Generalized parton distributions from lattice QCD and experimental data

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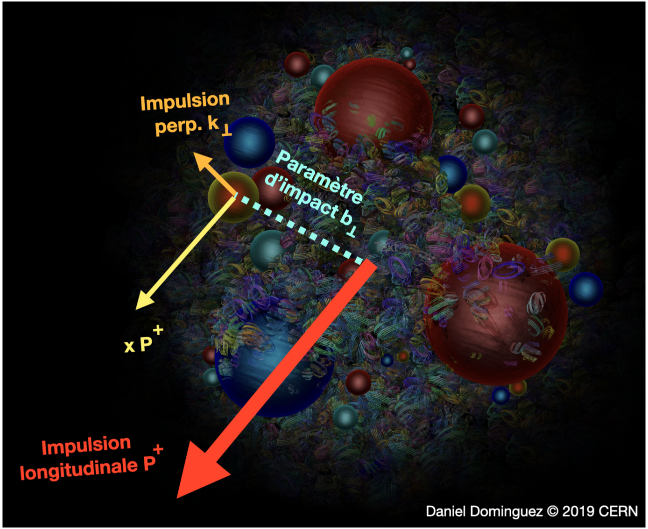
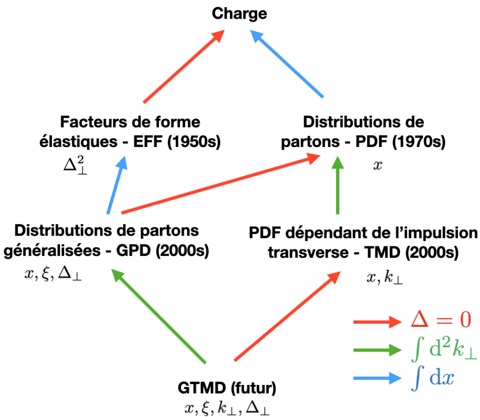
Overview

1. **Limits of GPD phenomenology from experimental data**
2. The Hadstruc GPD calculation on the lattice: **arXiv:2405.10304**
3. Perspectives

Limits of GPD phenomenology from experimental data





[Lorcé, Pasquini, Vanderhaeghen, 2011]

Generalized parton distributions (GPDs): [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle P_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| P_1 \right\rangle\Big|_{z_\perp=0,\ z^+=0}$$
$$= \frac{1}{2P^+}\bar{u}(P_2)\left(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right)u(P_1)$$

$$\Delta = P_2 - P_1\,,\quad t = \Delta^2\,,\quad P = \frac{1}{2}(P_1+P_2)\,,\quad \xi = \frac{P_1^+ - P_2^+}{P_1^+ + P_2^+} = -\frac{\Delta^+}{2P^+}$$

- 3D (x,ξ,t) vs 1D for PDFs
- more GPDs than PDFs
- $\to$ need to measure more (exclusive vs inclusive)
- $\to$ current GPD extractions are not data-driven

- **Hadron tomography** [Burkardt, 2003]:

$$I(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} H^q(x, \xi = 0, t = -\Delta_\perp^2)$$

- **Gravitational form factors** [Polyakov, 2003],[Lorcé et al, 2017]: radial energy / pressure

$$\langle P_2 | T_a^{\mu\nu} | P_1 \rangle = \bar{u}(P_2) \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu} \bar{C}_a(t)$$

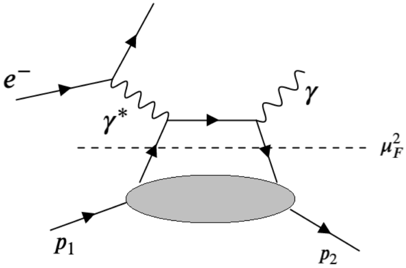
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M} [A_a(t) + B_a(t)] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t) \Bigg\} u(P_1)$$

$$\int_{-1}^{1} \mathrm{d}x \, x \, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2)$$

- **Proton's spin decomposition** [Ji, 1997]:

$$\frac{1}{2} = \sum_q \frac{1}{2} \int_{-1}^{1} dx \, x \left[H^q + E^q \right] \Big|_{t=0} + \frac{1}{2} \int_{-1}^{1} dx \left[H^g + E^g \right] \Big|_{t=0}$$

The problem with deeply virtual Compton scattering (DVCS)



"missing" variable:

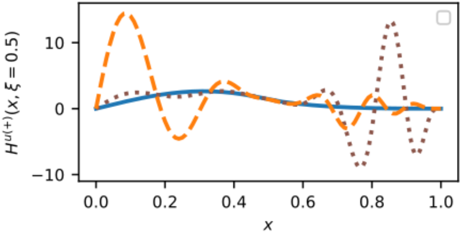
$$\text{experimental kinematics } \xi, t, [Q^2] \qquad \text{vs desired reconstruction} \qquad x, \xi, t, [\mu^2]$$

Can perturbative evolution bypass the issue? [Freund, 2000]

DVCS observables parametrized in terms of Compton form factors (CFFs) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \frac{H^a(x, \xi, t, \mu^2)}{|x|^{p_a}}$$

A (caricatural) case of uncertainty propagation at NLO: one needs to measure DVCS over a range from 1 to 100 GeV^2 with 10^{-5} relative accuracy to make the difference between those GPDs. [Bertone, HD, Mezrag, Moutarde, Sznajder, 2021]



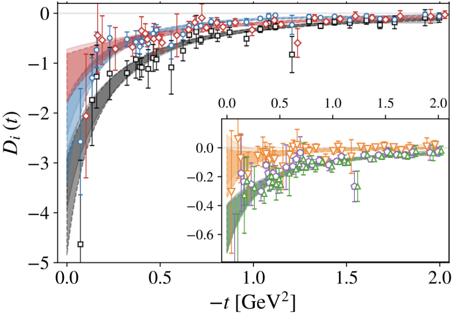
More specific studies:

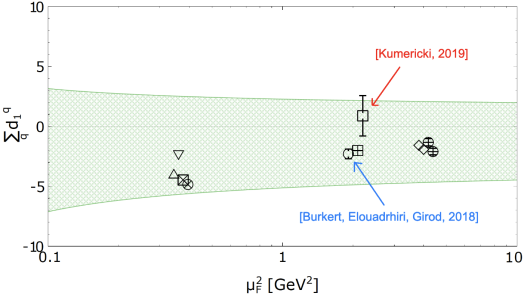
- using positivity [Pire, Soffer, Teryaev, 1998] and NNs in [HD, Grocholski, Moutarde, Sznajder, 2021]
- in the small x_B regime (Shuvaev transform) [HD, Winn, Bertone, 2023]

The GFFs can be accessed from the local matrix element

$$\langle P_2|\bar{\psi}D^{\{\mu}\gamma^{\nu\}}\psi|P_1\rangle$$

Calculations on the lattice have been available for 20 years **[Hagler, 2003]**. Recent result at $m_\pi = 170$ MeV **[Hackett, Pefkou, Shanahan, 2023]** vs experimental extraction **[HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, 2021]**





Overview

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On the lattice, no light-like separation available. Among several proposals:

- Large-momentum effective theory (LaMET) **[Ji, 2013]**: use a large hadron boost $P \to \infty$ to approach the light-like separation
- Short-distance factorization **[Radyushkin, 2017]**: use the OPE of non-local space-like operators in the limit $z^2 \to 0$.

Different strategy to approach the light-cone means different nature of power corrections

$$\langle P|\bar{\psi}(z)\gamma^\mu\psi(0)|P\rangle = P^\mu \mathcal{M}_\nu(\nu = P\cdot z, z^2) + z^\mu \mathcal{N}_\nu(\nu = P\cdot z, z^2)$$

$$\mathcal{M}_\nu(\nu, z^2) = \int_{-1}^{1} d\alpha\, C(\alpha, \mu^2 z^2)\mathcal{Q}(\alpha\nu, \mu^2) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

$$\mathcal{Q}(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{-i\nu x} f(x, \mu^2)$$

An observation

Ioffe-time GPDs are the best GPDs! **[Braun, Gornicki, Mankiewicz, 1995]**

x-dependent GPDs
non analytical form
triangular for x > xi (only needs larger momentum fractions)
needs all x range for x < xi

$$\frac{d}{d\ln(\mu^2)} H^q(x,\xi,\mu^2) = \int_0^1 \frac{dy}{y}\, C\left(\frac{x}{y},\frac{\xi}{x},\alpha_s(\mu^2)\right) H^q\left(y,\xi,\mu^2\right)$$

$$C\left(\alpha,\frac{\xi}{x},\alpha_s(\mu^2)\right) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left\{\theta(1-\alpha)\left[\left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \mathcal{O}(\xi^2)\right] + \theta(x\leq\xi)\left[-\left(\frac{1}{1-y}\right)_{++} + \ldots\right]\right\}$$

Ioffe-time GPDs
simple analytical form
triangular (only needs smaller Ioffe time range)

$$\frac{d}{d\ln(\mu^2)} H^q(\nu,\xi,\mu^2) = \int_0^1 d\alpha\, K(\alpha,\xi\nu,\alpha_s(\mu^2)) H^q(\alpha\nu,\xi,\mu^2)$$

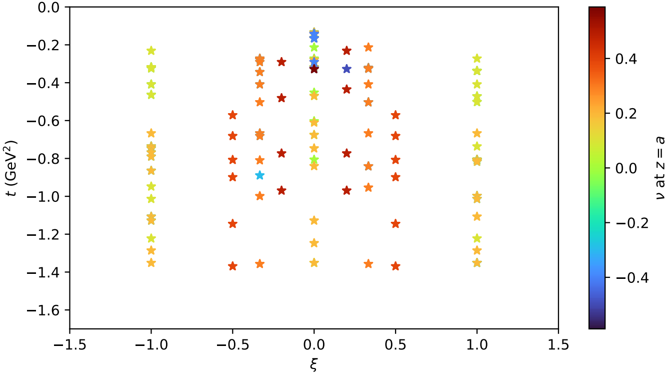
$$K(\alpha,\xi\nu,\alpha_s(\mu^2)) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left[\left(\frac{2\alpha}{1-\alpha}\right)_+ \cos\left(\bar{\alpha}\xi\nu\right) + \frac{\sin\left(\bar{\alpha}\xi\nu\right)}{\xi\nu} - \frac{\delta\left(1-\alpha\right)}{2}\right]$$

Conformal moments
simple analytical form
diagonal (each moment evolves independently)

$$O_n(\xi,\mu^2) \propto \int_{-1}^1 dx\, C_n^{(3/2)}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu^2)$$

$$O_n(\xi,\mu^2) = O_n(\xi,\mu_0^2)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_n/(2\pi\beta_0)}$$

ID	a (fm)	m_π (MeV)	β	$m_\pi L$	$L^3 \times N_T$	N_{cfg}	N_{srcs}	rk $(\mathcal{D})$
a094m358	0.094(1)	358(3)	6.3	5.4	$32^3 \times 64$	348	4	64



polynomiality of moments of GPDs (non-singlet case):

$$\int_{-1}^{1} x\, x^{n-1} H(x, \xi, t) = \sum_{k=0 \text{ even}}^{n-1} A_{n,k}(t)\, \xi^k$$

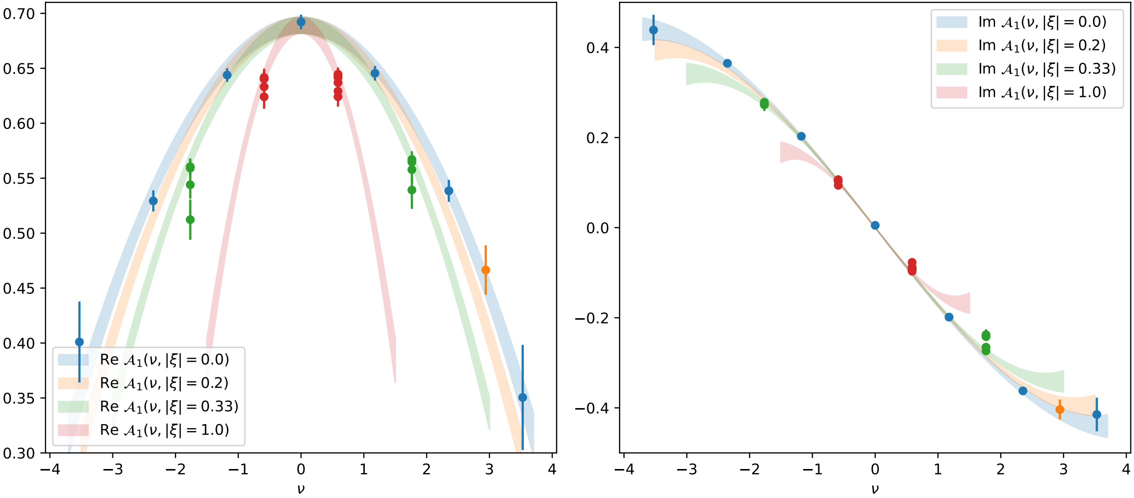
$$A_{1,0}(t) = F_1(t) \quad \text{(elastic form factor)}$$

Hence small Ioffe-time behavior:

$$\begin{aligned} H(\nu, \xi, t) &= \int dx\, e^{-ix\nu} H(x, \xi, t) \\ &= F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + \ldots + \text{power corrections} \end{aligned}$$

With momenta up to 1.4 GeV used in this study, we have signal up to $A_{4,0}$ and $A_{4,2}$.

$$\text{Dipole fit:} \;\; A_{n,k}(t) = A_{n,k}(t=0) \left(1 - \frac{t}{\Lambda_{n,k}^2}\right)^{-2}$$


0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
−4 −3 −2 −1 0 1 2 3 4
ν
Re $\mathcal{A}_1(\nu, |\xi| = 0.0)$
Re $\mathcal{A}_1(\nu, |\xi| = 0.2)$
Re $\mathcal{A}_1(\nu, |\xi| = 0.33)$
Re $\mathcal{A}_1(\nu, |\xi| = 1.0)$
0.4
0.2
0.0
−0.2
−0.4
−4 −3 −2 −1 0 1 2 3 4
ν
Im $\mathcal{A}_1(\nu, |\xi| = 0.0)$
Im $\mathcal{A}_1(\nu, |\xi| = 0.2)$
Im $\mathcal{A}_1(\nu, |\xi| = 0.33)$
Im $\mathcal{A}_1(\nu, |\xi| = 1.0)$

Our results

Pion mass = 0.36 GeV - Proton mass = 1.12 GeV
No continuum limit - signs of discretization errors / light-cone uncertainty
Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

GPD H^{u-d}

$A_{1,0}$	0.97(2)	
$A_{2,0}$	0.204(4)	
$A_{3,0}$	0.062(4)	$A_{3,2}$: 0.42(7)
$A_{4,0}$	0.06(1)	$A_{4,2}$: 0.5(2)

GPD E^{u-d}

$B_{1,0}$	3.44(4)	
$B_{2,0}$	0.36(2)	
$B_{3,0}$	0.07(2)	$B_{3,2}$: 0.9(6)
$B_{4,0}$	0.06(4)	$B_{4,2}$: 1.2(9)

Dipole mass (GeV)

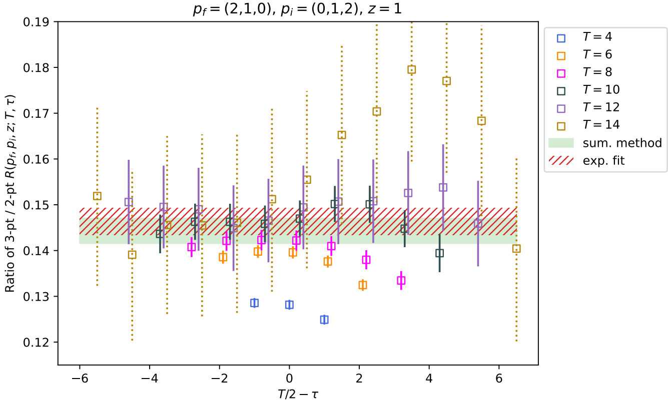
GPD H^{u-d}

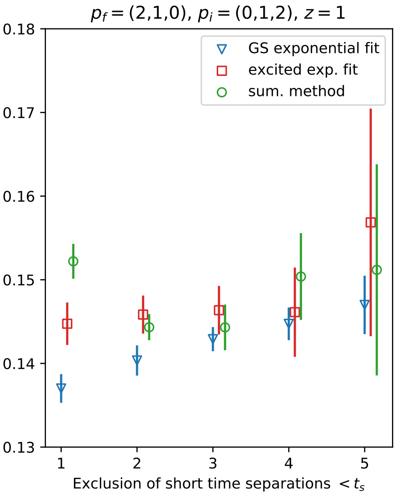
$A_{1,0}$	1.25(2)	
$A_{2,0}$	1.86(6)	
$A_{3,0}$	2.2(4)	$A_{3,2}$: 1.07(9)
$A_{4,0}$	Unreliable	$A_{4,2}$: 1.2(2)

GPD E^{u-d}

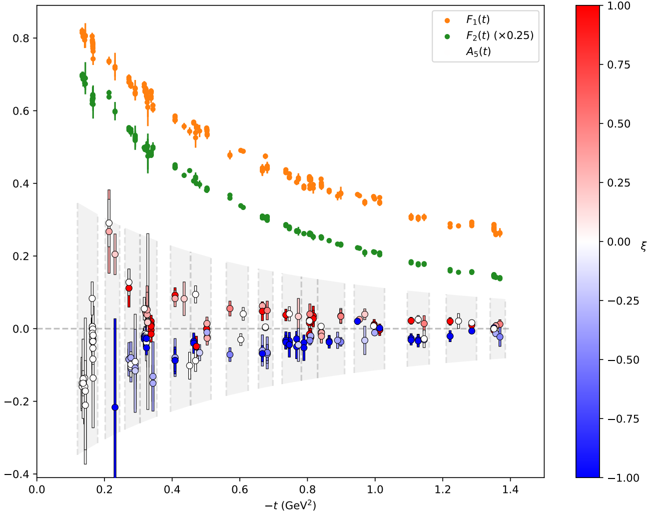
$B_{1,0}$	0.982(6)	
$B_{2,0}$	1.41(8)	
$B_{3,0}$	2.4(9)	$B_{3,2}$: 1.0(3)
$B_{4,0}$	Unreliable	$B_{4,2}$: 1.1(2)

Excited state contamination

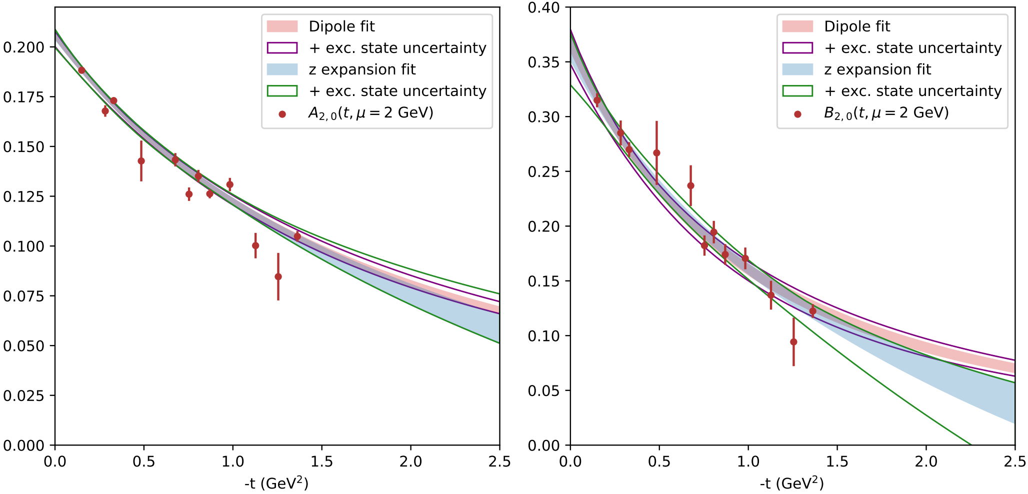




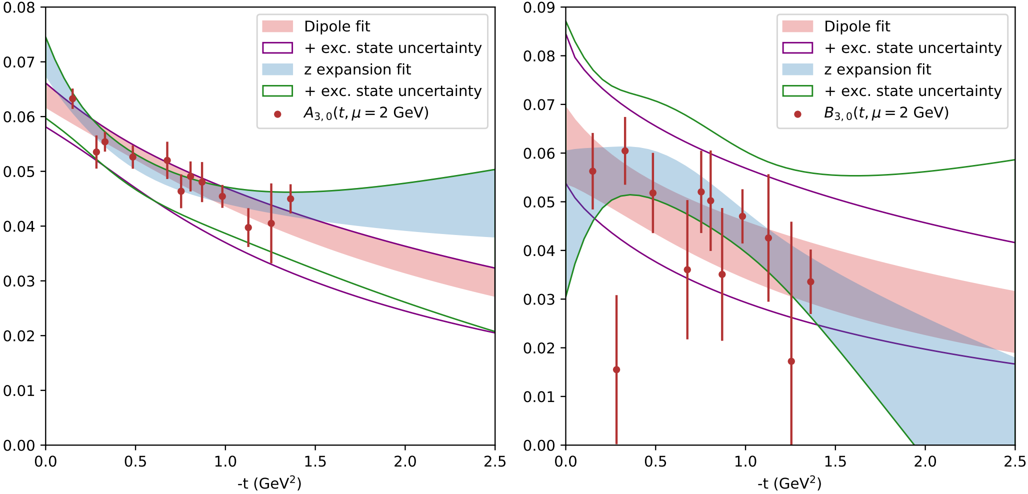
$$\langle p'|\bar{\psi}^q\gamma^\mu\psi^q|p\rangle\Big|_{z=0} = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} + A_5^q(t)\frac{\Delta^\mu}{2m}\right]u(p)$$

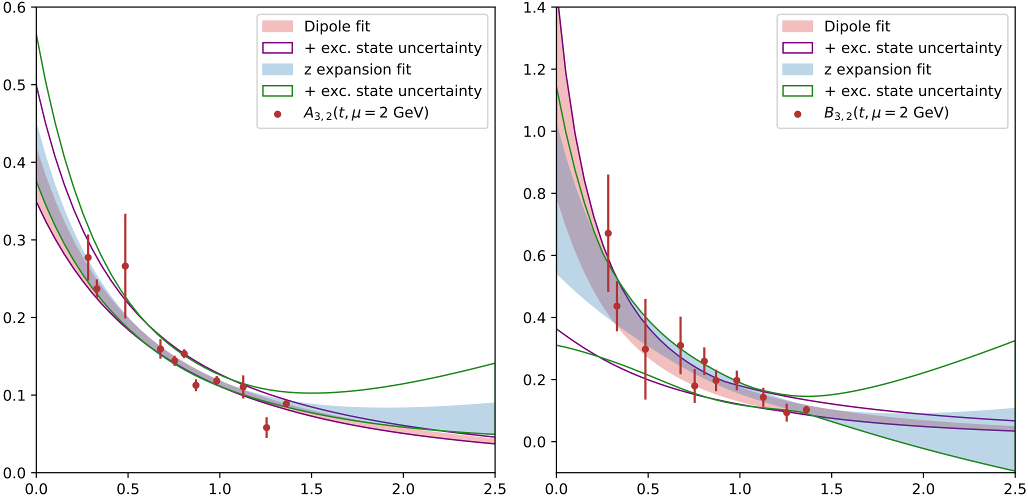


probable sign of lattice discretization in A_5 + enhanced sensitivity to excited state contamination

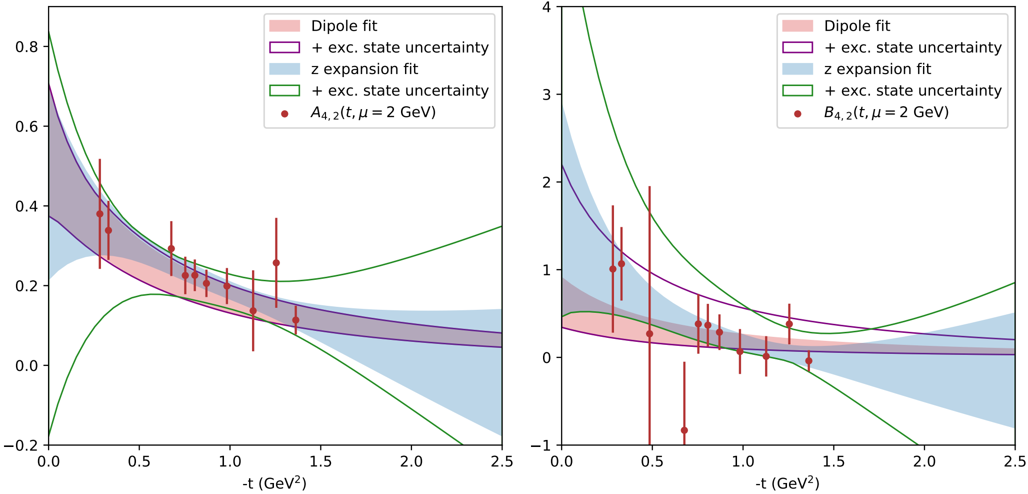


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{2,0}(t, \mu = 2 \text{ GeV})$
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{2,0}(t, \mu = 2 \text{ GeV})$
-t (GeV^2)


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,0}(t, \mu = 2\text{ GeV})$
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{3,0}(t, \mu = 2\text{ GeV})$

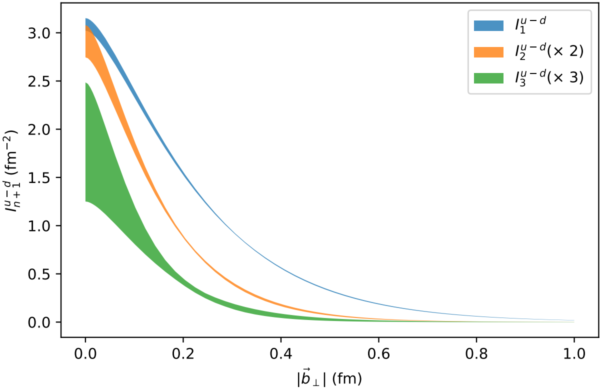


0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,2}(t, \mu = 2$ GeV)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{3,2}(t, \mu = 2$ GeV)


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{4,2}(t, \mu = 2$ GeV)
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{4,2}(t, \mu = 2$ GeV)
-t (GeV^2)

Impact parameter distribution: number density of unpolarized quarks in an unpolarized proton with mom. fraction x and radial distance to the center of longitudinal momentum $\vec{b}_\perp$: **model dependence!**

$$I(x, \vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2} \, e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$



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Perspectives on the lattice

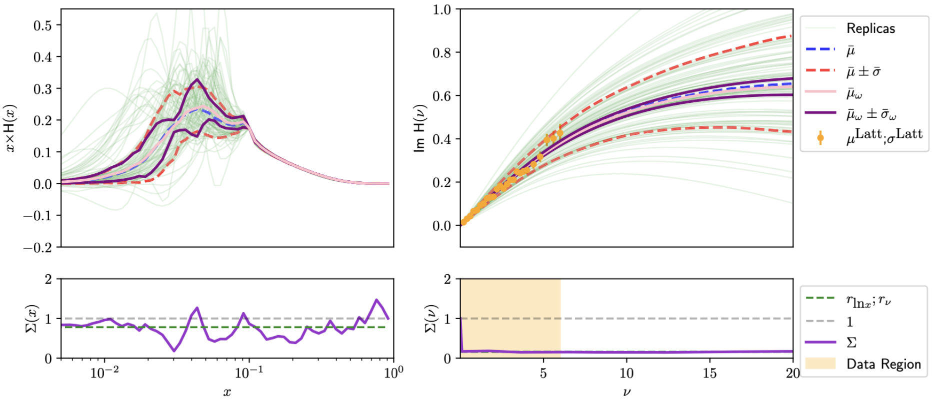
Uncertainties in lattice QCD:

- **Lattice discretization / power corrections**: evidence of issues in the EFFs, needs a continuum limit
- **Excited state / range in Ioffe time**: need a better assurance (GEVP) in order to produce reliable x-reconstruction / high-order moments using large momentum boost
- **Matching uncertainty / power corrections**: proposal to identify a regime of validity of factorization without the use of perturbation theory **[HD, Karpie, Monahan, Orginos, Zafeiropoulos, 2023]**
- **Pion mass / finite volume effects**

A joint lattice - experimental phenomenology

[Riberdy, HD, Mezrag, Sznajder, 2023]

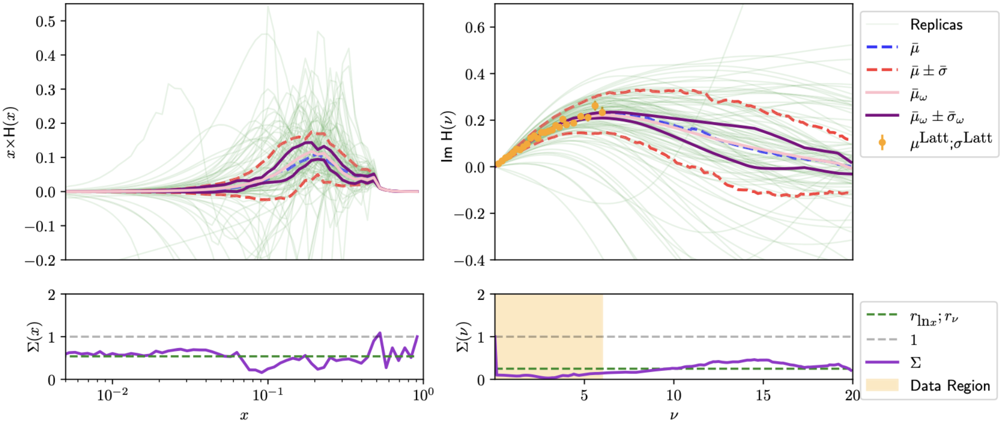
$\xi = 0.1$



A joint lattice - experimental phenomenology

[Riberdy, HD, Mezrag, Sznajder, 2023]

$\xi = 0.5$

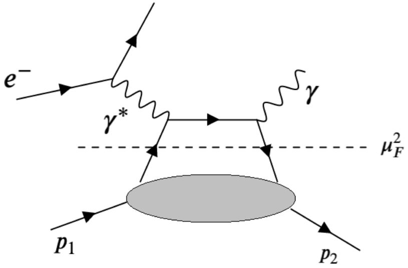


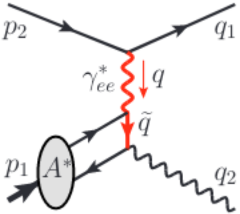
Conclusion

- We have a framework with excellent signal and very large kinematic coverage for GPDs
- There is still a lot of work on purely lattice systematics (but that's ok), power corrections and matching uncertainty is a bigger concern that we start to address
- We have quite a few innovative ideas that could produce considerable improvements, this is not the end even on the technology side, stay tuned!

Thank you for your attention!

The problem with deeply virtual Compton scattering (DVCS)





[Qiu, Yu, 2022]

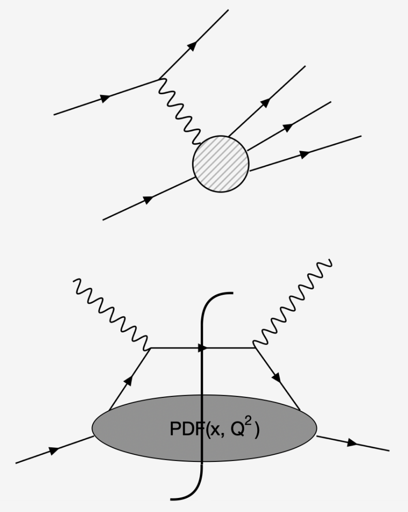
$$\tilde{q}^2 = \frac{Q^2 + q_2^2}{2\xi}\left[x - \xi\left(\frac{1 - q_2^2/Q^2}{1 + q_2^2/Q^2}\right)\right] + \mathcal{O}(t/Q^2)$$

real photon $q_2^2 = 0$: x and ξ not entangled with the virtuality Q^2.
Entanglement of x and Q^2 only through perturbative radiation: "missing" variable

$$x_B, t, [Q^2] \qquad \text{vs} \qquad x, \xi, t, [\mu^2]$$

DIS hadronic tensor (one-photon exchange approximation)

$$W^{\mu\nu} = \int d^4z\, e^{iq.z} \langle P| J^\mu(z) J^\nu(0) |P\rangle \propto \sum_X |\mathcal{M}(\gamma^* P \to X)|^2$$



In the Bjorken limit $-q^2 \to \infty$ with $x_B = -q^2/2P\cdot q$ fixed, stationary phase for

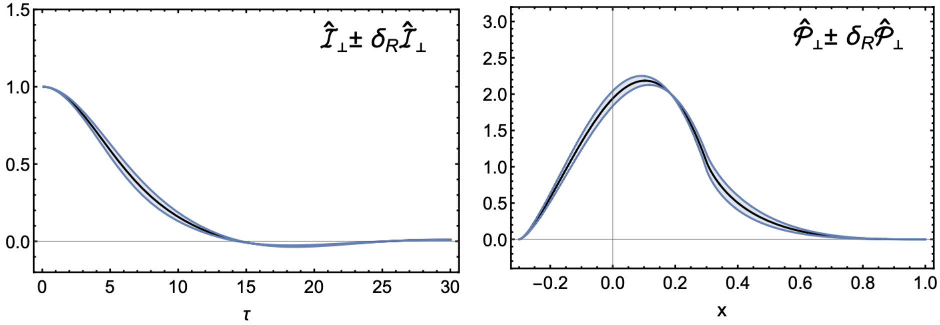
$$z^2 \leq \mathcal{O}(1/Q^2)\,, \quad q\cdot z \approx x_B P\cdot z$$

Hence a DIS-scheme definition of the PDF as:

$$q(x_B) = \int dz^- \; e^{ix_B P^+ z^-} \langle P| J^\mu(z^-) J^\nu(0) |P\rangle$$

Study of the analytical properties of $\mathcal{M}(\gamma^* P \to \gamma^* P)$ gives integrals of a PDF on all x_B in the Bjorken limit can be deformed to $1/x_B \to 0$ and therefore $z \to 0 \to$ **OPE of moments (and other integrals of x) with local operators**

Even with a space-like separation of 1 fm, the power-corrections at $\xi = 0.3$ might be very small (model-dependent estimate of non-perturbative higher-twist contributions through renormalon ambiguity) **[Braun, Koller, Schoenleber, 2024]**:



Situation much more complicated for LaMET formalism, with potentially divergent power-corrections when approaching $x \approx \xi$.

GPD matrix element:

$$\langle P_2|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^\mu\psi\left(\frac{z}{2}\right)|P_1\rangle = \bar{u}(P_2)\Big[\gamma^\mu A_1 + z^\mu A_2 + \sigma^{\mu\nu}z_\nu A_3 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}A_4 + \frac{\Delta^\mu}{2m}A_5 + \frac{i\sigma^{\alpha\beta}z_\alpha\Delta_\beta}{2m}\left(P^\mu A_6 + \Delta^\mu A_7 + z^\mu A_8\right)\Big]u(P_1)$$

Identify the terms that survive in the light-cone limit (non-singlet case):

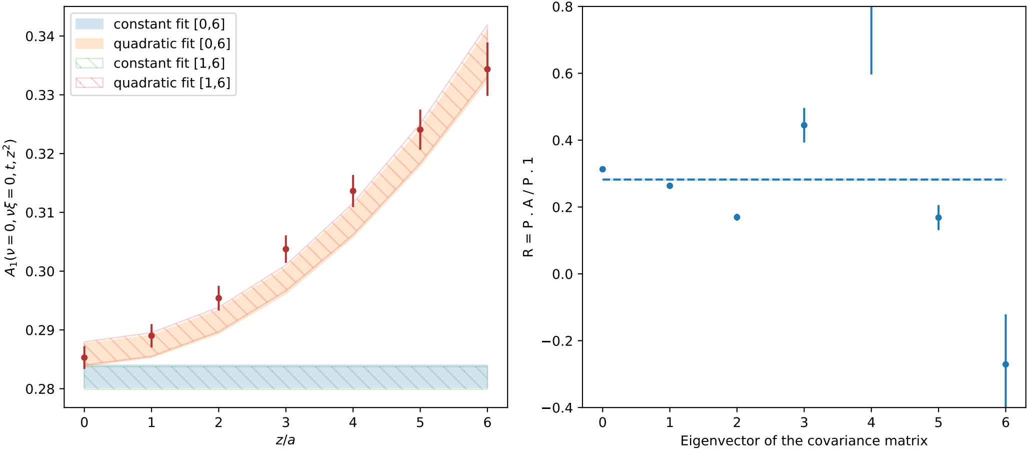
$$H(\nu,\xi,t,z^2) = A_1(\nu,\xi,t,z^2)$$
$$E(\nu,\xi,t,z^2) = A_4(\nu,\xi,t,z^2) + \nu A_6(\nu,\xi,t,z^2) - 2\xi\nu A_7(\nu,\xi,t,z^2)$$

Match to the light-cone limit

$$\begin{pmatrix} H \\ E \end{pmatrix}(\nu,\xi,t,\mu^2) = \int_{-1}^{1} d\alpha C(\alpha,\xi\nu,\mu^2 z^2)\begin{pmatrix} H \\ E \end{pmatrix}(\alpha\nu,\xi,t,z^2) + \text{power corrections}$$

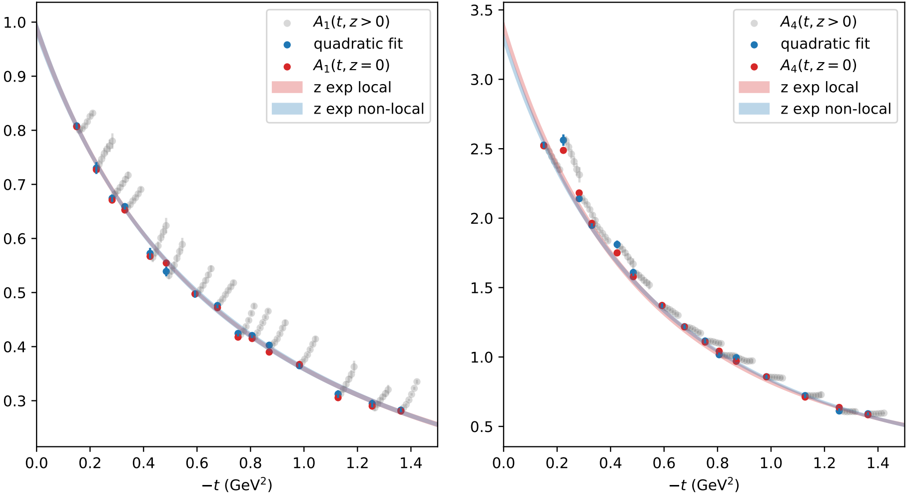
Extracting each amplitude A_k requires to measure matrix elements with various combinations of helicity and gamma structure (kinematic matrix inversion)

If $p_{f,z} = p_{i,z} = 0$, then $\nu = 0$ and $\nu\xi = 0$, so we have non-local data with signal only of the EFF

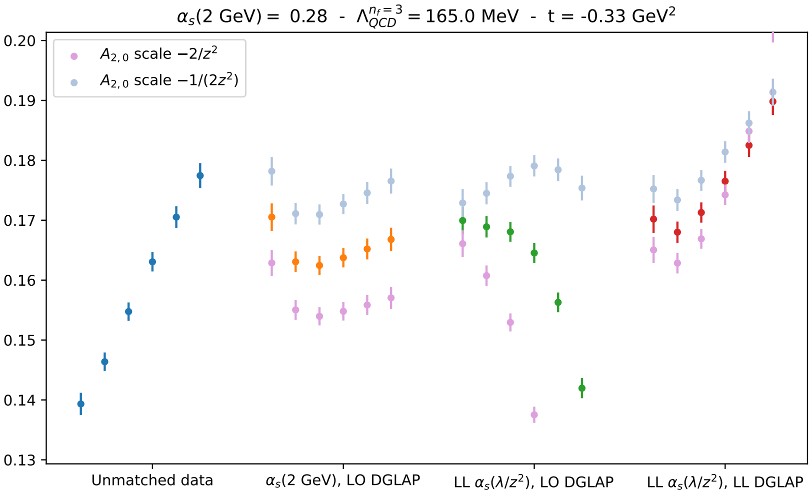


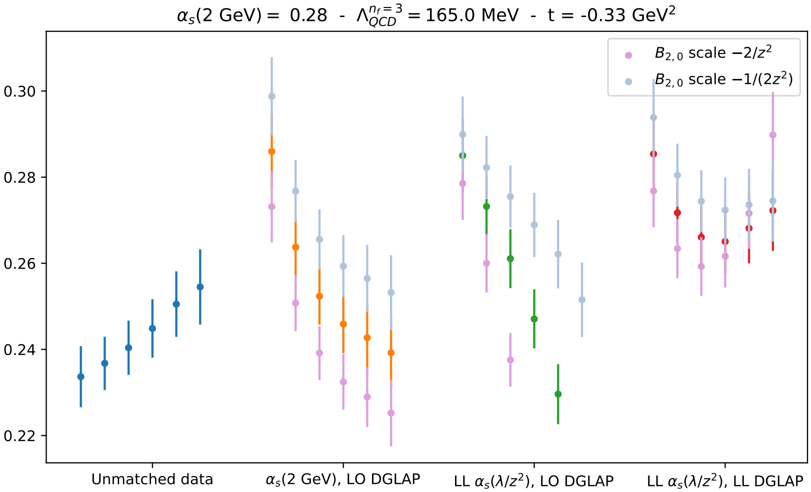
Candidates: lattice discretization + power corrections?

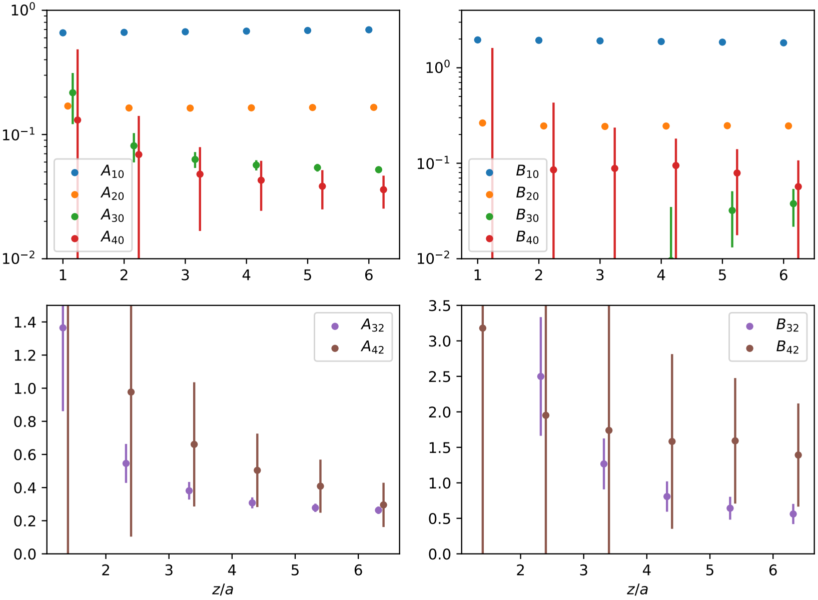
Full non-local EFF extraction perfectly compatible with the local one (with excited state uncertainty + binning).



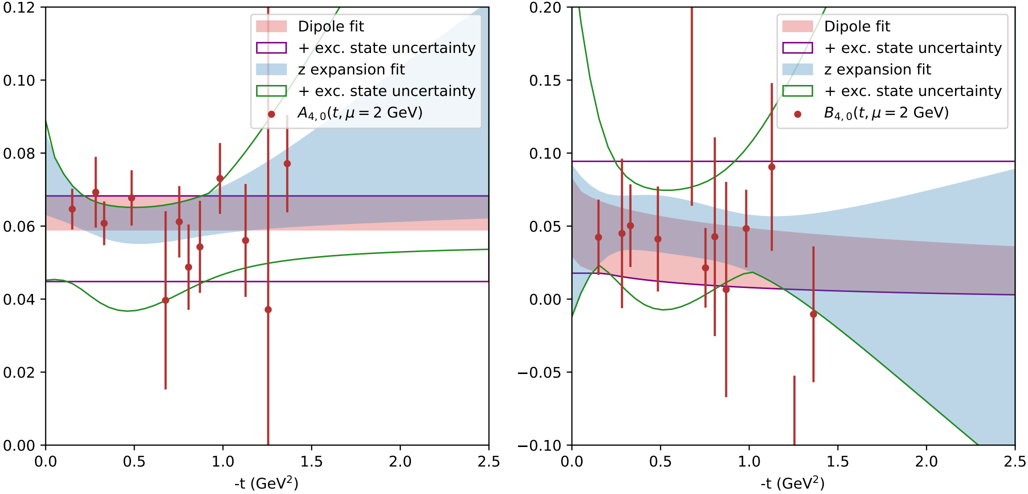
Perturbative matching uncertainty: if there is a strong leading-twist dominance up to separations of 1 fm, what is the perturbative matching kernel worth in this region? also the curse of precision




$\alpha_s(2\text{ GeV}) = 0.28$ - $\Lambda_{QCD}^{n_f=3} = 165.0$ MeV - t = -0.33 GeV^2
$B_{2,0}$ scale $-2/z^2$
$B_{2,0}$ scale $-1/(2z^2)$
0.30
0.28
0.26
0.24
0.22
Unmatched data
$\alpha_s(2\text{ GeV})$, LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LL DGLAP



10^0
10^{-1}
10^{-2}
A_{10}
A_{20}
A_{30}
A_{40}
B_{10}
B_{20}
B_{30}
B_{40}
A_{32}
A_{42}
B_{32}
B_{42}
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
2
3
4
5
6
z/a
z/a


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{4,0}(t, \mu = 2$ GeV)
$B_{4,0}(t, \mu = 2$ GeV)
-t (GeV^2)
-t (GeV^2)