

Entropic fluctuations in quantum two-time measurement framework

joint work in progress with T.Benoist, L.Bruneau, V.Jakšić,
C.-A.Pillet

Annalisa Panati et Claude-Alain Pillet
CPT, Université de Toulon

16th meeting of GdR Quantum Dynamics , CY Advanced Studies,
CY Cergy Paris Université, January 2024

Classical non equilibrium SM

Transient Fluctuation relation
Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement
statistics
Quantum steady functional
Results

Summary

Indirect measurement: setting
Results

Additional comments

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical System $(\mathcal{M}, \phi_t, \omega)$ dynamical system

ω reference state = probability measure on \mathcal{M}

$$\omega_t = \omega \circ \phi_{-t}$$

Relative entropy: $Ent(\nu|\mu) = -\nu(\log \frac{d\nu}{d\mu})$ (if ν and μ are absolutely continuous)

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

**Indirect
measurement:
setting**

Results

**Additional
comments**

Classical Non Equilibrium Statistical Mechanics

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Classical System $(\mathcal{M}, \phi_t, \omega)$ dynamical system

ω reference state = probability measure on \mathcal{M}

$$\omega_t = \omega \circ \phi_{-t}$$

Relative entropy: $Ent(\nu|\mu) = -\nu(\log \frac{d\nu}{d\mu})$ (if ν and μ are absolutely continuous)

$$\Sigma_t := Ent(\omega_t|\omega) - Ent(\omega|\omega) = -\int_0^t \omega_s(\sigma) ds$$

Σ_t entropy production

σ entropy production observable

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

**Indirect
measurement:
setting**

Results

**Additional
comments**

Classical Non Equilibrium Statistical Mechanics

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Classical System $(\mathcal{M}, \phi_t, \omega)$ dynamical system

ω reference state = probability measure on \mathcal{M}

$$\omega_t = \omega \circ \phi_{-t}$$

Relative entropy: $Ent(\nu|\mu) = -\nu(\log \frac{d\nu}{d\mu})$ (if ν and μ are absolutely continuous)

$$\Sigma_t := Ent(\omega_t|\omega) - Ent(\omega|\omega) = -\int_0^t \omega_s(\sigma) ds$$

Σ_t entropy production

σ entropy production observable

Σ_t can be viewed as a **random variable** on (\mathcal{M}, ω)

We denote its law by $\mathbb{P}_{\Sigma_t, \omega}$

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

**Indirect
measurement:
setting**

Results

**Additional
comments**

Classical Non Equilibrium Statistical Mechanics

Entropic
fluctuations in
quantum two-time
measurement
framework

**Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon**

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

Entropic
fluctuations in
quantum two-time
measurement
framework

**Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon**

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

Entropic
fluctuations in
quantum two-time
measurement
framework

**Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon**

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

(work in driven system [Bochkov-Kuzovlev '70s], [Jaryzinski '97], [Crooks '99] etc.)

Entropic
fluctuations in
quantum two-time
measurement
framework

**Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon**

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

(work in driven system [Bochkov-Kuzovlev '70s], [Jaryzinski '97], [Crooks '99] etc.)

Statistical refinement of thermodynamics second law

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

(work in driven system [Bochkov-Kuzovlev '70s], [Jaryzinski '97], [Crooks '99] etc.)

If $\mathbb{P}_{\Sigma_t, \omega}$ is the law of the random variable Σ_t on (\mathcal{M}, ω) and $\bar{\mathbb{P}}_{\Sigma_t, \omega}(s) = \mathbb{P}_{\Sigma_t, \omega}(-s)$.

If ω if equilibrium state at temperature β , or a multi-thermal state and the system is T.R.I.

$$\frac{d\bar{\mathbb{P}}_{\Sigma_t, \omega}}{d\mathbb{P}_{\Sigma_t, \omega}}(s) = e^{-s}$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

[Evans-Cohen-Morris '93] numerical experiments

[Evans-Searls '94] [Gallavotti Cohen '94] theoretical explanation

(work in driven system [Bochkov-Kuzovlev '70s], [Jaryzinski '97], [Crooks '99] etc.)

If $\mathbb{P}_{\Sigma_t, \omega}$ is the law of the random variable Σ_t on (\mathcal{M}, ω) and $\bar{\mathbb{P}}_{\Sigma_t, \omega}(s) = \mathbb{P}_{\Sigma_t, \omega}(-s)$.

If ω if equilibrium state at temperature β , or a multi-thermal state and the system is T.R.I.

$$\frac{d\bar{\mathbb{P}}_{\Sigma_t, \omega}}{d\mathbb{P}_{\Sigma_t, \omega}}(s) = e^{-s}$$

This is called classical transient (ES) fluctuation relation

Entropic fluctuations in quantum two-time measurement framework

Annalisa Panati et Claude-Alain Pillet
CPT, Université de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Classical non equilibrium statistical mechanics

Transient fluctuation relation

Classical transient (ES) fluctuation relation

$$\frac{d\bar{\mathbb{P}}_{\Sigma_t, \omega}}{d\mathbb{P}_{\Sigma_t, \omega}}(s) = e^{-s}$$

equivalent to

$$\mathcal{F}_{\omega, t}^{cl}(\alpha) = \mathcal{F}_{\omega, t}^{cl}(1 - \alpha)$$

with

$$\mathcal{F}_{\omega, t}^{cl}(\alpha) = \int e^{-\alpha s} d\mathbb{P}_{\Sigma_t, \omega}(s) = \omega(e^{-\alpha \Sigma_t})$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical non equilibrium statistical mechanics

Transient fluctuation relation

Classical transient (ES) fluctuation relation

$$\frac{d\bar{\mathbb{P}}_{\Sigma_t, \omega}}{d\mathbb{P}_{\Sigma_t, \omega}}(s) = e^{-s}$$

equivalent to

$$\mathcal{F}_{\omega, t}^{cl}(\alpha) = \mathcal{F}_{\omega, t}^{cl}(1 - \alpha)$$

with

$$\mathcal{F}_{\omega, t}^{cl}(\alpha) = \int e^{-\alpha s} d\mathbb{P}_{\Sigma_t, \omega}(s) = \omega(e^{-\alpha \Sigma_t})$$

The related object **cumulant generating function**

$$e_{\omega, t}^{cl}(\alpha) = \frac{1}{t} \log \mathcal{F}_{\omega, t}^{cl}(\alpha)$$

is a well known object in large deviation theory.

Large time fluctuations can be described (via Gärtner-Ellis theorem) using properties of the limit

$$e_{\omega, +}^{cl}(\alpha) := \lim_{t \rightarrow \infty} e_{\omega, t}^{cl}(\alpha)$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

**Transient Fluctuation
relation**

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Define similarly as before

$$\mathcal{F}_{\omega_{NESS}, t}^{cl}(\alpha) := \omega_{NESS}(e^{-\alpha \Sigma_t})$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional
Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Define similarly as before

$$\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) := \omega_{NESS}(e^{-\alpha \Sigma_t})$$

Large time fluctuations behaviour is encoded by

$$e_{\omega_{NESS},+}^{cl}(\alpha) := \frac{1}{t} \log \mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha)$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Define similarly as before

$$\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) := \omega_{NESS}(e^{-\alpha \Sigma_t})$$

Large time fluctuations behaviour is encoded by

$$e_{\omega_{NESS},+}^{cl}(\alpha) := \frac{1}{t} \log \mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha)$$

In general:

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Define similarly as before

$$\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) := \omega_{NESS}(e^{-\alpha \Sigma_t})$$

Large time fluctuations behaviour is encoded by

$$e_{\omega_{NESS},+}^{cl}(\alpha) := \frac{1}{t} \log \mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha)$$

In general:

- ▶ for t finite $\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) \neq \mathcal{F}_{\omega,t}^{cl}(\alpha)$ and $\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) \neq \mathcal{F}_{\omega_{NESS},t}^{cl}(1 - \alpha)$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Steady fluctuation relation

Classical steady(GC) fluctuation relation

The initial state of the system is a *non-equilibrium steady state* (NESS) $\omega_{NESS} = \lim_{T \rightarrow \infty} \omega \circ \phi_{-T}$ (existence assumed).

Define similarly as before

$$\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) := \omega_{NESS}(e^{-\alpha \Sigma_t})$$

Large time fluctuations behaviour is encoded by

$$e_{\omega_{NESS},+}^{cl}(\alpha) := \frac{1}{t} \log \mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha)$$

In general:

- ▶ for t finite $\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) \neq \mathcal{F}_{\omega,t}^{cl}(\alpha)$ and $\mathcal{F}_{\omega_{NESS},t}^{cl}(\alpha) \neq \mathcal{F}_{\omega_{NESS},t}^{cl}(1 - \alpha)$
- ▶ for $t \rightarrow \infty$ the following is expected to hold under strong ergodic hypothesis

$$e_{\omega_{NESS},+}^{cl}(\alpha) = e_{\omega,+}^{cl}(\alpha)$$

proven in some (paradigmatic) models ; in [JPR11] advocated as Principle of Regular Entropic Fluctuation

Entropic fluctuations in quantum two-time measurement framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$
The initial state of the system is ω_T .

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

The initial state of the system is ω_T .

Define similarly as before

$$\mathcal{F}_{T,t}^{cl}(\alpha) := \omega_T(e^{-\alpha \Sigma_t})$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

The initial state of the system is ω_T .

Define similarly as before

$$\mathcal{F}_{T,t}^{cl}(\alpha) := \omega_T(e^{-\alpha \Sigma_t})$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

The initial state of the system is ω_T .

Define similarly as before

$$\mathcal{F}_{T,t}^{cl}(\alpha) := \omega_T(e^{-\alpha \Sigma_t})$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

The initial state of the system is ω_T .

Define similarly as before

$$\mathcal{F}_{T,t}^{cl}(\alpha) := \omega_T(e^{-\alpha\Sigma_t})$$

- ▶ for t finite $\mathcal{F}_{T,t}^{cl}(\alpha) \neq \mathcal{F}_{\omega,t}^{cl}(\alpha)$ unless $\omega = \omega_T$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Classical Non Equilibrium Statistical Mechanics

Classical: with respect to $\omega_T = \omega \circ \phi_{-T}$

The initial state of the system is ω_T .

Define similarly as before

$$\mathcal{F}_{T,t}^{cl}(\alpha) := \omega_T(e^{-\alpha \Sigma_t})$$

- ▶ for t finite $\mathcal{F}_{T,t}^{cl}(\alpha) \neq \mathcal{F}_{\omega,t}^{cl}(\alpha)$ unless $\omega = \omega_T$
- ▶ for $t \rightarrow \infty$ one has in many cases (for example on compact spaces)

$$e_{\omega_{T,+}}^{cl}(\alpha) := \lim_{t \rightarrow \infty} e_{T,t}^{cl}(\alpha) = e_{\omega,+}^{cl}(\alpha)$$

Principle of Regular Entropic Fluctuation equivalent to a limit exchange property

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

**Steady Fluctuation
relation**

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional
Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
 $(\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$
 $(\mathcal{O}_{\mathcal{S}}, \tau_{\mathcal{S}, t}, \omega_{\mathcal{S}})$, where $\omega_{\mathcal{S}}$ some state

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
 $(\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$
 $(\mathcal{O}_{\mathcal{S}}, \tau_{\mathcal{S}, t}, \omega_{\mathcal{S}})$, where $\omega_{\mathcal{S}}$ some state

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$

$(\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_{\mathcal{S}} = N$

$(\mathcal{O}_{\mathcal{S}}, \tau_{\mathcal{S}, t}, \omega_{\mathcal{S}})$, where is $\omega_{\mathcal{S}}$ some state

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_S = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
 $(\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_S = N$
 $(\mathcal{O}_S, \tau_{\mathcal{S}, t}, \omega_S)$, where is ω_S some state

Initial state: $\omega := \omega_S \otimes (\omega_{\beta_1} \otimes \dots \otimes \omega_{\beta_M}) = \omega_S \otimes \omega_{\mathcal{R}}$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_S = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
($\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j}$), where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_S = N$
($\mathcal{O}_S, \tau_{\mathcal{S}, t}, \omega_S$), where is ω_S some state

Initial state: $\omega := \omega_S \otimes (\omega_{\beta_1} \otimes \dots \otimes \omega_{\beta_M}) = \omega_S \otimes \omega_{\mathcal{R}}$

Free Dynamics: $\tau_t^\omega(A) := \tau_{\mathcal{S}, t} \otimes (\tau_{\mathcal{R}_1, t} \otimes \dots \otimes \tau_{\mathcal{R}_M, t}) =: \tau_{\mathcal{S}, t} \otimes \tau_{\mathcal{R}, t}$
with generator δ_ω

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
coupled directly or through a small system \mathcal{S} , $\dim \mathcal{H}_S = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots, \mathcal{R}_M$
($\mathcal{O}_j, \tau_{\mathcal{R}_j, t}, \omega_{\beta_j}$), where ω_{β_j} is β_j KMS state for τ_t^j

Small system \mathcal{S} , $\dim \mathcal{H}_S = N$
($\mathcal{O}_S, \tau_{\mathcal{S}, t}, \omega_S$), where is ω_S some state

Initial state: $\omega := \omega_S \otimes (\omega_{\beta_1} \otimes \dots \otimes \omega_{\beta_M}) = \omega_S \otimes \omega_{\mathcal{R}}$

Free Dynamics: $\tau_t^\omega(A) := \tau_{\mathcal{S}, t} \otimes (\tau_{\mathcal{R}_1, t} \otimes \dots \otimes \tau_{\mathcal{R}_M, t}) =: \tau_{\mathcal{S}, t} \otimes \tau_{\mathcal{R}, t}$
with generator δ_ω

Perturbed dynamics: $\tau_t(A)$ with generator $\delta_\omega = \delta_\omega + i[-, V]$

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$

and consider its spectral measure on \mathcal{H} and consider the spectral

measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$
and consider its spectral measure on \mathcal{H} and consider the spectral measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

leads to *NO-fluctuation relation!!!!*

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 1: "Naive quantization"

—attempted in work related literature [Bochkov-Kuzovlev '70s-'80s]

—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$
and consider its spectral measure on \mathcal{H} and consider the spectral measure μ_{Σ_t}

From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω ;

$$\Sigma_t = \log \Delta_{\omega_{-t}|\omega} - \log \Delta_{\omega|\omega}$$

Define

$$\mathcal{F}_t^{naive}(\alpha) = \omega(e^{-\alpha \Sigma_t}) = \int e^{-\alpha s} d\mu_{\Sigma_t}$$

leads to *NO-fluctuation relation!!!!*

Remark: Σ_t is constructed as difference at time t and at time 0 of a given observable.

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Mathematical point of view: Phase space contraction

What is relevant is

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega)$$

In a GNS representation

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega_\omega, e^{-\alpha \log \Delta_{\omega_{-t}|\omega}} \Omega_\omega) = (\Omega_\omega, \Delta_{\omega_{-t}|\omega}^{-\alpha} \Omega_\omega)$$

In terms of cocycle

$$\mathcal{F}_{\omega,t}(\alpha) = \omega([D_{\omega_{-t}} : D_\omega]^\alpha)$$

leads to *fluctuation relation*!!!!

Let $\mathbb{P}_{\omega,t}$ be such that

$$\mathcal{F}_{\omega,t}(\alpha) = \int e^{-\alpha s} d\mathbb{P}_{\omega,t}.$$

What does $\mathbb{P}_{\omega,t}$ means?

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements this approach leads to *fluctuation relation*

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements this approach leads to *fluctuation relation*

At the level of averages and variances, there is no difference!

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements this approach leads to *fluctuation relation*

At the level of averages and variances, there is no difference!

The success of TTM come with a price: unexpected phenomena (with no classical counterpart) due to the invasive role of measurement [Benoist-P.Raquépas19, Benoist-P.Pautrat 20] and *now*

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

**Quantum transient
functional and
Two-time
measurement statistics**

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Protocol:

- $t = 0$, we measure A (outcome a_j)
- evolve for time t
- measure again at time t (outcome a_k)

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j}

associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

In confined system:

$$\mathbb{P}_{A,t}(\phi) =$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

In confined system:

$$\mathbb{P}_{A,t}(\phi) = \text{tr}(\rho P_{a_j})$$

Fact/Problem: *the measurement perturbs the state, the initial state reduces to ρ_{am}*

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

In confined system:

$$\mathbb{P}_{A,t}(\phi) = \text{tr}(\rho P_{a_j})$$

with

$$\rho_{am} = \frac{1}{\text{tr}(\rho P_{a_j})} P_{a_j} \rho P_{a_j}.$$

Fact/Problem: *the measurement perturbs the state, the initial state reduces to ρ_{am}*

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

In confined system:

$$\mathbb{P}_{A,t}(\phi) = \text{tr}(\rho P_{a_j}) \text{tr}(e^{-itH} \rho_{am} e^{itH} P_{a_k})$$

with

$$\rho_{am} = \frac{1}{\text{tr}(\rho P_{a_j})} P_{a_j} \rho P_{a_j}.$$

Fact/Problem: *the measurement perturbs the state, the initial state reduces to ρ_{am}*

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee, Lesovik '96]

Confined systems: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Given an observable A : $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

Two-time measurement distribution of A :

$\mathbb{P}_{A,t}(\phi)$ = probability of measuring a change in A equal to ϕ .

In confined system:

$$\mathbb{P}_{A,t}(\phi) = \sum_{a_k - a_j = \phi} \text{tr}(\rho P_{a_j}) \text{tr}(e^{-itH} \rho_{am} e^{itH} P_{a_k})$$

with

$$\rho_{am} = \frac{1}{\text{tr}(\rho P_{a_j})} P_{a_j} \rho P_{a_j}.$$

Fact/Problem: *the measurement perturbs the state, the initial state reduces to ρ_{am}*

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics -thermodynamic limit

This defines $\mathcal{F}_{\omega,t}^{(n)}(\alpha) = \int e^{-\alpha s} d\mathbb{P}_{S,t}^{(n)}(s)$.

Theorem

It is possible to rewrite $\mathcal{F}_{\omega,t}^{(n)}(\alpha)$ in term of algebraic objects that survive the limit $n \rightarrow \infty$. Under standard hypothesis,

$\lim_{(n) \rightarrow \infty} \mathcal{F}_{\omega,t}^{(n)}(\alpha) =: \mathcal{F}_{\omega,t}(\alpha)$ is well defined and correspond to the same formal expression.

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantization: transient fluctuation relation

Two-time measurement statistics -thermodynamic limit

This defines $\mathcal{F}_{\omega,t}^{(n)}(\alpha) = \int e^{-\alpha s} d\mathbb{P}_{S,t}^{(n)}(s)$.

Theorem

It is possible to rewrite $\mathcal{F}_{\omega,t}^{(n)}(\alpha)$ in term of algebraic objects that survive the limit $n \rightarrow \infty$. Under standard hypothesis, $\lim_{(n) \rightarrow \infty} \mathcal{F}_{\omega,t}^{(n)}(\alpha) =: \mathcal{F}_{\omega,t}(\alpha)$ is well defined and correspond to the same formal expression.

And that expression is same as before!!!

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega_{\omega}, e^{-\alpha \log \Delta_{\omega-t|\omega}} \Omega_{\omega}) = (\Omega_{\omega}, \Delta_{\omega-t|\omega}^{-\alpha} \Omega_{\omega})$$

In other words

$\mathcal{F}_{\omega,t}(\alpha) = \mathcal{F}_{\omega,t}^{\text{ttm}}(\alpha)$ obtained though two-time measurement protocol

$\mathcal{F}_{\omega,t}(\alpha) = \mathcal{F}_{\omega,t}^{\text{PSC}}(\alpha)$ obtained in the spirit of quantum phase phase contraction

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantum steady entropic functional: our proposal

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

How to quantize steady FR? Difficulties:

- ▶ A measurement would destroy the steady state
- ▶ steady state exists only in the thermodynamic limit
- ▶ in the thermodynamic limit, non-normality of the steady state (to the initial state)

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

**Quantum steady
functional**

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantum steady entropic functional: our proposal

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Quantum steady entropic functional: our proposal

ω_{NESS} as an idealization of ω_T at an unknown very large time (see remark about classical case)

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

**Quantum steady
functional**

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantum steady entropic functional: our proposal

ω_{NESS} as an idealization of ω_T at an unknown very large time (see remark about classical case)

Two time measurement framework (start with finite dimensional approximation $\dim \mathcal{H} = n$)

- start with ω initial state as in the transient case
- perform the first measurement at an unknown very large time T
- let the system evolve for time t
- perform the measurement at an unknown very large time $T + t$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

**Quantum steady
functional**

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantum steady entropic functional: our proposal

ω_{NESS} as an idealization of ω_T at an unknown very large time (see remark about classical case)

Two time measurement framework (start with finite dimensional approximation $\dim \mathcal{H} = n$)

- start with ω initial state as in the transient case
 - perform the first measurement at an unknown very large time T
 - let the system evolve for time t
 - perform the measurement at an unknown very large time $T + t$
- This defines

$$\mathbb{P}_{T,t}^{(n)}, \quad \mathcal{F}_{\omega_{T,t}}^{(n)}(\alpha) = \int e^{-t\alpha s} d\mathbb{P}_{T,t}^{(n)}(s)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Quantum steady entropic functional: our proposal

Proposition (Thermodynamic limit)

(T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23))

It is possible to rewrite $\mathcal{F}_{\omega_T,t}^{(n)}(\alpha)$ in term of algebraic objects that survive the limit. Under standard hypothesis,

$\mathcal{F}_{\omega_T,t}(\alpha) := \lim_{n \rightarrow \infty} \mathcal{F}_{\omega_T,t}^{(n)}(\alpha)$ is well defined correspond to the same formal expression.

$$\mathcal{F}_{\omega_T,t}(\alpha) = \lim_{R \rightarrow \infty} \int_0^R \omega_T(\zeta_\theta[D_{\omega_{-t}} : D_\omega]^\alpha) d\theta$$

with $\zeta_\theta(-)$ modular dynamic associated to ω (free dynamics up to a temperature time scale factor)

Recall:

$$\mathcal{F}_{\omega,t}(\alpha) = \omega([D_{\omega_{-t}} : D_\omega]^\alpha)$$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Quantum steady entropic functional: our proposal

Proposition (Thermodynamic limit)

(T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23))

It is possible to rewrite $\mathcal{F}_{\omega_T,t}^{(n)}(\alpha)$ in term of algebraic objects that survive the limit. Under standard hypothesis,

$\mathcal{F}_{\omega_T,t}(\alpha) := \lim_{n \rightarrow \infty} \mathcal{F}_{\omega_T,t}^{(n)}(\alpha)$ is well defined correspond to the same formal expression.

$$\mathcal{F}_{\omega_T,t}(\alpha) = \lim_{R \rightarrow \infty} \int_0^R \omega_T(\zeta_\theta[D_{\omega_{-t}} : D_\omega]^\alpha) d\theta$$

with $\zeta_\theta(-)$ modular dynamic associated to ω (free dynamics up to a temperature time scale factor)

Recall:

$$\mathcal{F}_{\omega,t}(\alpha) = \omega([D_{\omega_{-t}} : D_\omega]^\alpha)$$

Definition

$$\mathcal{F}_{\omega_{NESS},t}(\alpha) := \lim_{T \rightarrow \infty} \mathcal{F}_{\omega_T,t}(\alpha)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Quantum steady entropic functional: our proposal: results

Theorem (direct coupling, no S , T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau_{\mathcal{R},t}^0$ is ergodic. Then

$$\mathcal{F}_{\omega_T,t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$$

for all $T \in \mathbb{R}$.

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Quantum steady entropic functional: our proposal: results

Theorem (direct coupling, no S , T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau_{\mathcal{R},t}^0$ is ergodic. Then

$$\mathcal{F}_{\omega_T,t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$$

for all $T \in \mathbb{R}$.

Consequences:

Quantum steady entropic functional: our proposal: results

Theorem (direct coupling, no S , T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau_{\mathcal{R},t}^0$ is ergodic. Then

$$\mathcal{F}_{\omega_T,t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$$

for all $T \in \mathbb{R}$.

Consequences:

► $\mathcal{F}_{\omega_{NESS},t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$

Quantum steady entropic functional: our proposal: results

Theorem (direct coupling, no S , T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau_{\mathcal{R},t}^0$ is ergodic. Then

$$\mathcal{F}_{\omega_T,t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$$

for all $T \in \mathbb{R}$.

Consequences:

- ▶ $\mathcal{F}_{\omega_{NESS},t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$
- ▶ If the symmetry true for $\mathcal{F}_{\omega,t}(\alpha)$, then for the steady functional $\mathcal{F}_{\omega_{NESS},t}(\alpha)$ also satisfy the symmetry AT FINITE TIME t ;
- ▶ if $e_{\omega_{NESS},+}(\alpha)$, $e_{\omega,+}(\alpha)$ exist, they are equal.

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Quantum steady entropic functional: our proposal: results

Theorem (coupling through \mathcal{S} ,
T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are through a small system S ; assume the dynamics $\tau_{\mathcal{R},t}^0$ is ergodic. Then

$$e_{\omega_{NESS,+}}(\alpha) = e_{\omega,+}(\alpha)$$

Remark

In both theorems:

1. No additional hypothesis on the perturbed dynamics; strong coupling allowed.
2. Underlying mechanism: invasive measurement role; memory erasing effect of first measurement; stability of the fluctuation
3. General proof, with algebraic methods (no need for resonance analysis model by model)
4. We are able to cope with NESS in the thermodynamic limit.

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Proof in abstract terms

Consider $(\mathcal{O}, \tau_t, \omega)$ with ω a τ_t invariant.

$(\mathcal{O}, \tau_t, \omega)$ is ergodic iff for any ω normal state ν and any $A \in \mathcal{O}$

$$\lim_{T \rightarrow \infty} \int_0^T \nu(\tau_t(A)) dt$$

Apply this property to

$$\mathcal{F}_{\omega_T, t}(\alpha) = \lim_{R \rightarrow \infty} \int_0^R \omega_T(\zeta_\theta[D_{\omega_{-t}} : D_\omega]^\alpha) d\theta$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Proof in terms of Liouvillean

Consider the GNS representation associated to ω ; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Proof in terms of Liouvillean

Consider the GNS representation associated to ω ; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

Liouvillean: any operator such that $\pi_\omega(\tau_t(A)) = e^{itL}\pi_\omega(A)e^{-itL}$
not uniquely defined.

L_∞ such that $L_\infty\Omega = \Omega$

L_α deformed Liouvillean, L_0 liouvillean for the free dynamics

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Proof in terms of Liouvillean

Consider the GNS representation associated to ω ; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

Liouvillean: any operator such that $\pi_\omega(\tau_t(A)) = e^{itL}\pi_\omega(A)e^{-itL}$
not uniquely defined.

L_∞ such that $L_\infty\Omega = \Omega$

L_α deformed Liouvillean, L_0 liouvillean for the free dynamics

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega, \Delta_{\omega_t}^\alpha |_\omega \Omega) = (\Omega, e^{-itL_\alpha} \Omega)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Proof in terms of Liouvillean

Consider the GNS representation associated to ω ; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

Liouvillean: any operator such that $\pi_\omega(\tau_t(A)) = e^{itL}\pi_\omega(A)e^{-itL}$
not uniquely defined.

L_∞ such that $L_\infty\Omega = \Omega$

L_α deformed Liouvillean, L_0 liouvillean for the free dynamics

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega, \Delta_{\omega_t|\omega}^\alpha \Omega) = (\Omega, e^{-itL_\alpha} \Omega)$$

$$\begin{aligned}\mathcal{F}_{\omega_T,t}(\alpha) &= \lim_{A \rightarrow \infty} \frac{1}{A} \int_0^A (\Omega_T, e^{i\theta L_0} \Delta_{\omega_t|\omega}^\alpha e^{-i\theta L_0} \Omega_T) d\theta \\ &= (\Omega, e^{iT L_\infty} \mathbf{1}_{\{0\}}(-\beta L_0) e^{-itL_\alpha} \Omega)\end{aligned}$$

Direct coupling: If the dynamics on \mathcal{R} is ergodic, 0 is a simple eigenvalue for L_0 and $\mathbf{1}_{\{0\}}(L_0) = |\Omega\rangle\langle\Omega|$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Proof in terms of Liouvillean

Consider the GNS representation associated to ω ; $(\mathcal{H}_\omega, \pi_\omega, \Omega)$

Liouvillean: any operator such that $\pi_\omega(\tau_t(A)) = e^{itL}\pi_\omega(A)e^{-itL}$
not uniquely defined.

L_∞ such that $L_\infty\Omega = \Omega$

L_α deformed Liouvillean, L_0 liouvillean for the free dynamics

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega, \Delta_{\omega_t|\omega}^\alpha \Omega) = (\Omega, e^{-itL_\alpha} \Omega)$$

$$\begin{aligned}\mathcal{F}_{\omega_T,t}(\alpha) &= \lim_{A \rightarrow \infty} \frac{1}{A} \int_0^A (\Omega_T, e^{i\theta L_0} \Delta_{\omega_t|\omega}^\alpha e^{-i\theta L_0} \Omega_T) d\theta \\ &= (\Omega, e^{iT L_\infty} \mathbf{1}_{\{0\}}(-\beta L_0) e^{-itL_\alpha} \Omega)\end{aligned}$$

Direct coupling: If the dynamics on \mathcal{R} is ergodic, 0 is a simple eigenvalue for L_0 and $\mathbf{1}_{\{0\}}(L_0) = |\Omega\rangle\langle\Omega|$

Coupling through a small system \mathcal{S} : If the dynamics on \mathcal{R} is ergodic, 0 is a simple eigenvalue for L_0 and $\ker(L) = \ker(L_S) \otimes \Omega_{\mathcal{R}}$; equality is attained in the long time limit $t \rightarrow \infty$

Summing up so far

- ▶ we have introduced classical entropic functional and we have review the quantization in the transient case
- ▶ we have introduced a proposal for quantum steady (GC) entropic functional $\mathcal{F}_{\omega_{NESS},t}(\alpha)$
- ▶ we have shown a "memory erasing effect" / stability due to the first measurement ; this can be interpreted as stability in the quantum case of the fluctuations
- ▶ we have shown $e_{\omega_{NESS},+}(\alpha) = e_{\omega,+}(\alpha)$ under very minimal ergodicity hypothesis
- ▶ direct measurement on infinitely extended or (very large) reservoir is an idealization. Are we able to write similar (less general) results ins the framework of an indirect measurement using resonance theory ? YES, next subject.

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Indirect Measurement: Ancilla State Tomography

The Entropic Ancilla State Tomography (EAST)

Confined system: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Entropic
fluctuations in
quantum two-time
measurement
framework

**Annalisa Panati et
Claude-Alain Pillet**
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Indirect Measurement: Ancilla State Tomography

The Entropic Ancilla State Tomography (EAST)

Confined system: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Couple $\mathcal{S} + \mathcal{R}$ with an auxiliary system called "ancilla"

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Indirect Measurement: Ancilla State Tomography

The Entropic Ancilla State Tomography (EAST)

Confined system: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Couple $\mathcal{S} + \mathcal{R}$ with an auxiliary system called "ancilla"

ancilla = spin 1/2

$(\mathfrak{H}_a, H, \rho_a)$ with $\mathfrak{H}_a = \mathbb{C}^2$

Denote by v_{\pm} the eigenvectors of the Pauli matrix σ_z associated to the eigenvalues ± 1 . ρ_a a density matrix such that $\langle v_+, \rho_a v_- \rangle \neq 0$.

Consider the extended system

$$\widehat{\mathcal{H}} = \mathcal{H} \otimes \mathbb{C}^2$$

$$\widehat{\omega} = \omega \otimes \rho_a$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Indirect Measurement: Ancilla State Tomography

The Entropic Ancilla State Tomography (EAST)

Confined system: described by (\mathcal{H}, H, ρ) $\dim \mathcal{H} < \infty$

Couple $\mathcal{S} + \mathcal{R}$ with an auxiliary system called "ancilla"

ancilla = spin 1/2

$(\mathfrak{H}_a, H, \rho_a)$ with $\mathfrak{H}_a = \mathbb{C}^2$

Denote by v_{\pm} the eigenvectors of the Pauli matrix σ_z associated to the eigenvalues ± 1 . ρ_a a density matrix such that $\langle v_+, \rho_a v_- \rangle \neq 0$.

Consider the extended system

$$\widehat{H} = \mathcal{H} \otimes \mathbb{C}^2$$

$$\widehat{\omega} = \omega \otimes \rho_a$$

We introduce family of Hamiltonians (for each α)

$$\widehat{H}_\alpha = e^{\frac{\alpha}{2} \log \omega \otimes \sigma_z} (H \otimes \mathbb{1}) e^{-\frac{\alpha}{2} \log \omega \otimes \sigma_z}$$

$P := |v_-\rangle\langle v_+|$, a simple computation gives

$$\text{tr}(e^{-it\widehat{H}_\alpha} \widehat{\nu} e^{it\widehat{H}_\alpha} \mathbb{1} \otimes P) = \langle v_+, \rho_a v_- \rangle \mathcal{F}_{\omega, t}(\alpha)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Indirect Measurement: Ancilla State Tomography

Entropic fluctuations in quantum two-time measurement framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

In the general case:

$$\hat{O} := O \otimes O_{\mathbb{C}^2}$$

$$\hat{\tau}^t = \tau^t \otimes \text{id} \quad \hat{\tau}_\alpha^t \text{ perturbed dynamics corresponding to } \hat{H}_\alpha$$

$$\hat{\omega}(\hat{\tau}_\alpha^t(\mathbb{1} \otimes P)) = \langle v_+, \rho_a v_- \rangle \mathcal{F}_{\omega,t}^{\text{ancilla}}(\alpha).$$

$$\begin{aligned} \mathcal{F}_{\omega,t}^{\text{ancilla}}(\alpha) &:= \omega \left(([D\omega_{-t} : D\omega])^{\frac{\alpha}{2}*} [D\omega_{-t} : D\omega]^{\frac{\alpha}{2}} \right) \\ &= \omega \left([D\omega_{-t} : D\omega]^{\frac{\alpha}{2}} \right) = \mathcal{F}_{\omega,t}(\alpha) \end{aligned}$$

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Indirect Measurement: Ancilla State Tomography

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

If the initial state is ω the multi-thermal state

$$\mathcal{F}_{\omega,t}^{\text{ttm}}(\alpha) = \mathcal{F}_{\omega,t}^{\text{ancilla}}(\alpha)$$

But for general ν , in particular if $\nu = \omega_T$ the functionals

$$\mathcal{F}_{\omega_T,t}(\alpha) \neq \mathcal{F}_{\omega_T,t}^{\text{ancilla}}(\alpha)$$

This indirect measurement protocol is not "invariant" if with respect of the time T .

Is the protocol "stable"? Meaning that in the limit $t \rightarrow +\infty$ the corresponding cumulant generating function will be the same?

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

**Indirect
measurement:
setting**

Results

Additional
comments

Indirect Measurement: Ancilla State Tomography

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Answer is : YES! but not in general

For ω_T finite, true under additional hypothesis

For ω_{NESS} expected under additional chaotic property of the dynamics, we can prove it for Spin-Fermion/boson model.

One can argue for Ancilla ST, the situation is parallel to the classical case.

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

Indirect Measurement: Ancilla State Tomography, results

General results:

(AnV) For some $\vartheta > 0$ the map

$$\mathbb{R} \ni s \mapsto \zeta_\omega^\theta(V) \in \mathcal{O}$$

has an analytic extension to the complex strip

$$\mathfrak{S}(\vartheta) = \{z \in \mathbb{C} \mid \operatorname{Re} z - 1 \setminus 2 \mid < \vartheta\}.$$

Theorem (T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '24)

Suppose that **(AnV)** holds with $\theta > \frac{1}{2}$. Set

$$C_T := e^{T(\|\zeta_\omega^{-i\frac{1}{2}}(V)\| + \|V\|)} \quad D_T := e^{-T(\|\zeta_\omega^{i\frac{1}{2}}(V)\| + \|V\|)}.$$

Then for any $\alpha \in (-\vartheta, \vartheta)$,

$$D_T \mathcal{F}_{\omega,t}(\alpha) \leq \mathcal{F}_{\omega_T,t}^{\text{ancilla}}(\alpha) \leq C_T \mathcal{F}_{\omega,t}(\alpha).$$

Consequence (stability of the measurement protocol for finite T):

$$e_{\omega,+}(\alpha) = e_{\omega_T,+}^{\text{ancilla}}(\alpha)$$

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Indirect Measurement: Ancilla State Tomography: results

Entropic fluctuations in quantum two-time measurement framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Model dependent results

Theorem (T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '24)

*For the Spin-Fermion. Suppose that **(AnV)** holds with $\theta > \frac{1}{2}$ and some additional regularity hypothesis. Then for any $\alpha \in (-\vartheta, \vartheta)$,*

$$e_{\omega_{NESS},t}(\alpha) = e_{\omega,t}(\alpha).$$

Plan

Classical non equilibrium SM

Transient Fluctuation relation

Steady Fluctuation relation

Quantum Non Equilibrium SM

Quantum transient functional and Two-time measurement statistics

Quantum steady functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

A comment on entropic functionals at $T > 0$

For $T = 0$, i.e. with respect to ω

$$\mathcal{F}_{\omega,t}(\alpha) = \mathcal{F}_{\omega,t}^{\text{psc}}(\alpha) = \mathcal{F}_{\omega,t}^{\text{ttm}}(\alpha) = \mathcal{F}_{\omega,t}^{\text{ancilla}}(\alpha)$$

$$\mathcal{F}_{\omega,t}^{\text{psc}}(\alpha) = \mathcal{F}_{\omega,t}^{\text{ttm}}(\alpha) = \omega([D\omega_{-t} : D\omega]^\alpha)$$

$$\mathcal{F}_{\omega,t}^{\text{ancilla}}(\alpha) = \omega\left([D\omega_{-t} : D\omega]^{\frac{\alpha}{2}} [D\omega_{-t} : D\omega]^{\frac{\alpha}{2}}\right)$$

For $T > 0$, i.e. with respect to ω_T

$$\mathcal{F}_{\omega_T,t}^{\text{psc}}(\alpha) \neq \mathcal{F}_{\omega_T,t}^{\text{ttm}}(\alpha) \neq \mathcal{F}_{\omega_T,t}^{\text{ancilla}}(\alpha)$$

$$\mathcal{F}_{\omega_T,t}^{\text{ttm}}(\alpha) = \lim_{R \rightarrow \infty} \int_0^R \omega_T(\zeta_\theta [D\omega_{-t} : D\omega]^\alpha) d\theta$$

$$\mathcal{F}_{\omega_T,t}^{\text{ancilla}}(\alpha) = \omega_T([D\omega_{-t} : D\omega]^{\frac{\alpha}{2}} [D\omega_{-t} : D\omega]^{\frac{\alpha}{2}})$$

$$\mathcal{F}_{\omega_T,t}^{\text{psc}}(\alpha) = \omega_T([D\omega_{-t} : D\omega]^\alpha)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments

A comment on entropic functionals at $T > 0$

Equality are restored in the large time limit under some conditions

If reservoir are ergodic and (AnV) is verified

$$e_{\omega,+}(\alpha) = \lim_{t \rightarrow \infty} \frac{1}{t} \log \mathcal{F}_{\omega_T, t}^{\text{ttm}}(\alpha) = \lim_{t \rightarrow \infty} \frac{1}{t} \log \mathcal{F}_{\omega_T, t}^{\text{ancilla}}(\alpha)$$

Plan

Classical non equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect measurement: setting

Results

Additional comments

Thank you for your attention

Entropic
fluctuations in
quantum two-time
measurement
framework

Annalisa Panati et
Claude-Alain Pillet
CPT, Université
de Toulon

Plan

Classical non
equilibrium SM

Transient Fluctuation
relation

Steady Fluctuation
relation

Quantum Non
Equilibrium SM

Quantum transient
functional and
Two-time
measurement statistics

Quantum steady
functional

Results

Summary

Indirect
measurement:
setting

Results

Additional
comments