Random matrices and Free probability

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IMT

Programme Scientifique

Marwa Banna
Titre: Notions of Non-Commutative Independence
Résumé: In this talk, I will start by illustrating matrix models relative to free, infinitesimal free, monotone, and c-free independencies. Notions of independence play a key role in studying joint distributions of non-commutative random variables and hence in studying limiting distributions of the associated random matrix models. I will illustrate in particular recent results relative to the monotone case.
Just as in the classical setting, to each notion of independence corresponds a central limit theorem. The second part of the talk will focus on the operator-valued setting, where I show quantitative results for the relative operator-valued central limit theorems.
Based on joint works with Arizmendi, Gilliers, Mai, & Tseng.

Denis Bernard
Title: Structured random matrices and cyclic cumulants : a free probability approach (inspired by noisy many-body quantum systems)
Abstract:
We shall discuss a new class of large structured random matrices characterised by the properties of their cyclic cumulants. This class is remarkably stable under non-linear operations. We shall present a simple algorithm, based on an extremization problem, to compute the spectrum of sub-blocks of such matrices, and explain the connection between such algorithm and operator valued free probability. This class of random matrices emerged via the study of the quantum symmetric simple exclusion process or, more generally, noisy many-body quantum systems.
[Work done in collaboration with Ludwig Hruza].

Philippe Biane
Free cumulants everywhere
abstract:
I will give examples of free cumulants appearing in various questions such as matrix integrals, map enumeration, characters of symmetric groups, braid enumeration and a quantum version of the simple exclusion process.

Charles Bordenave
Title: Large deviations for macroscopic observables of heavy-tailed matrices
Abstract: This is an ongoing joint work with Alice Guionnet and Camille Male. We consider a finite collection of Hermitian heavy-tailed random matrices of dimension N. Our model include the Lévy matrices introduced by Bouchaud and Cizeau or sparse random matrices with O(1) non-zeroes entries per row. When represented as weighted graphs on N vertices, these matrices have local weak limits in the Benjamini-Schramm topology. Thanks to this representation, we establish large deviations principle for macroscopic observables of such collection of matrices. These observable include the empirical distribution of the eigenvalues and empirical distribution of the neighborhood distribution.

Gaëtan Borot
Freeness to all orders
I will describe a theory of free cumulants to all orders (g,n), both at the level of combinatorics (surfaced permutations) and generating

series (higher R-transform machinery). Freeness up to order (g,n) is then defined by the additivity of free cumulants up to order (g,n). (0,1) is the usual freeness, (1/2,1) is infinitesimal freeness, (0,n) is the n-order freeness of Collins-Mingo-Speicher-Sniady. This theory is adapted to address all-order (in particular, beyond leading order) asymptotic expansions in unitarily invariant ensembles of random hermitian matrices. I will discuss its application to GUE + deterministic.

François Chapon
Title: Outliers of perturbations of banded Toeplitz matrices
Abstract: This is an ongoing work in collaboration with Charles Bordenave and Mireille Capitaine. Toeplitz matrices are non-normal matrices whose spectral analysis in high dimensions is well understood. The spectrum of these matrices is in particular very sensitive to small perturbations. In this talk, we will focus on banded Toeplitz matrices, whose symbol is given by a Laurent polynomial, and which are perturbed by a random matrix. The goal is to describe "outliers", which are eigenvalues that lie outside the support of the limiting distribution of the perturbed matrix as the dimension tends to infinity. The presence of outliers in some region of the complex plane is specifically related to the winding number of the curve determined by the symbol in that region.

Sandrine Dallaporta
Rate of convergence of empirical measures of hyperuniform point processes
This talk is concerned with the empirical measure of a random point process in R^d, such as the eigenvalues of a random matrix or a Coulomb gas. In several cases, this empirical measure converges towards a deterministic measure. In order to quantify the rate of convergence, we are interested in the p-Wasserstein distance between this random measure and its mean, particularly in dimension 2. We obtain a bound for this distance under some assumption on the p-th centered moment of the number of points in squares, which amounts to hyperuniformity when p=2. In addition, hyperuniform determinantal point processes will satisfy the required assumptions for any p>=1.
This is a joint work with Raphaël Butez (Université de Lille) and David García-Zelada (Sorbonne Université).

Slim Kammoun
Title: Universality of the Large Deviations of the Longest Increasing Subsequence of Random Permutations
Abstract: The length of the longest increasing subsequence of a uniform permutation displays phenomena similar to those observed in the largest eigenvalue of the Gaussian Orthogonal Ensemble, such as Tracy-Widom fluctuations and large deviations with two speeds.
In this presentation, we establish the universality of the large deviations for both speeds for a class of random permutations with a conjugacy invariant distribution and a low number of cycles.
This work is based on a joint work with A. Guionnet.

Mylène Maïda
Title : Random partitions and topological expansion of 2D Yang-Mills partition function
Abstract : In the fifties, Chen Ning Yang and Robert Mills made a major breakthrough in quantum field theory by extending the concept of gauge theory to non-abelian groups. Since then, the study of Yang-Mills theory has been a very active field of research both in mathematics and physics. In particular, in the last two decades, significant progress has been made on the rigorous mathematical understanding of the theory on two-dimensional manifolds with gauge group U(N) or SU(N), and of their limit as N grows to infinity. If this talk, I will how the probabilistic study of well chosen random partitions allows us to give rigorous proofs of some topological expansions of the

partition function predicted by physicists Gross and Taylor in the nineties. This is joint work with Thibaut Lemoine (Université de Lille).

Joseph Najnudel
Title: Secular coefficients and the holomorphic multiplicative chaos.
Abstract: We study the coefficients of the characteristic polynomial of unitary matrices drawn from the Circular Beta Ensemble. When the inverse temperature parameter beta is strictly larger than 4, we obtain a new class of limiting distributions that arise when both the order of the coefficient and the dimension of the matrix goes to infinity. For beta equal to 2, we solve an open problem of Diaconis and Gamburd by showing that the middle coefficient tends to zero in probability when the dimension goes to infinity. We introduce a new stochastic object associated to the coefficients of the characteristic polynomial, which we call Holomorphic Multiplicative Chaos (HMC). Viewing the HMC as a random distribution, we prove a sharp result about its regularity in an appropriate Sobolev space. Our proofs expose and exploit several novel connections with other areas, including random permutations, Tauberian theorems and combinatorics.

Pierre Youssef
Title: Tensors of free variables and random quantum channels.
Abstract: We study tensor products of free random variables and establish a corresponding central limit theorem. This framework appears naturally as the limiting behavior of some random matrix models associated with quantum channels having random Kraus operators. We study the limiting spectral distribution of those random quantum channels and provide an estimate on the extreme eigenvalues. In particular, those estimates imply that many generic random constructions of quantum channels produce quantum expanders. Based on joint works with Cécilia Lancien and Patrick Oliveira Santos.