

Delocalization of the Laplace eigenmodes on Anosov surfaces

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The eigenmodes of the Laplace-Beltrami operator on a smooth compact Riemannian manifold (M, g) can exhibit various localization properties in the high frequency regime, which strongly depend on the properties of the geodesic flow. We will focus on situations where this flow is strongly chaotic (Anosov), e.g. if the sectional curvature of (M, g) is negative. The Quantum Ergodicity theorem then states that almost all the eigenmodes become equidistributed on M , in the high frequency limit. The Quantum Unique Ergodicity conjecture claims that this behaviour admits no exception. What can be said about possible "exceptional" eigenstates?

In the case of compact surfaces with Anosov geodesic flow, we prove that all eigenmodes fully delocalize across M : for any open set Ω on M , the L^2 mass on Ω of any eigenstate is uniformly bounded from below. This is in contrast with, e.g., the case of eigenstates on the round sphere, which may be strongly concentrated along a closed geodesic.

The proof uses various methods of semiclassical analysis, the structure of stable and unstable manifolds of the Anosov flow, and a Fractal Uncertainty Principle due to Bourgain-Dyatlov.

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