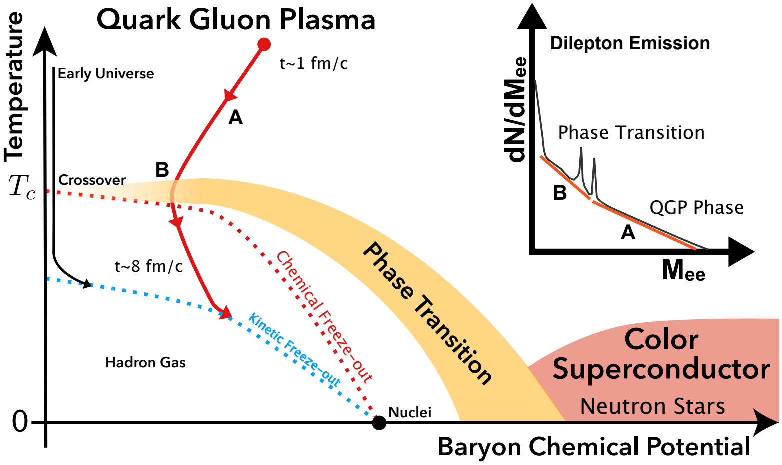


Electromagnetic probes

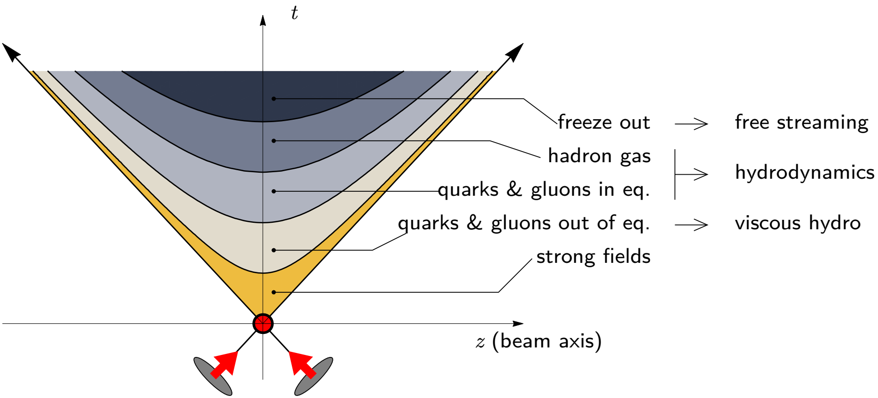
Invariant mass spectrum of *dileptons pairs*, e.g. from $q\bar{q} \to \gamma^* \to e^+e^-$



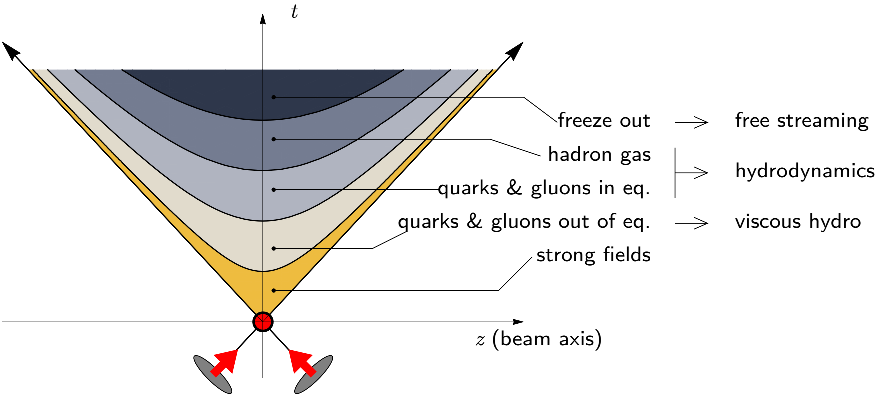
Intermediate Mass Range (IMR) $= 1 \ldots 3$ GeV



[STAR collaboration (2024)]



$\tau \sim 0.2$ fm/c	production of light quarks & gluons
$1 - 2$ fm/c	thermalisation rapid (?)
$2 - 10$ fm/c	**quark-gluon plasma**
$10 - 20$ fm/c	hadron gas
$\tau \to \infty$	dilute, no further interactions



$\tau \sim 0.2$ fm/c	production of light quarks & gluons	beginning
$1 - 2$ fm/c	thermalisation rapid (?)	
$2 - 10$ fm/c	**quark-gluon plasma**	middle
$10 - 20$ fm/c	hadron gas	end
$\tau \to \infty$	dilute, no further interactions	

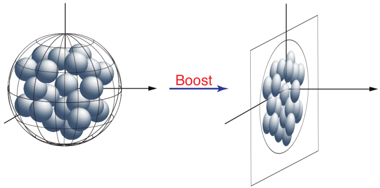
Beginning: (initial conditions)

Pedestrian approach:

sample nucleons with,
Monte Carlo (Glauber)

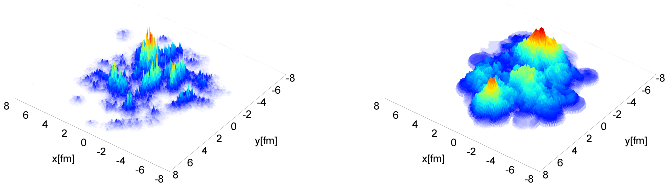
public code: $\mathrm{T_RENTO}$

[Moreland, Bernhard, Bass (2014)]



For the connoisseur: IP-Glasma / KLN / EKRT / ...

(classical YM action in 2D, sat. scale Q_s ... valid at high-E)

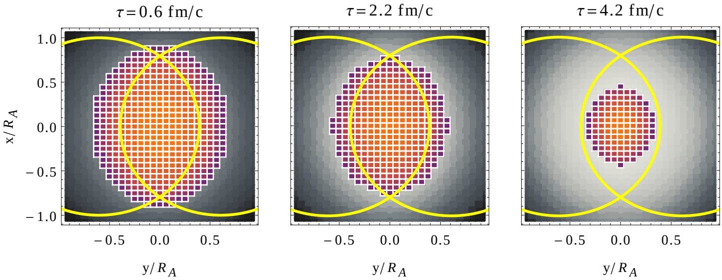


[Eskola, Kajantie, Ruuskanen, Tuominen (1999)]

[Schenke, Tribedy, Venugopalan (2012)]

from IC: get energy density $e(x, y, \ldots)$ at τ_0 [Kurkela, et al. (2019)]

... then discretize & evolve in spacetime:



VISH2+1 = **VIS**cous **H**ydrodynamics in (2+1) dim. [Song, Heinz (2008)]
(using SHASTA = SHarp And Smooth Transport Algorithm)

MUSIC = **MUS**(cl) for **I**on **C**ollisons [Schenke, Jeon, Gale (2010)]
(MUSCL = Monotonic Upstream-centered Schemes for Conservation Laws)

End: ("particlization")

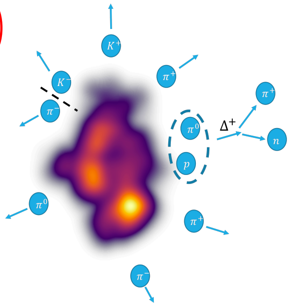
convert $T^{\mu\nu}(X)$ and $J^{\mu}(X)$ into hadrons [Huovinen, Petersen (2012)]

(in a way that conserves E and $\boldsymbol{p}$) [Cooper, Frye (1974)]

freeze-out: $E\frac{\mathrm{d}N}{\mathrm{d}^3\boldsymbol{p}} = \int_\Sigma \mathrm{d}\sigma_\mu P^\mu f_{\mathrm{B/F}}\left(\frac{P\cdot u(X)}{T(X)}\right)$

(for MUSIC, this is done with iS3D:

https://github.com/derekeverett/iS3D)



... then hadronic transport, e.g.:

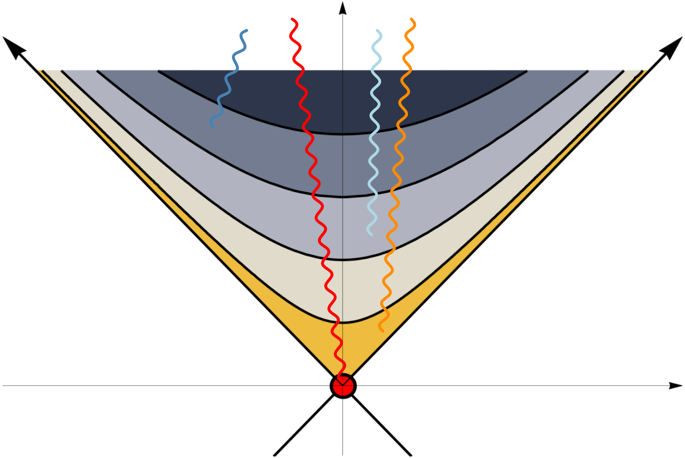
UrQMD = **U**ltra-**r**elativistic **Q**uantum **M**olecular **D**ynamics

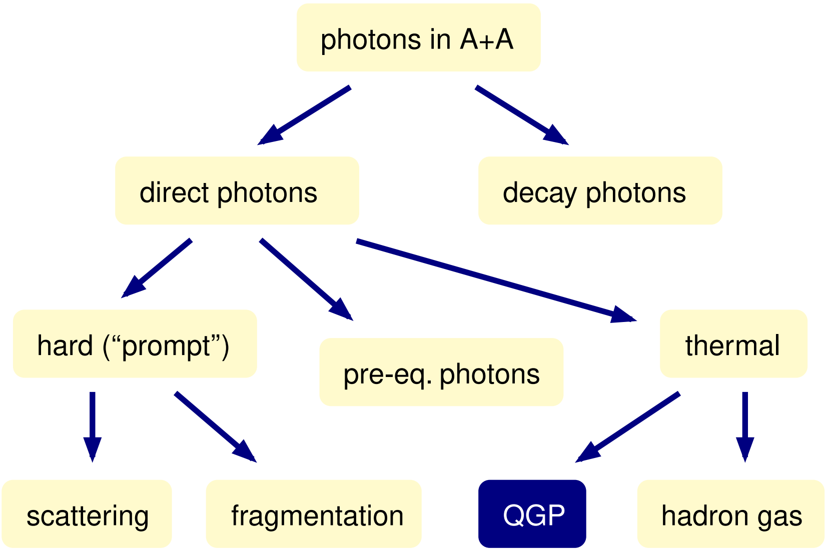
[Bleicher, et al. (1999)]

SMASH = **S**imulating **M**any **A**ccelerated **S**trongly interacting **H**adrons

[Weil, et al. (2016)]

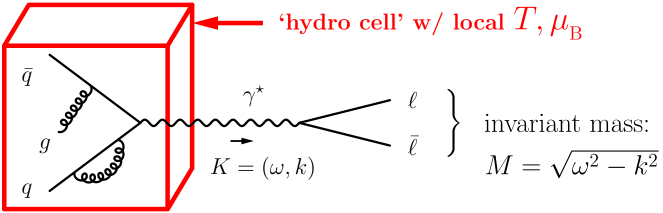
photons in A+A




photons in A+A
direct photons
decay photons
hard (“prompt”)
pre-eq. photons
thermal
scattering
fragmentation
QGP
hadron gas

Basic relations from pert. theory

[McLerran, Toimela (1995)]
[Gale, Kapusta (1991)]



Emission rate per unit volume, $\Gamma_{\ell\bar{\ell}}$, of an **equilibrated** QGP

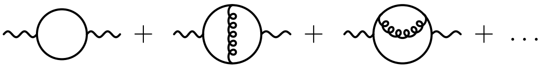
$$\frac{d\Gamma_{\ell\bar{\ell}}}{d\omega\, d^3\boldsymbol{k}} = \frac{\alpha_{\rm em}^2 \sum_{f=1}^{n_f} Q_f^2}{3\pi^3 M^2 \left(e^{\omega/T} - 1\right)} \times B\Big(\frac{m_\ell^2}{M^2}\Big) \times \rho_{\rm V}(\omega, k)$$

- Quark charge-fractions: Q_f (in units of the electron charge)
- Kinematic factor: $B(x) \equiv (1+2x)\Theta(1-4x)\sqrt{1-4x}$
- Spectral function $\rho_{\rm V} \equiv \rho_\mu^{\ \mu}$

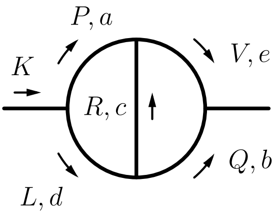
$$\rho_{\mu\nu}(\omega, k) = \mathrm{Im}\Big[\,\Pi_{\mu\nu}^{\rm ret}(\omega + i0^+, k)\Big]$$

QCD corrections

$$\Pi^{\mu\nu} = \Big[\textstyle\sum_{l=0}^{\infty} g_s^{2l}\, \Pi^{\mu\nu}_{(l)} \Big] + O(e^2)\ ; \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

$=$ 

Project 2-loop result onto 'basis' of **master diagrams** and evaluate:

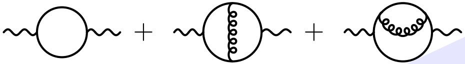


$$\rho^{(m,n)}_{abcde}(\omega, \boldsymbol{k}) \equiv \mathrm{Im} \,\,\SumInt_{P,Q} \frac{p_0^m\, q_0^n}{P^{2a} Q^{2b} (K-P-Q)^{2c} (K-P)^{2d} (K-Q)^{2e}}$$

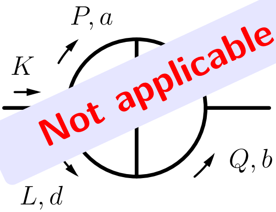
[1910.07552]

QCD corrections

$$\Pi^{\mu\nu} \;=\; \Big[\sum_{l=0}^{\infty} g_s^{2l}\, \Pi^{\mu\nu}_{(l)} \Big] \;+\; O(e^2)\;; \qquad \alpha_s \;=\; \frac{g_s^2}{4\pi}$$

$$= \quad$$  $$+ \ \cdots$$

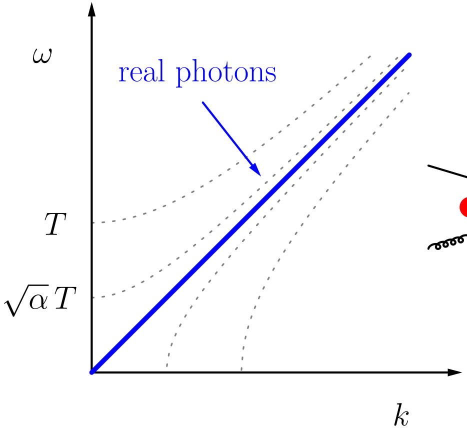
Project 2-loop result onto ‘basis’ of **master di** … evaluate:



Not applicable for $K^2 \to 0$!

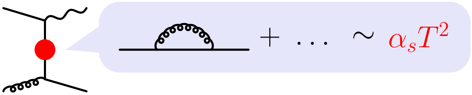
$$\rho^{(m,n)}_{abcde}(\omega, \boldsymbol{k}) \;\equiv\; \mathrm{Im} \SumInt_{P,Q} \frac{p_0^m\, q_0^n}{P^{2a} Q^{2b} (K-P-Q)^{2c} (K-P)^{2d} (K-Q)^{2e}}$$

[1910.07552]



Thermal Screening

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{-\pi\,\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}}{3s^2}\,\frac{t^2+s^2}{t\,s}$$

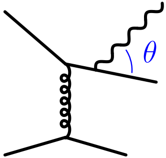


[Kapusta, Lichard, Seibert (1991)]

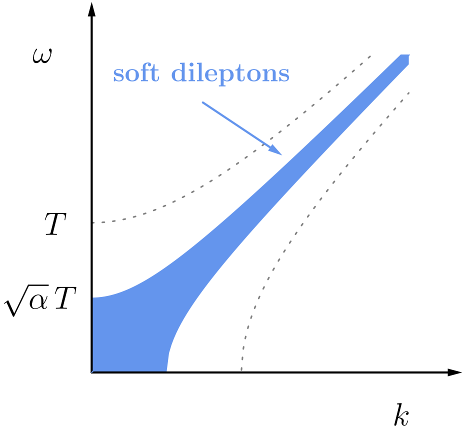
[Baier, *et al* (1992)]

Landau-Pomeranchuk-Migdal (*LPM*)

$$\int \frac{\mathrm{d}\cos\theta}{E(1-\cos\theta)} = \infty$$



LO: [Arnold, Moore, Yaffe (2001)] , NLO: [Ghiglieri, *et al* (2013)]



LO: [Aurenche, *et al* (2002)]

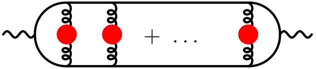
NLO: [Ghiglieri, Moore (2014)]

light-like correlator

$\Downarrow$ [Caron-Huot (2009)]

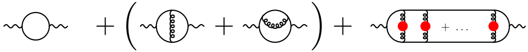
Effective Field Theory

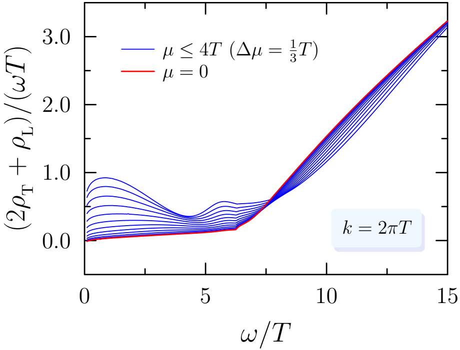
'ladder diagrams' for $M^2 \ll T^2 \rightarrow$ LPM effect + *Hard Thermal Loops*

$\rho_{\mu\nu}(\omega, \boldsymbol{k}) \quad = \quad \mathrm{Im}\Big[\mu$  $\nu\Big]$

Combine LPM effect with strict 2-loop truncation, [Ghisoiu, Laine (2014)]

$$\rho(\omega,k)\Big|^{\rm NLO}_{\rm resummed} = \rho\Big|^{1-\rm loop} + \rho\Big|^{2-\rm loop} + \rho\Big|^{\rm LPM}_{\text{beyond } O(g^2)}$$

$=$ 

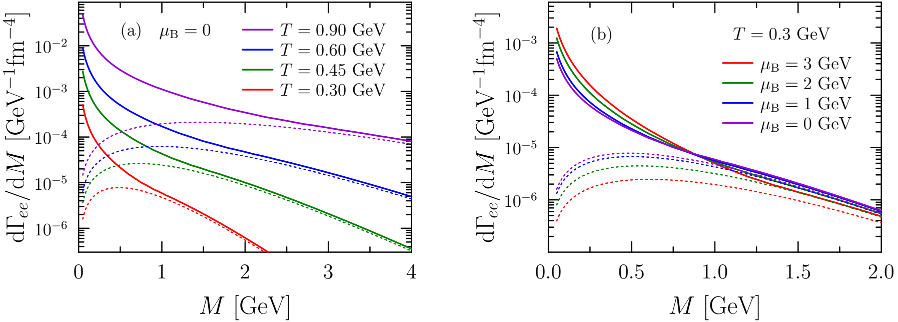


NB: Spectral fncs. can be checked with Euclidean corr. computable on the lattice:

$$G(\tau) = \int_0^\infty d\omega\, \rho(\omega)\, \mathcal{K}(\omega,\tau)$$

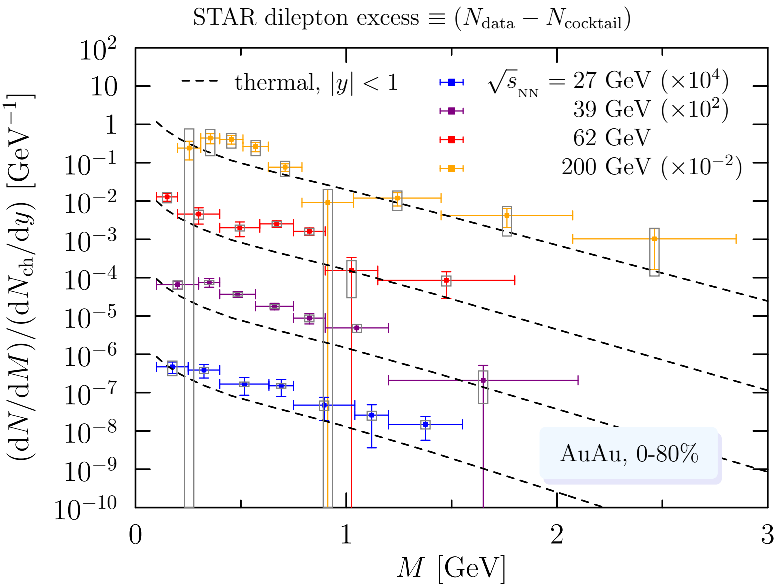
$\Rightarrow$ see [GJ, Laine (2019)] and [Ali, et al. (2024)]

($\rho_{\rm V}$ determined for $\omega > k$ in [Laine (2013)] , and $\omega < k$ in [GJ (2019)]) rate from a **static source**: $\dfrac{d\Gamma_{\ell\bar{\ell}}}{dM} = \int_{\boldsymbol{k}} \dfrac{M}{\sqrt{M^2+k^2}} \dfrac{d\Gamma_{\ell\bar{\ell}}}{d\omega\, d^3\boldsymbol{k}}$



- dileptons are a good thermometer!
- ... but a poor "baryometer"

$*$ in these, and subsequent, plots: $\alpha_s = 0.3$



*see also: [Burnier, Gastaldi (2015)] (LHC energies) for large $M \gg T$ and μ_{B}: $$\frac{\mathrm{d}\Gamma_{\ell\bar{\ell}}}{\mathrm{d}M} \propto (M T_{\mathrm{eff}})^{3/2} \exp\left(-M/T_{\mathrm{eff}}\right)$$

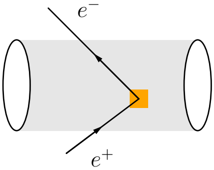
$\Rightarrow$ determine T_{eff} from the 'inverse slope' of the spectrum

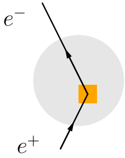
What physics does this *effective* temperature represent?

---

in simulations we can access the full history, so the method can be tested!

$$\frac{\mathrm{d}N_{\ell\bar{\ell}}}{\mathrm{d}^4 K'} = \int \mathrm{d}t \int \mathrm{d}^3\boldsymbol{x}\, \frac{\mathrm{d}\Gamma_{\ell\bar{\ell}}}{\mathrm{d}\omega\,\mathrm{d}^3\boldsymbol{k}}\bigg|_{K^\mu = \Lambda^{\mu\nu} K'_\nu}$$

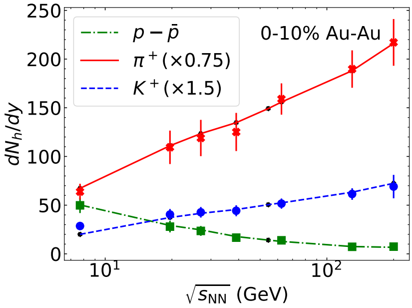


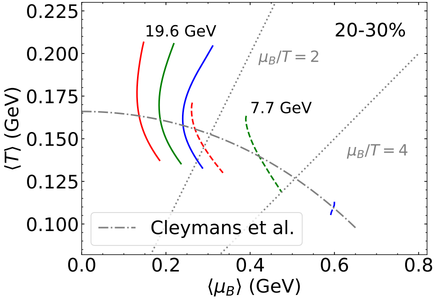


note: $\rho_{\mathrm{V}}(\omega, k)$ evaluated at $\omega = K'_\mu u^\mu$, $k = \sqrt{(K'_\mu u^\mu)^2 - M^2}$

Callibrating the thermometer

- MC-Glauber initial conditions (at finite $\mu_{\rm B}$)
- Hydro with MUSIC (including viscous corrections)
- Equation of state: NEOS-B (neglects strangeness and μ_e)
- Hydro stops at $e_{\rm fo} = 0.26$ GeV/fm^3
- Freeze-out (iS3D) and hadronic scatterings w/ UrQMD





code public: https://github.com/LipeiDu/DileptonEmission

Callibrating the thermometer

- MC-Glauber initial conditions (at finite μ_B)
- Hydro with MUSIC (including viscous corrections)
- Equation of state: NEOS-B (neglects strangeness and μ_e)
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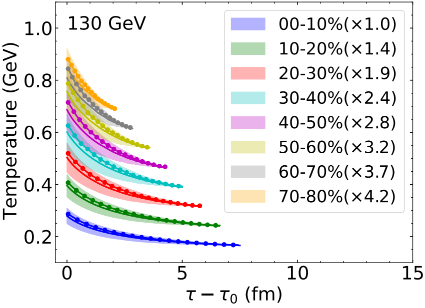
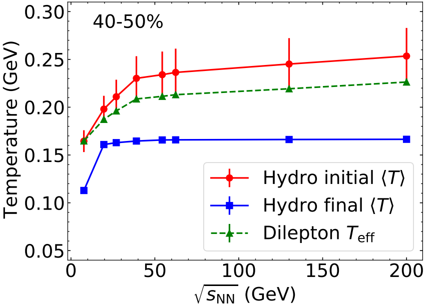
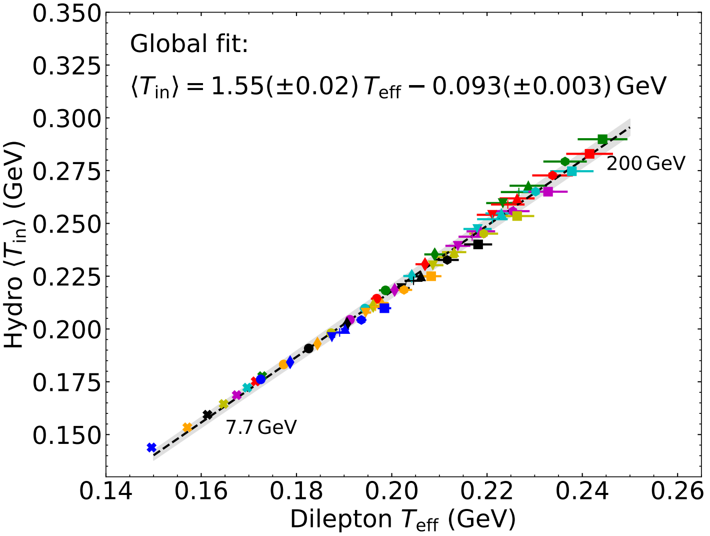




code public: https://github.com/LipeiDu/DileptonEmission

T_{eff} represents the *initial* temperature!

[Churchill, et al. (2024)]



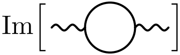
... predicted over 40 years ago (!!)

[Kajantie, Miettinen (1981)]

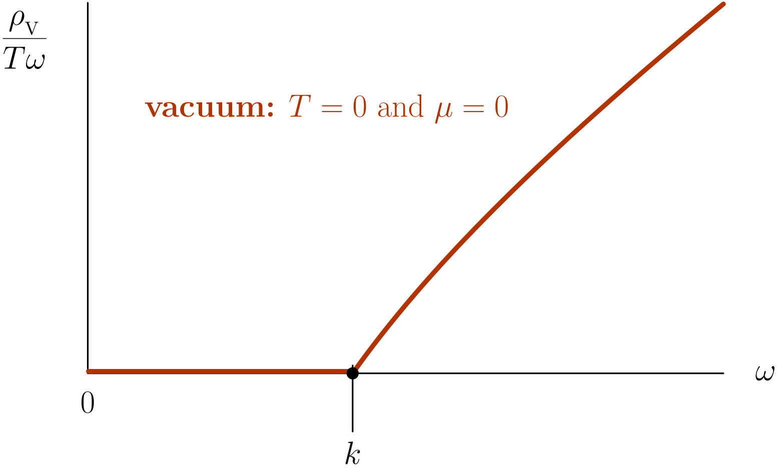
Summary

Arxiv: 2211.09575
2311.06675
2311.06951

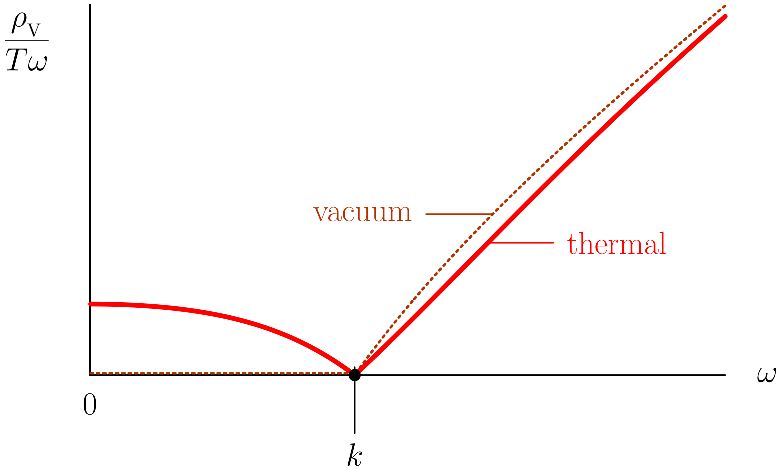
- thermal dilepton yields at NLO+LPM
  $\Rightarrow$ predicted from first principles, at finite T and $\mu_{\rm B}$
- extracted 'effective' temperature
  $\Rightarrow$ linear relationship between $T_{\rm eff}$ and $T_{\rm in}$



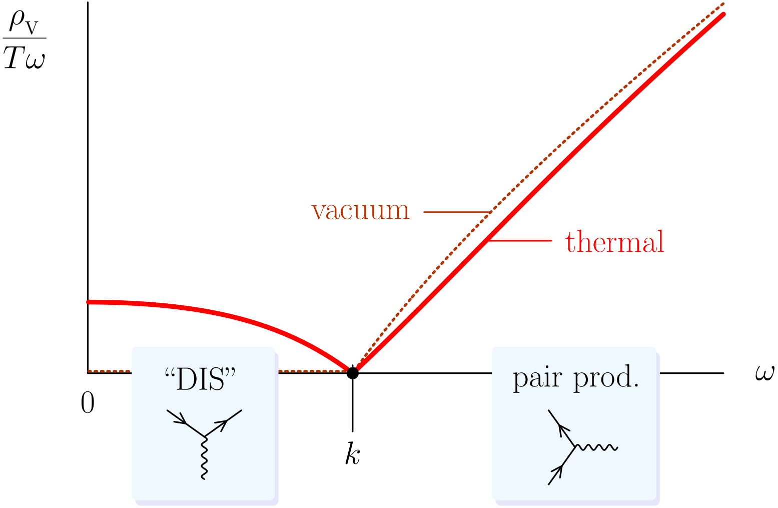
$\xrightarrow{T=0} \frac{N K^2}{4\pi} \Theta(K^2)$



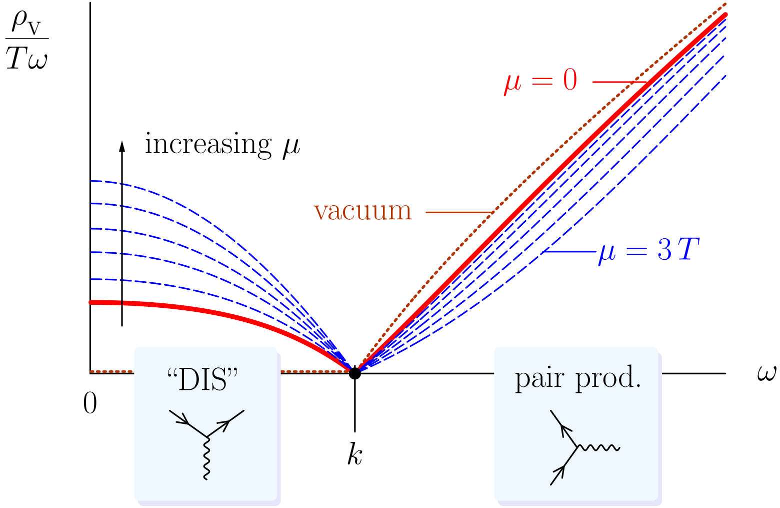
$$\mathrm{Im}\left[\text{~~}\bigcirc\text{~~}\right] \xrightarrow{T>0} \frac{NK^2}{4\pi}\left\{\frac{2T}{k}\log\left[\frac{1+e^{-\frac{1}{2}(\omega+k)/T}}{1+e^{-\frac{1}{2}|\omega-k|/T}}\right]+\Theta(K^2)\right\}$$

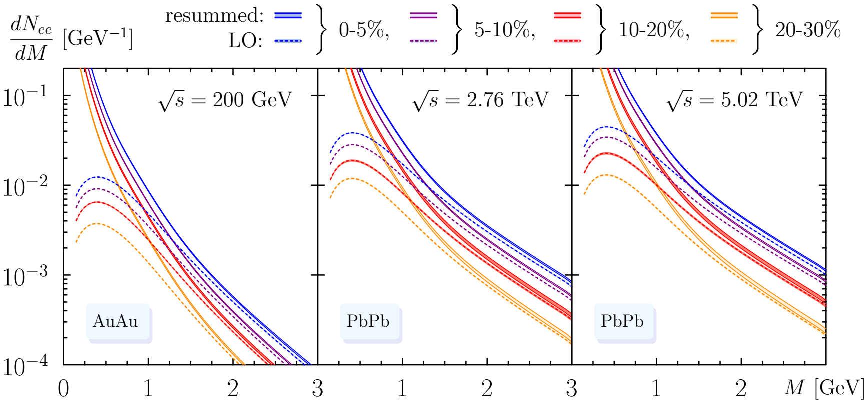


$$\mathrm{Im}\Big[\ \text{(loop diagram)}\ \Big] \xrightarrow[\mu=0]{T>0} \frac{N K^2}{4\pi}\left\{\frac{2T}{k}\log\left[\frac{1+e^{-\frac{1}{2}(\omega+k)/T}}{1+e^{-\frac{1}{2}|\omega-k|/T}}\right]+\Theta(K^2)\right\}$$



$$\mathrm{Im}\left[\text{(loop)}\right] = \frac{NK^2}{4\pi}\left\{\frac{T}{k}\sum_{\nu=\pm\mu}\log\left[\frac{1+e^{(\nu-\frac{1}{2}(\omega+k))/T}}{1+e^{(\nu-\frac{1}{2}|\omega-k|)/T}}\right]+\Theta(K^2)\right\}$$

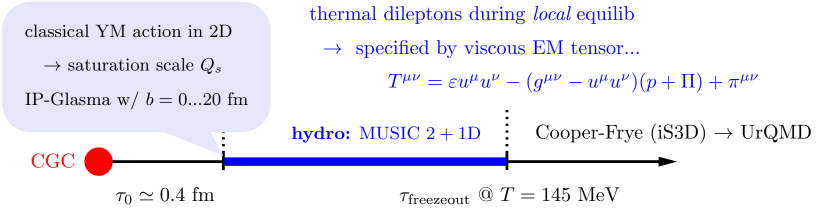


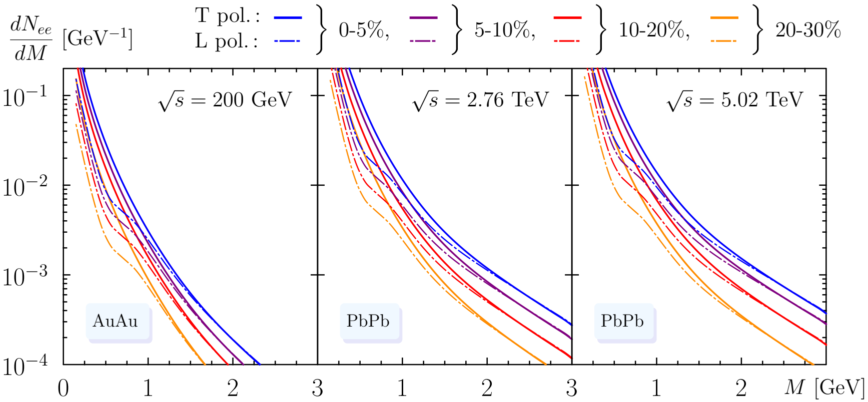


thermal dileptons during *local* equilib

$\rightarrow$ specified by viscous EM tensor...

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (g^{\mu\nu} - u^\mu u^\nu)(p + \Pi) + \pi^{\mu\nu}$$





classical YM action in 2D

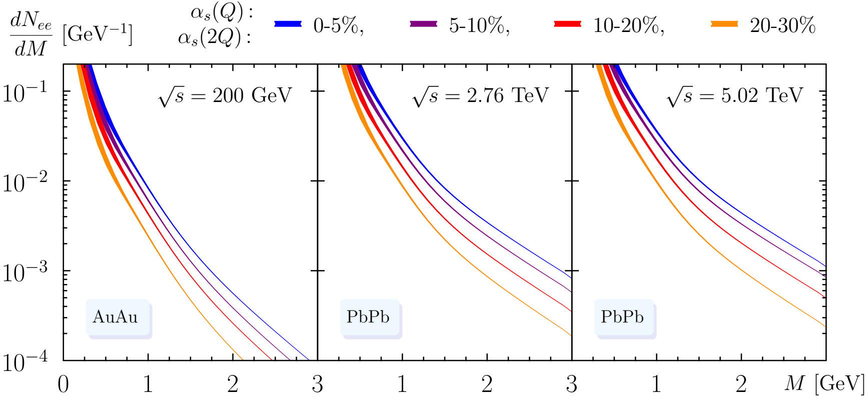
→ saturation scale Q_s

IP-Glasma w/ $b = 0...20$ fm

thermal dileptons during *local* equilib

→ specified by viscous EM tensor...

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (g^{\mu\nu} - u^\mu u^\nu)(p + \Pi) + \pi^{\mu\nu}$$



classical YM action in 2D

$\rightarrow$ saturation scale Q_s

IP-Glasma w/ $b = 0...20$ fm

thermal dileptons during *local* equilib

$\rightarrow$ specified by viscous EM tensor...

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (g^{\mu\nu} - u^\mu u^\nu)(p + \Pi) + \pi^{\mu\nu}$$

Considerations for non-zero μ_B

- chemical equilibrium $\Rightarrow \mu \equiv \mu_\mathrm{q} = \frac{1}{3}\mu_\mathrm{B}$

- Debye mass m_D and the 'asymptotic' quark mass m_∞

$$m_D^2 \equiv g^2\left[\left(\tfrac{1}{2}\,n_f + N\right)\frac{T^2}{3} + n_f\,\frac{\mu^2}{2\pi^2}\right]$$

$$m_\infty^2 \equiv g^2\,\frac{C_F}{4}\left(T^2 + \frac{\mu^2}{\pi^2}\right)$$

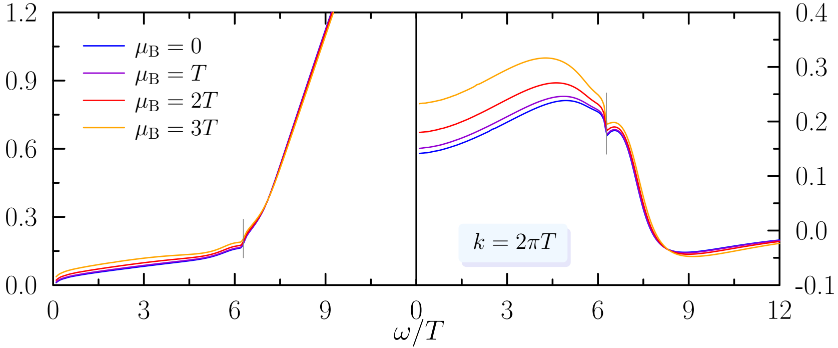
- large frequency limit: enhancement ↘

$$\rho_\mathrm{V} \simeq \frac{NM^2}{4\pi} + 4g^2 C_F N\left\{\frac{3M^2}{4(4\pi)^3} + \frac{\pi\left(\omega^2 + \frac{k^2}{3}\right)}{36M^4}\left(T^4 + \frac{6}{\pi^2}T^2\mu^2 + \frac{3}{\pi^4}\mu^4\right)\right\}$$

NEW RESULTS: the **full** effect of μ_B on $\rho(\omega,k)\big|^{\mathrm{NLO}}_{\mathrm{resummed}}$ …

What we find

pQCD spectral functions $T = 280$ MeV and $n_f = 3$:



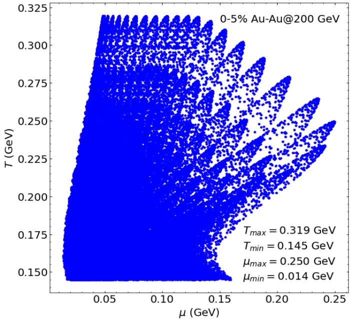
Spectral functions, Left: $\rho_V/(\omega T) = (2\rho_T + \rho_L)/(\omega T)$

Right: $\rho_H/(\omega T) = 2(\rho_T - \rho_L)/(\omega T)$

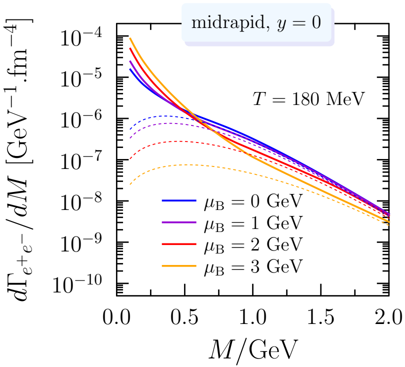
Impact on yield (non-zero μ_B)

BES $\Rightarrow$ probe baryon rich region work w/ Churchill, Du, Gale, Jeon

MUSIC: [Schenke, Jeon, Gale (2010)]



(↑ figure by Lipei Du)

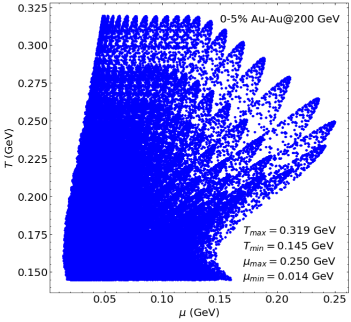


$\Rightarrow$ compensation of LO suppression & NLO enhancement! ...

Impact on yield (non-zero μ_B)

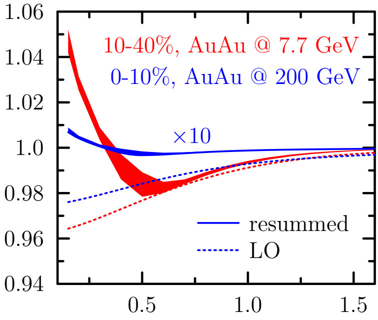
BES $\Rightarrow$ probe baryon rich region work w/ Churchill, Du, Gale, Jeon

MUSIC: [Schenke, Jeon, Gale (2010)]



(↑ figure by Lipei Du)

ratio: $\dfrac{dN_{e^+e^-}/dM|_{\mu>0}}{dN_{e^+e^-}/dM|_{\mu=0}}$



smooth MC-Glauber initial conditions + baryon diffusion