Linear power corrections to top quark production processes

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based on JHEP05(2023)153, JHEP01(2024)074
with **S. Makarov**, **K. Melnikov**, **P. Nason**

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Part I

Introduction:
Renormalons & Power Corrections

precision at colliders

- improved experimental precision at colliders

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- *master equation* for hadron colliders:

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 \, f_{i/p}(x_1) \, f_{j/p}(x_2) \, d\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2) \tag{1}$$

- $f_{i/p}$: parton distribution function (PDF)
- $d\hat{\sigma}_{ij}$: partonic cross-section

cross-sections: QCD corrections

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 \, f_{i/p}(x_1) \, f_{j/p}(x_2) \, d\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2) \tag{2}$$

- partonic cross-section can be represented by Feynman diagrams

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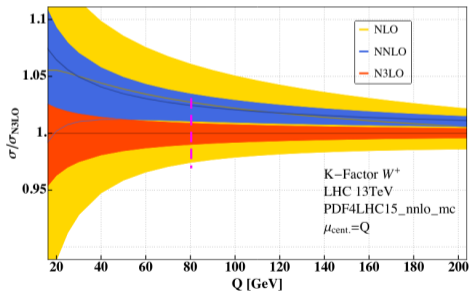
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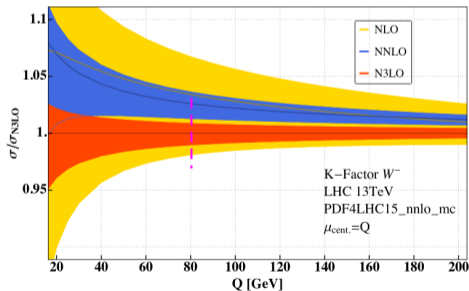
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Drell-Yan $N^3LO \sim 1\%$ correction

[Duhr, Dulat, Mistlberger; JHEP 11 (2020) 143]

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$$Q \sim 30 - 100 \text{ GeV}$$

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$\rightarrow$ **non-perturbative corrections may become relevant!**
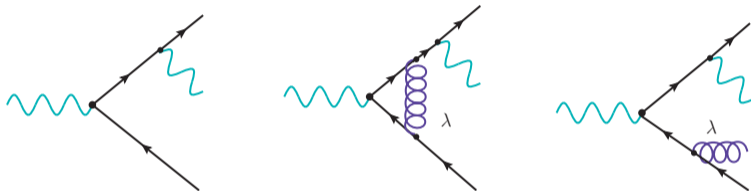
linear power corrections: renormalon calculus

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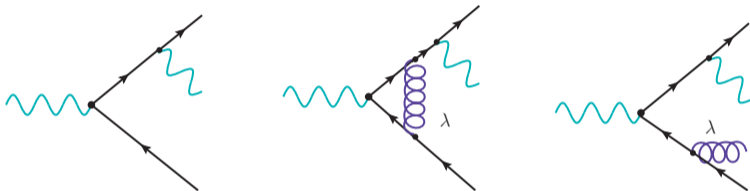
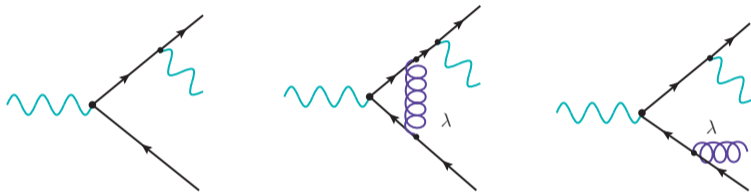# linear power corrections: renormalon calculus

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- direct relation between $\lambda^p \to \Lambda_{QCD}^p$
- for phenomenological applications only linear terms λ/Q are relevant, higher orders in λ are surpressed by $\mathcal{O}(\Lambda_{QCD}^2/Q^2)$

non-perturbative physics

In order to compute linear power corrections with renormalon calculus,

- compute NLO corrections with a massive gluon of mass λ
- can be applied only to processes with no gluons in the Born process

Recent progresses with renormalon calculus,

- linear power corrections to event shape variables $\rightarrow$ can be used for α_s extraction

  [F. Caola, S. Ferrario Ravasio, G. Limatola, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 12 (2022) 062]

- linear power corrections to top quark processes at the LHC $\rightarrow$ may be used for m_t extract.

  [S. Makarov, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 05 (2023) 153 & JHEP 01 (2024) 074]

Top mass determination

- most precise top mass measurements at LHC come from "direct measurements"
  - $\rightarrow$ kinematic reconstruction of measured top quark decay products

- currently, typical error quoted to be around $\Delta m_t \sim 500$ MeV
  - $\rightarrow$ future (HL-LHC), envisaged uncertainty $\Delta m_t \sim 200$ MeV

- direct measurements $\rightarrow$ measured top quark mass is the value of the top mass parameter in the Monte Carlo generator
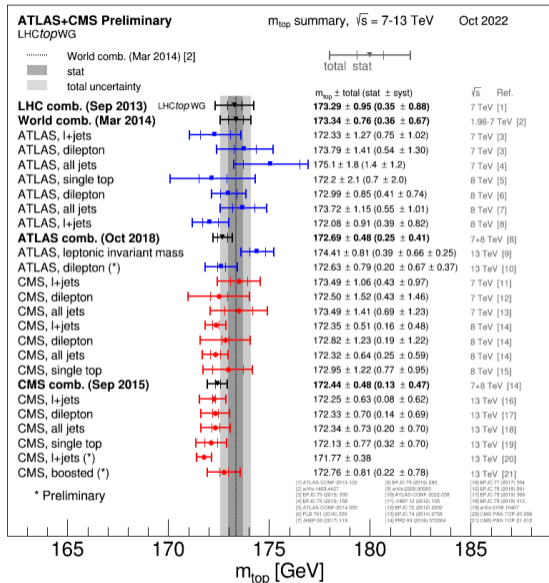
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$\rightarrow$ investigate mass scheme-dependence on linear power corrections to top quark production cross-sections and observables
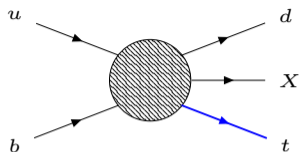
Part II

Linear power corrections to top quark processes

[S. Makarov, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 05 (2023) 153 & JHEP 01 (2024) 074]

Massive case: presence or absence of linear power correction

For processes with *massive* quarks:
- single top production,
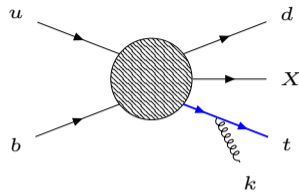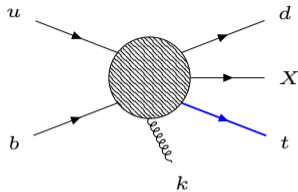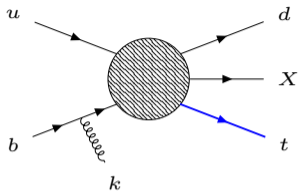- $t\bar{t}$ production,
- ...

we can make the following statements on presence or absence of linear power corrections:

[Makarov, Melnikov, Nason, Ozcelik, JHEP 05 (2023) 153]
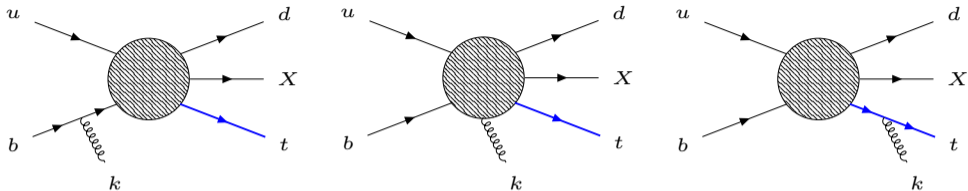
- *virtual* corrections **do** induce linear corrections
  - in the <u>soft</u> limit at next-to-soft approximation

- *real* corrections **do** induce linear corrections
  - in the <u>soft</u> limit at *next-to-soft* approximation

- need amplitudes at *next-to-soft* approximation
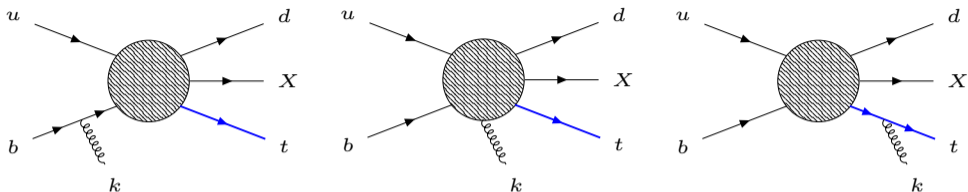 $\rightarrow$ can use *Low-Burnett-Kroll* theorem

Real radiation: Low-Burnett-Kroll theorem

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- the reduced real radiation amplitude $\mathcal{M}^\mu$ reads:

$$\mathcal{M}^\mu = \bar{u}(q_t)\gamma^\mu \frac{\slashed{q}_t + \slashed{k} + m_t}{d_t} \mathsf{N}(q_t + k, p_b, q_d, ...) u(p_b)$$
$$+ \bar{u}(q_t)\mathsf{N}(q_t, p_b - k, q_d, ...)\frac{\slashed{p}_b - \slashed{k}}{d_b}\gamma^\mu u(p_b) + \mathcal{M}^\mu_{\mathrm{reg}}(q_t, p_b, q_d, ..|k), \tag{4}$$

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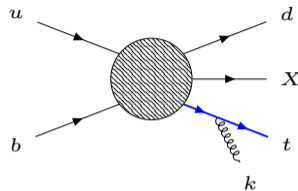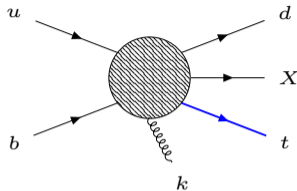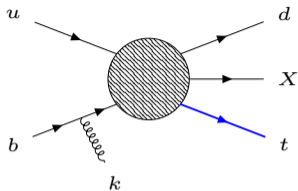

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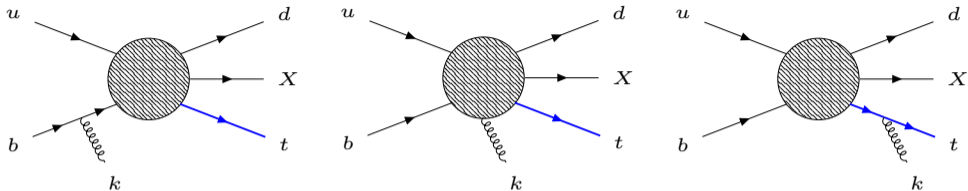
- need $\mathcal{M}^\mu_{\mathrm{reg}}$ up to sub-leading term within $\mathcal{M}$

Real radiation: Low-Burnett-Kroll theorem

- use condition: $k_\mu \mathcal{M}^\mu = 0$

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$$\mathcal{M}_{\text{reg}}^\mu(q_t, p_b, q_d, ...|k = 0) = -\bar{u}_t \left[\frac{\partial \mathsf{N}(q_t, p_b, q_d, ...)}{\partial q_{t,\mu}} + \frac{\partial \mathsf{N}(q_t, p_b, q_d, ...)}{\partial p_{b,\mu}} \right] u_b. \quad (4)$$
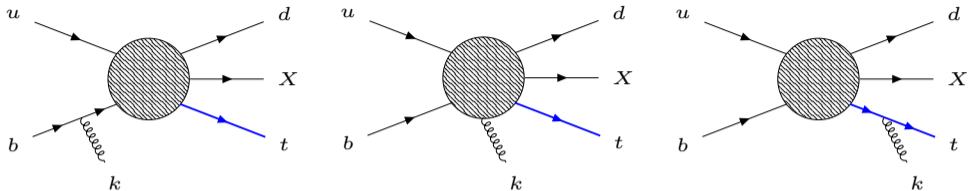
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- we now have all the ingredients
  $\rightarrow$ construct $|\mathcal{M}|^2$ up to *next-to-soft* approximation

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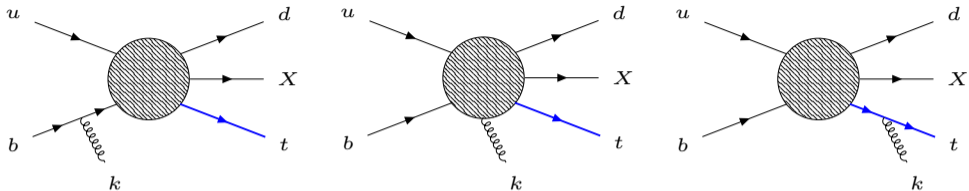
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$$|\mathcal{M}|^2 = -J^\mu J_\mu F_{\text{LO}}(q_t, p_b, q_d, ...) - J_\mu L^\mu F_{\text{LO}}(q_t, p_b, q_d, ...). \tag{4}$$

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$$J^\mu = J_t^\mu + J_b^\mu, \quad L^\mu = L_t^\mu - L_b^\mu, \quad J_t^\mu = \frac{2q_t + k^\mu}{d_t}, \quad J_b^\mu = \frac{2p_b^\mu - k^\mu}{d_b},$$

$$L_t^\mu = J_t^\mu k^\nu \frac{\partial}{\partial q_t^\nu} - \frac{\partial}{\partial q_{t,\mu}}, \quad L_b^\mu = J_b^\mu k^\nu \frac{\partial}{\partial p_b^\nu} + \frac{\partial}{\partial p_{b,\mu}}$$
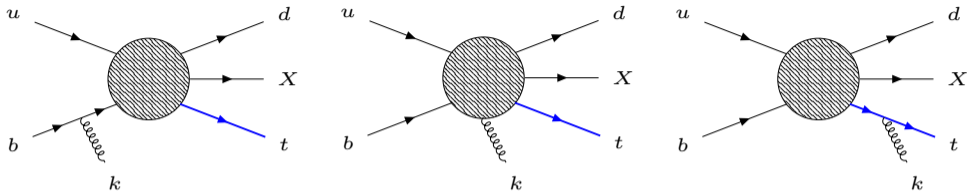
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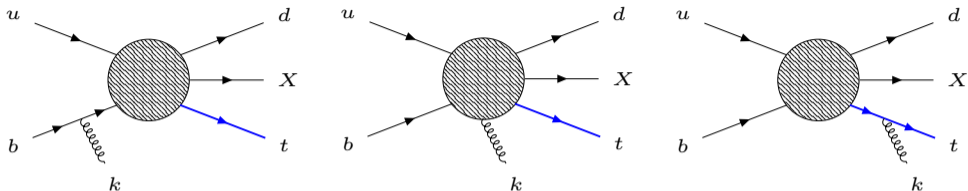
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- $|\mathcal{M}|^2$ in the next-to-soft approximation can be related to the leading-order F_{LO} in a *process-independent* manner

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- perform momentum mapping:

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d, \qquad q_d = p_d - \frac{p_t k}{p_t p_d} p_d \tag{5}$$

$$q_t + q_d + k + p_X = p_t + p_d + p_X \tag{6}$$

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- can now write

$$\text{dLips}(p_u, p_b; q_d, q_t, p_X, k) = \text{dLips}(p_u, p_b; p_d, p_t, p_X) \frac{d^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times$$
$$\times \left[1 + \frac{p_d k}{p_t p_d} - \frac{p_t k}{p_t p_d} \right] + \mathcal{O}(\lambda^2) \tag{7}$$
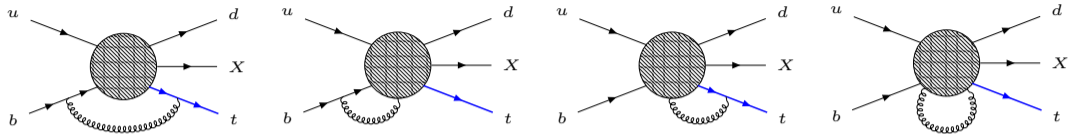
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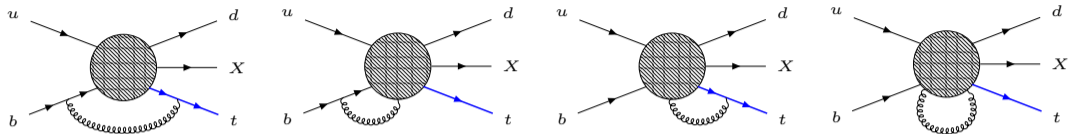
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$$\delta[\mathcal{MM}^+]_{\text{virt}} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2} \left[2J_t^\alpha J_{b,\alpha} F_{\text{LO}} + J_t^\alpha J_{b,\alpha} k^\mu D_{p,\mu} F_{\text{LO}} - (J_t^\alpha + J_b^\alpha) D_{p,\alpha} F_{\text{LO}} \right.$$
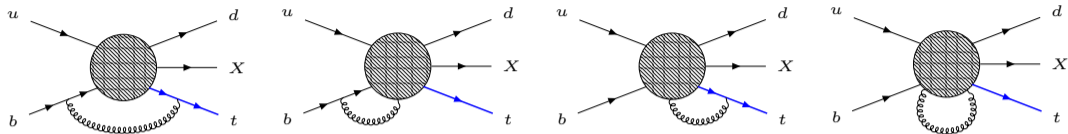
$$\left. + J_t^\alpha \text{Tr} \left[(D_{p,\alpha} \not{p}_t) \mathsf{N} \not{p}_b \bar{\mathsf{N}} \right] + J_b^\alpha \text{Tr} \left[(\not{p}_t + m_t) \mathsf{N} (D_{p,\alpha} \not{p}_b) \bar{\mathsf{N}} \right] \right].$$

(8)

$$D_p^\mu = \frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}}$$

Real & virtual contributions

- we can now integrate out gluon momentum and obtain for the real and virtual contributions:

$$
\begin{aligned}
\mathcal{T}_\lambda\left[\sigma_\mathrm{R}\right] =& \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \mathrm{dLips}_{\mathrm{LO}} \Bigg[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) \\
&- \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \Bigg] F_{\mathrm{LO}}.
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\mathcal{T}_\lambda\left[\sigma_V\right] =& - \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \mathrm{dLips}_{\mathrm{LO}} \Bigg[\mathrm{Tr}\left[\slashed{p}_t \mathsf{N} \slashed{p}_b \bar{\mathsf{N}} \right] \\
&+ \left(\frac{2 p_t p_b - m_t^2}{p_t p_b} - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right) F_{\mathrm{LO}} \Bigg],
\end{aligned}
\tag{10}
$$

Renormalisation contribution

- we need to renormalise the heavy-quark wavefunction and mass in the on-shell scheme:

$$
\begin{aligned}
Z_m =& 1 + \frac{C_F g_s^2 m_t^{-2\epsilon} \Gamma(1+\epsilon)}{(4\pi)^{d/2}} \left[-\frac{3}{\epsilon} - 4 + \frac{2\pi\lambda}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right) \right], \\
Z_2 =& 1 + \frac{C_F g_s^2 m_t^{-2\epsilon} \Gamma(1+\epsilon)}{(4\pi)^{d/2}} \left[-\frac{1}{\epsilon} - 4 + 4\ln\frac{m_t}{\lambda} + \frac{3\lambda\pi}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right) \right].
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\end{aligned}
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$$

- the renormalisation contribution reads

$$
\begin{aligned}
\mathcal{T}_\lambda\left[\sigma_{\mathrm{ren}}\right] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \mathrm{dLips}_{\mathrm{LO}} \Bigg[& \frac{3}{2} F_{\mathrm{LO}} + m_t \mathrm{Tr}\left[(\slashed{p}_t + m_t)\frac{\partial \mathsf{N}}{\partial m_t}\slashed{p}_b\bar{\mathsf{N}} \right] \\
& + m_t \mathrm{Tr}\left[(\slashed{p}_t + m_t)\mathsf{N}\slashed{p}_b\frac{\partial \bar{\mathsf{N}}}{\partial m_t} \right] \Bigg].
\end{aligned}
\tag{12}
$$

Top quark mass scheme redefinition

- we now perform a mass scheme redefinition from pole-mass scheme to short-distance mass scheme

$$m_t = \tilde{m}_t \left(1 - \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \right). \tag{13}$$

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$$\sigma_{\mathrm{LO}}(m_t) - \sigma_{\mathrm{LO}}(\tilde{m}_t) = \delta\sigma_{\mathrm{mass}}^{\mathrm{expl}} + \delta\sigma_{\mathrm{mass}}^{\mathrm{impl}} = \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \int \mathrm{dLips}_{\mathrm{LO}} \times$$

$$\times \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\mathrm{LO}} - m_t \mathrm{Tr} \left[1 \mathsf{N} \not{p}_b \bar{\mathsf{N}} \right] \tag{14}$$

$$- m_t \mathrm{Tr} \left[(\not{p}_t + m_t) \left(\frac{\partial \mathsf{N}}{\partial m_t} \not{p}_b \bar{\mathsf{N}} + \mathsf{N} \not{p}_b \frac{\partial \bar{\mathsf{N}}}{\partial m_t} \right) \right] \right].$$

Combined contributions

- combining all contributions, we obtain in the short-distance mass scheme:

$$\delta\sigma_{\mathrm{NLO}} = \sigma_R + \sigma_V + \sigma_{\mathrm{ren}} + \delta\sigma_{\mathrm{mass}}^{\mathrm{expl}} + \delta\sigma_{\mathrm{mass}}^{\mathrm{impl}} \qquad (15)$$

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- what about observables, e.g. kinematic distributions, ... ?

Kinematic observables

- observable that depends on the momentum of the top quark

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- now compute $\mathcal{T}_\lambda[O_X]$ and obtain for the observable

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- for observables, linear power corrections **do not vanish** within the short-distance mass scheme
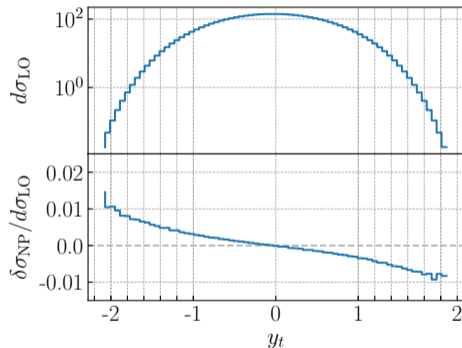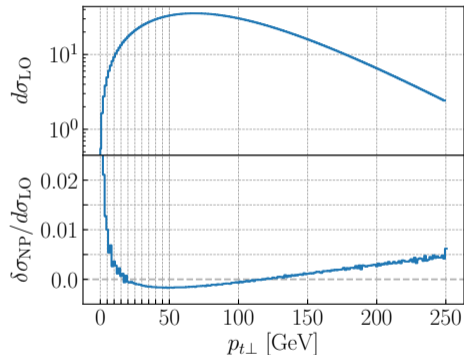
Top quark pair production at the Tevatron

- the dominant channel for $t\bar{t}$ production at the Tevatron is via $q\bar{q}$ fusion

- tree-level diagram contains gluon but it is **off-shell**

- can generalise previous calculation to dipole pairs

- for total cross-sections, there are **no linear power corrections** for $t\bar{t}$ production type processes within the short-distance mass scheme

- for observables, linear power corrections **do not vanish** within the short-distance mass scheme
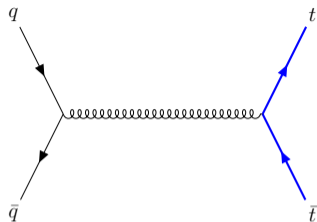
Top quark pair production at the Tevatron: $p_{t\perp}$, y_t, $s_{t\bar{t}}$
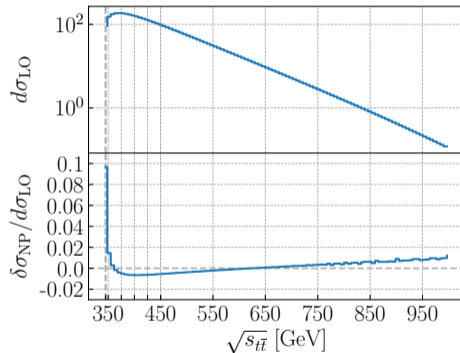
- it is now straightforward to compute the linear shift for $p_{t\perp}$, y_t and $s_{t\bar{t}}$

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Part III

Conclusions & Outlook

Summary

- improved understanding of analytic structure of linear power corrections

- computed linear power corrections for event-shape variables (C-parameter, thrust)

- for top quark production processes, **there are <u>no</u> linear power corrections** to total cross-sections within the short-distance mass scheme ($\overline{\text{MS}}$-scheme)

- however for observables ($p_{t\perp}$, y_t, $s_{t\bar{t}}$), **there <u>are</u> linear power corrections** within the short-distance mass scheme ($\overline{\text{MS}}$-scheme)

- computed expressions for observables, e.g. $p_{t\perp}$, y_t and $s_{t\bar{t}}$, ...

- similar results and conclusions for other (abelian) type processes ...
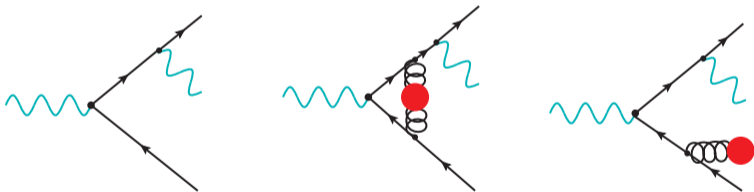
Thank you for attention!

Backup

non-perturbative physics: Renormalons

- Renormalon model identifies simple class of diagrams that dominate in the large n_f limit

  [Beneke, Braun, Dokshitzer, Marchesini, Smye, Webber, etc.]
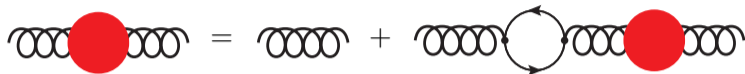
- example: 3-jet event $Z^*/\gamma^* \to q\bar{q}\gamma$

- each diagram can be computed perturbatively,

$$d\sigma = d\sigma^{(0)} + \left(\frac{\alpha_s}{\pi}\right) d\sigma^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 n_f \, d\sigma^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 n_f^2 \, d\sigma^{(3)} + ... \quad (20)$$

non-perturbative physics: Renormalons

- can resum leading-n_f contributions via integral

$$\int_0^Q dk \, k^{p-1} \alpha_s(k) = \alpha_s(Q) \, Q^p \sum_{n=0}^\infty \underbrace{\left(\frac{\beta_0}{2\pi} \alpha_s(Q)\right)^n \frac{1}{p^{n+1}} n!}_{\text{factorial growth}}, \tag{21}$$

$$\text{with} \quad \alpha_s(\mu) = \frac{1}{\frac{\beta_0}{2\pi} \log \frac{\mu}{\Lambda_{\text{QCD}}}}, \quad \beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f. \tag{22}$$

- series is not Borel summable, ambiguity given by

$$\int dk \, k^{p-1} \frac{2\pi}{\beta_0} \frac{\Lambda_{\text{QCD}}}{k - \Lambda_{\text{QCD}}} = \pm 2\pi i \frac{2\pi}{\beta_0} \Lambda_{\text{QCD}}^p$$

$\rightarrow$ ambiguity removed by non-perturbative power corrections $\Lambda_{\text{QCD}}^p / Q^p$

Massless case: presence or absence of linear power correction

For processes with *massless* quarks:

- three-jet events,
- ...,

one can make following statements on presence or absence of linear power corrections:

[Caola, Ferrario Ravasio, Limatola, Melnikov, Nason, JHEP 01 (2022) 093]

- *virtual* corrections **do not** induce linear corrections

- *real* corrections
  - hard region **does not** induce linear corrections
  - <u>soft</u> radiation at next-to-soft approximation may lead to linear corrections
    - <u>condition</u>: observable $\mathcal{O}_{\mathcal{O}(k)}$ must exhibit non-analytic dependence on λ,
      e.g. event-shape variables, ...

C-parameter: linear power corrections

linear power corrections to C-parameter: [F. Caola et al, JHEP 12 (2022) 062]

$$\mathcal{T}_\lambda[I_C] = \frac{15}{128\pi} \frac{s_{12}^3}{1-z_3} \left(\frac{\lambda}{q}\right) \left[\frac{(1+z_3)}{2} K\left(c_{12}^2\right) - (1-z_1 z_2) E\left(c_{12}^2\right)\right] \tag{23}$$
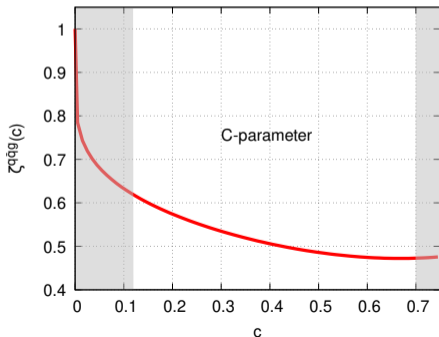
$\rightarrow$ find agreement previous with results (2-jet limit, ...)

$\rightarrow$ have **analytic** results for entire 3-jet region and these are superior to numerical methods

$\rightarrow$ could be used for α_s determinations

[P. Nason, G. Zanderighi, JHEP 06 (2023) 058]

Top quark pair production at the Tevatron: $p_{t\perp}$, y_t, $s_{t\bar{t}}$

- it is now straightforward to compute the linear shift for $p_{t\perp}$, y_t and $s_{t\bar{t}}$

$$\frac{\delta_{\mathrm{NP}}[p_{t\perp}]}{p_{t\perp}} = \frac{\alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \frac{(2C_F - C_A\tau)}{2(1-\tau)} \qquad \text{with } \tau = \frac{4m_t^2}{s_{t\bar{t}}} \text{ and } \lambda\alpha_s \sim 0.3 \text{ GeV}$$

$$\delta_{\mathrm{NP}}[y_t] = \frac{\alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \left[(3C_A - 8C_F)\,\tau\cosh^2 y_t - (C_A - 2C_F)\frac{\tau(2-\tau)}{4(1-\tau)}\sinh(2y_t) \right]$$

$$\frac{\delta_{\mathrm{NP}}[s_{t\bar{t}}]}{s_{t\bar{t}}} = \frac{\alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \left[2C_F(1-\tau) - C_A\,\tau\cosh(2y_t) + (3C_A - 8C_F)\sinh(2y_t) \right]$$

- can now compute non-perturbative shift at Tevatron ($\sqrt{s} = 1.8$ TeV)