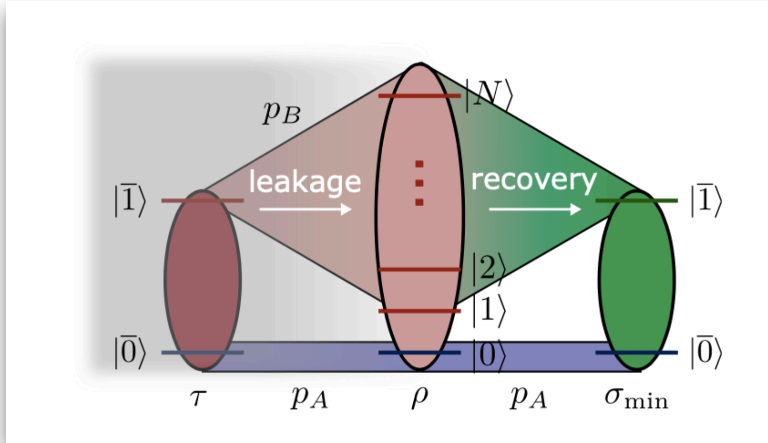
GDR DynQua conference
CY Cergy Paris Université
1 February 2024

Recovery of qubit state after noisy leakage in high-dimensional space



Janet Anders
University of Exeter & University of Potsdam

joint work with:

+ many many inputs from

Sofia Sevitz
Uni Potsdam

Nathanaël Cottet
(Alice&Bob)

Benjamin Huard

EPSRC Engineering and Physical Sciences Research Council
DFG Deutsche Forschungsgemeinschaft
cost EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Quantum thermodynamics - Motivation

MICROSCOPIC WORLD

- atoms, electrons, photons

MACROSCOPIC WORLD

- gases, fluids, solids
- pistons and weights

1nm/1amu → 1m/1kg

Quantum thermodynamics

- include small ensemble sizes
- include non-equilibrium properties
- include quantum properties

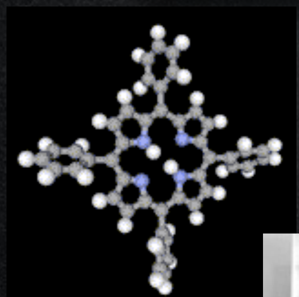
Quantum Mech

- superpositions
- quantum correlations

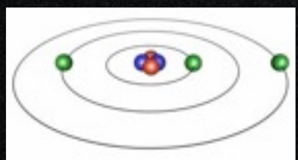
cs

- ork, heat, entropy
- 1st law, 2nd law, 3rd law
- Carnot efficiency, engines

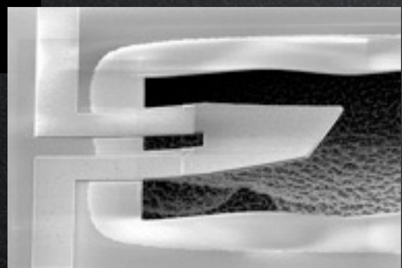
photon



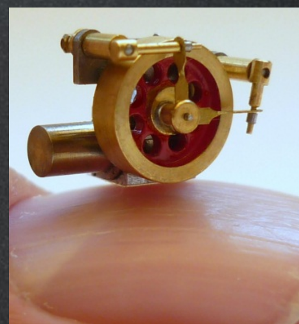
bio-molecule



atom



micro-meter resonator



nature physics

ARTICLES

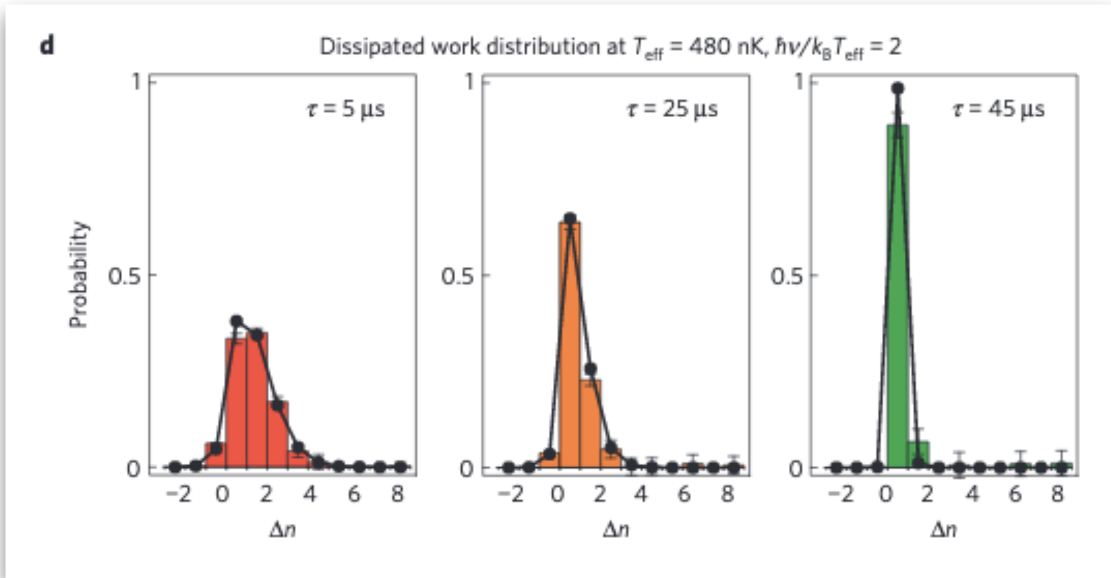
PUBLISHED ONLINE: 22 DECEMBER 2014 | DOI: 10.1038/NPHYS3197

Experimental test of the quantum Jarzynski equality with a trapped-ion system

Shuoming An[1], Jing-Ning Zhang[1], Mark Um[1], Dingshun Lv[1], Yao Lu[1], Junhua Zhang[1], Zhang-Qi Yin[1], H. T. Quan[2,3]* and Kihwan Kim[1]*

extension of classical fluctuation relations to quantum regime

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$



OPEN

Coherence and measurement in quantum thermodynamics

P. Kammerlander[1] & J. Anders[2]

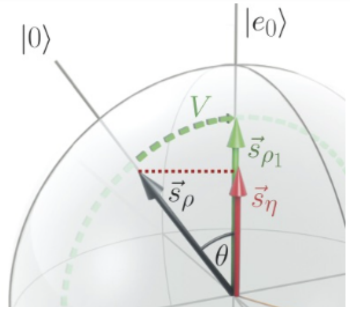
Received: 03 February 2016
Accepted: 09 February 2016
Published: 26 February 2016

Thermodynamics is a highly successful macroscopic theory widely used across the natural sciences and for the construction of everyday devices, from car engines to solar cells. With thermodynamics predating quantum theory, research now aims to uncover the thermodynamic laws that govern finite size systems which may in addition host quantum effects. Recent theoretical breakthroughs include the characterisation of the efficiency of quantum thermal engines, the extension of classical

$$\langle W_{ext}^{max} \rangle = k_B T \, [\, S(\eta_H) - S(\rho) \,] > 0$$

for initial states with coherences

coherences are a source of work
(or heat, depending on how they are used)



npj | Quantum Information

www.nature.com/npjqi

ARTICLE OPEN

The role of quantum measurement in stochastic thermodynamics

Cyril Elouard[1], David A. Herrera-Martí[1], Maxime Clusel[2] and Alexia Auffèves[1]

This article sets up a new formalism to investigate stochastic thermodynamics in the quantum regime, where stochasticity and irreversibility primarily come from quantum measurement. In the absence of any bath, we define a purely quantum component to heat exchange, that corresponds to energy fluctuations caused by quantum measurement. Energetic and entropic signatures of measurement-induced irreversibility are then explored for canonical experiments of quantum optics, and the energetic cost of counter-acting decoherence is studied on a simple state-stabilizing protocol. By placing quantum measurement in a central position, our formalism contributes to bridge a gap between experimental quantum optics and quantum thermodynamics, and opens new paths to characterize the energetic features of quantum processing.

npj Quantum Information (2017)3:9 ; doi:10.1038/s41534-017-0008-4

PHYSICAL REVIEW X **5**, 031044 (2015)

Equivalence of Quantum Heat Machines, and Quantum-Thermodynamic Signatures

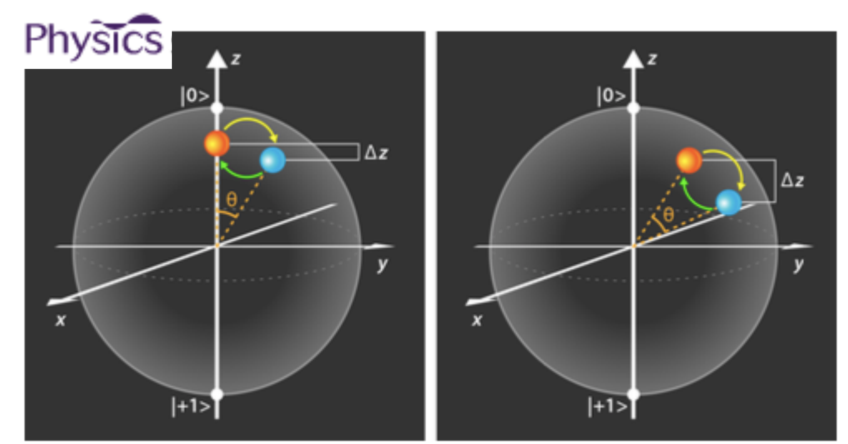
Raam Uzdin,* Amikam Levy, and Ronnie Kosloff
Fritz Haber Research Center for Molecular Dynamics,

Featured in Physics   Editors' Suggestion

Experimental Demonstration of Quantum Effects in the Operation of Microscopic Heat Engines

James Klatzow, Jonas N. Becker, Patrick M. Ledingham, Christian Weinzetl, Krzysztof T. Kaczmarek, Dylan J. Saunders, Joshua Nunn, Ian A. Walmsley, Raam Uzdin, and Eilon Poem
Phys. Rev. Lett. **122**, 110601 – Published 20 March 2019

Physics See Viewpoint: Powering an Engine with Quantum Coherence



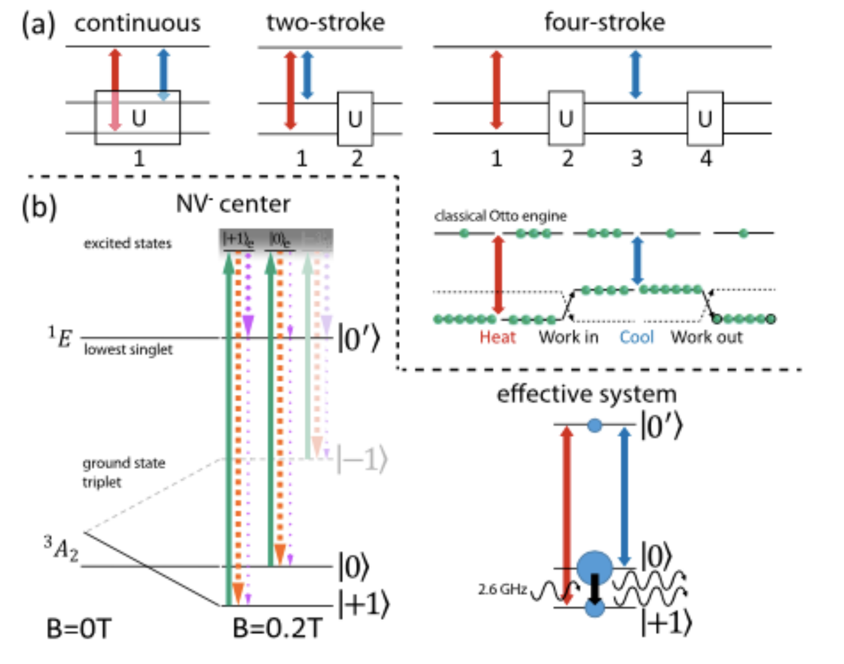


FIG. 1. Quantum heat engine schematics. (a) Three basic heat

Outline

- Physical encoding and noise
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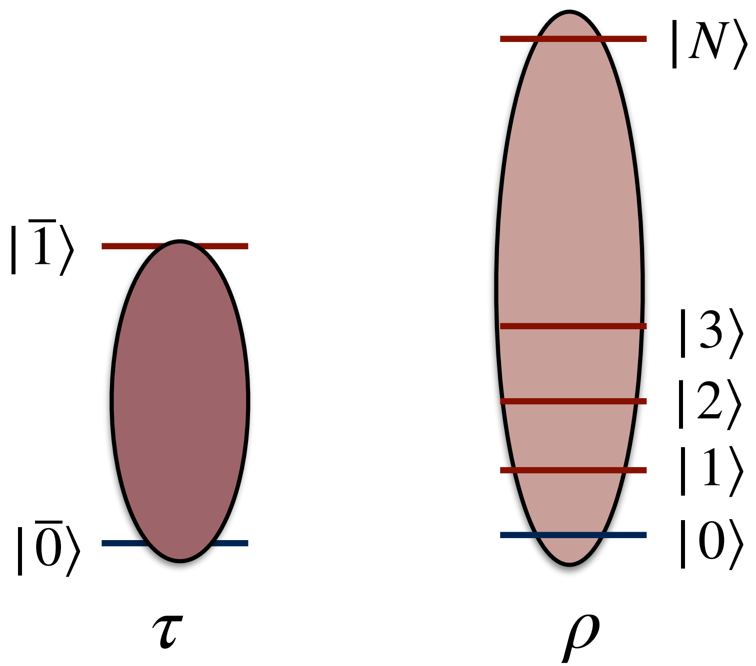
- Summary

Noise



e.g.

$$|\bar{0}\rangle = |0\rangle$$

$$|\bar{1}\rangle = |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{N} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

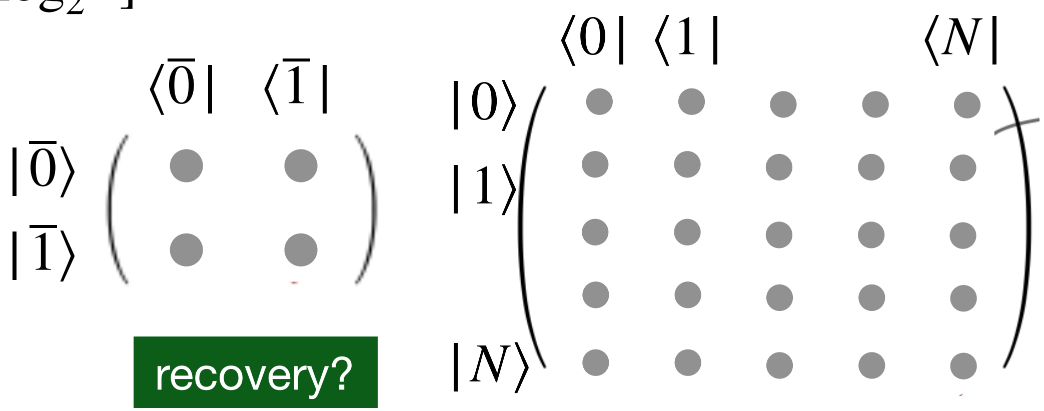
with $|\alpha| \gg 1$

so that $\langle 0|\alpha\rangle = e^{-|\alpha|^2/2} \approx 0$

von Neumann entropy

$$S(\cdot) = -\mathrm{tr}[\cdot \log_2 \cdot]$$

$S(\tau) \leq 1$



$$S(\tau) \leq S(\rho)$$

$$S(\rho) \leq \log_2(N+1)$$

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Example

University of Exeter · Universität Potsdam

Quantum Maxwell demon experiment

PNAS

Observing a quantum Maxwell demon at work

Nathanaël Cottet[a,1], Sébastien Jezouin[a,1], Landry Bretheau[a], Philippe Campagne-Ibarcq[a], Quentin Ficheux[a], Janet Anders[b], Alexia Auffèves[c], Rémi Azouit[d,e], Pierre Rouchon[d,e], and Benjamin Huard[a,f,2]

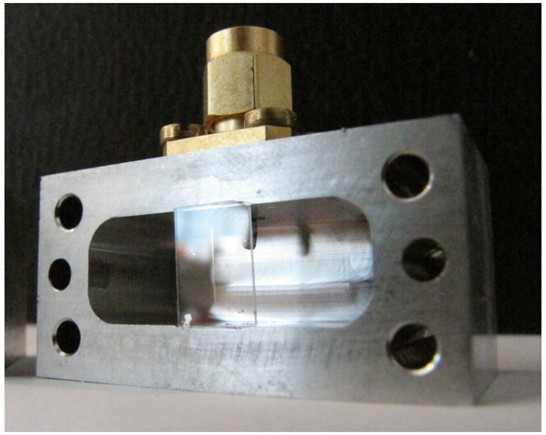
[a]Laboratoire Pierre Aigrain, Ecole Normale Supérieure, PSL Research University, CNRS, Université Pierre et Marie Curie, Sorbonne Universités, Université Paris Diderot, Sorbonne Paris-Cité, 75231 Paris Cedex 05, France; [b]Physics and Astronomy, College of Engineering, Mathematics, and Physical Sciences University of Exeter, Exeter EX4 4QL, United Kingdom; [c]Institut Néel, UPR2940 CNRS and Université Grenoble Alpes, 38042 Grenoble, France; [d]Centre Automatique et Systèmes, Mines ParisTech, PSL Research University, 75272 Paris Cedex 6, France; [e]Quantic Team, INRIA Paris, 75012 Paris, France; and [f]Laboratoire de Physique, Ecole Normale Supérieure de Lyon, 69364 Lyon Cedex 7, France

Edited by Steven M. Girvin, Yale University, New Haven, CT, and approved June 5, 2017 (received for review March 23, 2017)

Cottet, …, Huard, PNAS (2017)

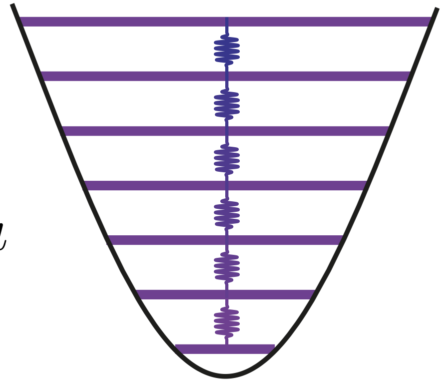
Quantum Maxwell demon experiment

University of Exeter · Universität Potsdam

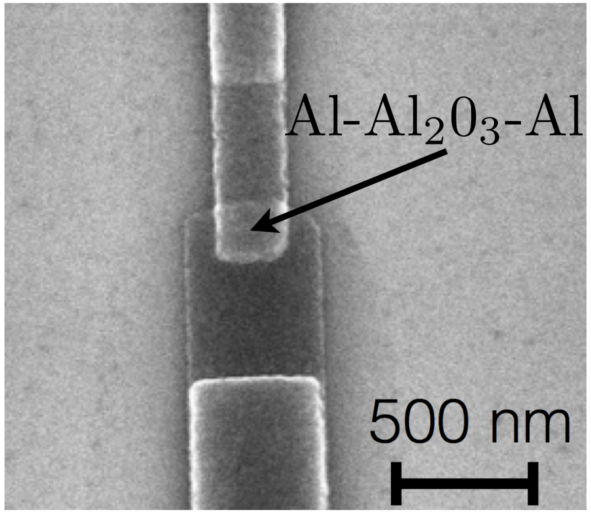


✦ microwave cavity = c = D

$$H_c \approx \hbar\omega_c\, a^\dagger\, a$$

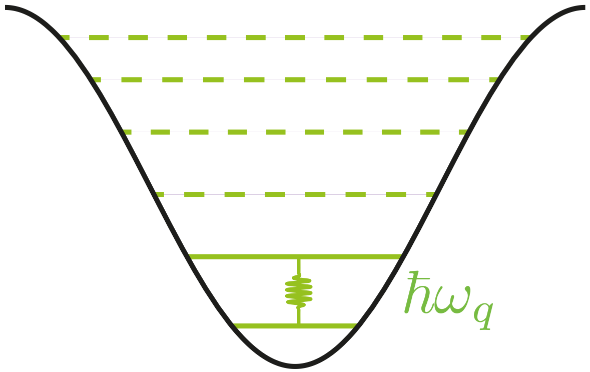


large physical space:
photon number basis



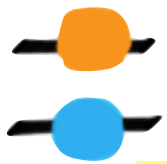
✦ superconducting transmon qubit = q = S

$$\hat{H} = \frac{\hat{q}^2}{2C_J} - E_J \cos\frac{\hat{\phi}}{\hbar/2e} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\phi}^2}{2L_J} + H_{\text{non-lin}}(\hat{\phi})$$



$$H_q \approx 0\,|g\rangle\langle g| + \hbar\omega_q\,|e\rangle\langle e|$$

Maxwell demon recording information

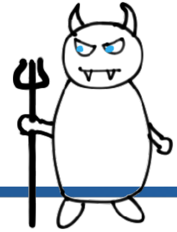


$|\alpha\rangle$ $|0\rangle$

qubit encoded in large space

Cottet, …, Huard, PNAS (2017)

Reading the demon’s mind



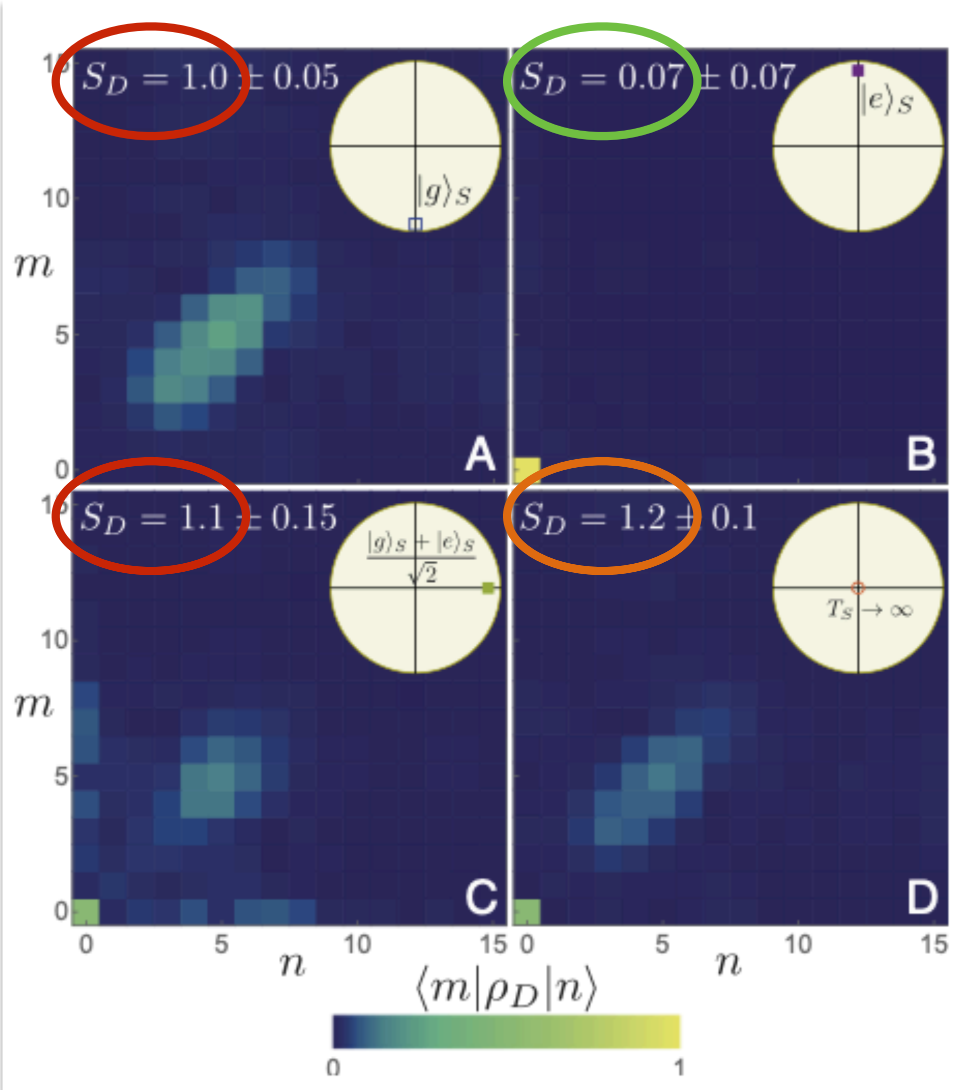
Tomography of the demon (cavity) state after the feedback protocol.

expected:

$|\alpha\rangle \qquad S = 0$

expected:

$\frac{|0\rangle + |\alpha\rangle}{\sqrt{2}} \qquad S = 0$



expected:

$|0\rangle \qquad S = 0$

expected:

$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|\alpha\rangle\langle\alpha|$

$S = \log_2 2 = 1$

Cottet, …, Huard, PNAS (2017)

Janet Anders janet@qipc.org

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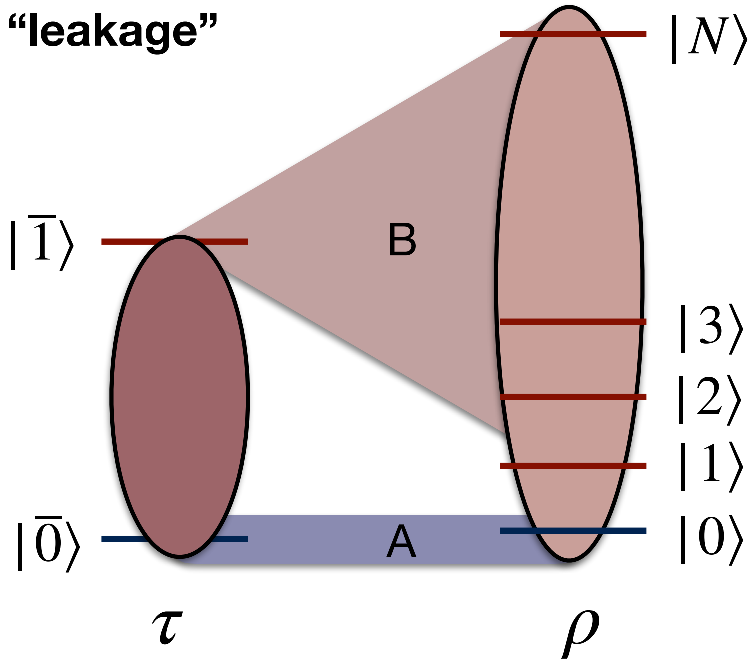
Noise

Assume:

subspace-confined "leakage"



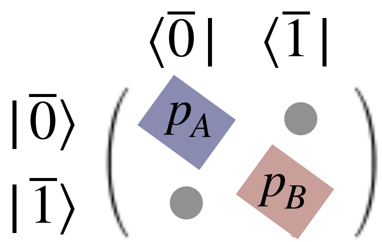
e.g.

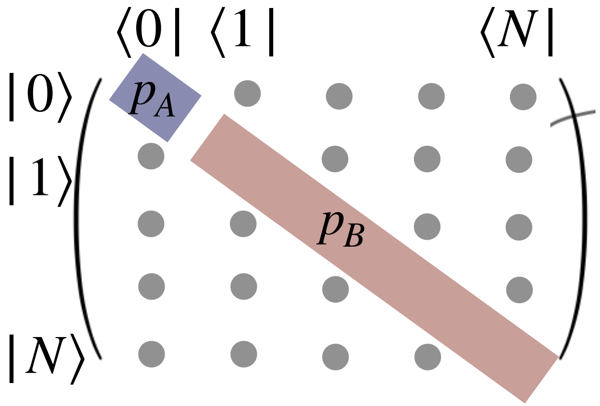
$$|\bar{0}\rangle = |0\rangle$$

$$|\bar{1}\rangle = |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{N} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

with $|\alpha| \gg 1$

so that $\langle 0|\alpha\rangle = e^{-|\alpha|^2/2} \approx 0$

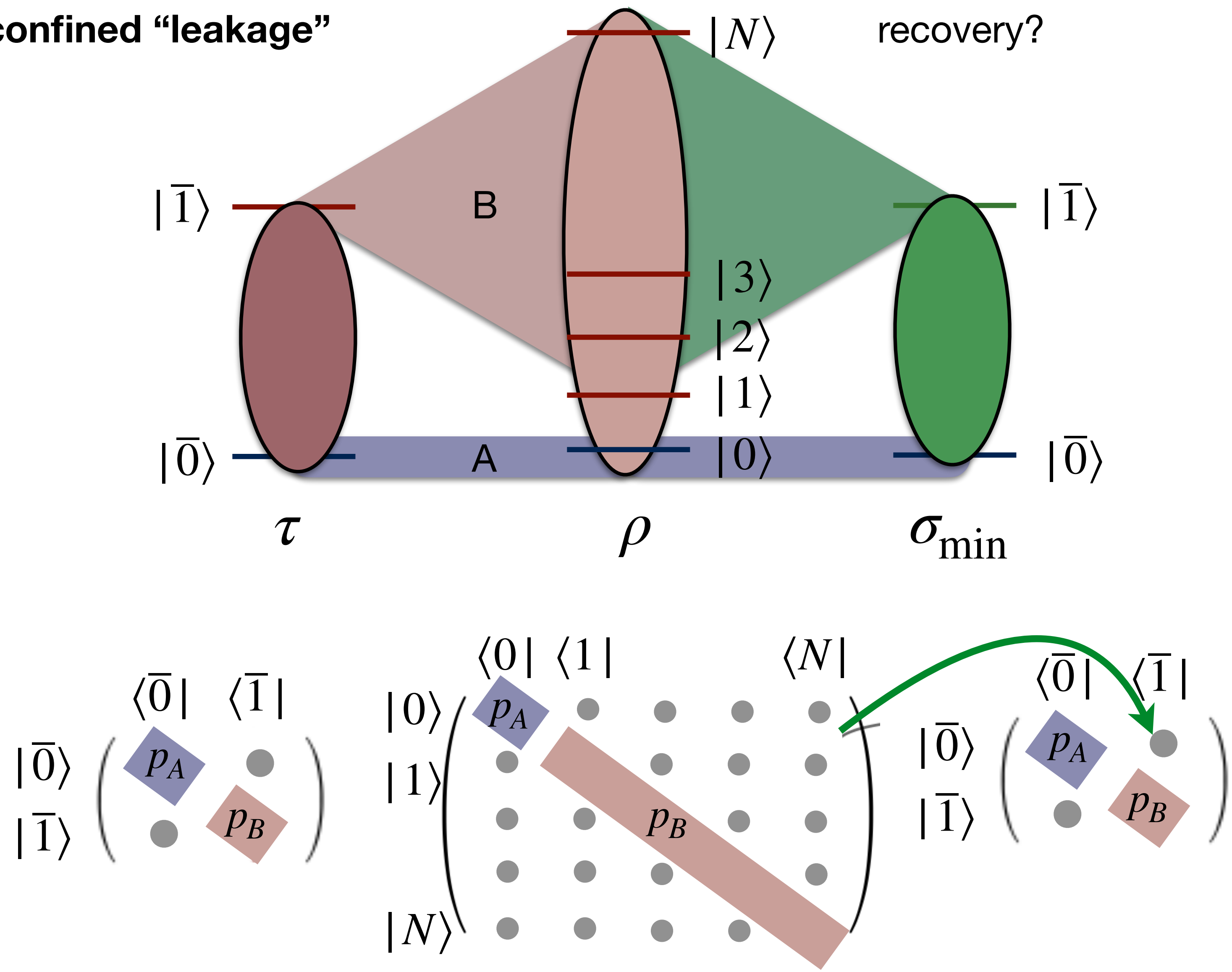




Recovery?

Assume:

subspace-confined “leakage”

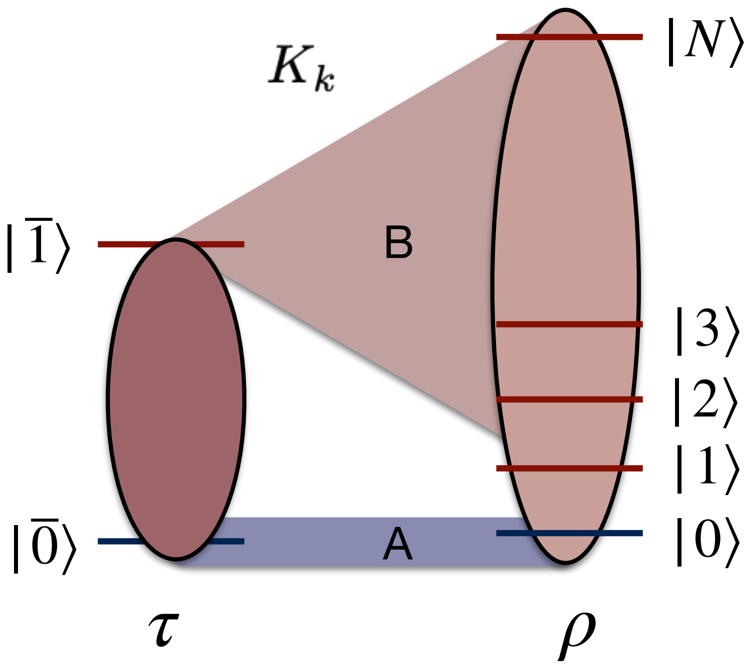


Formalising the noise

Experimental state arises from noise acting on initial qubit state:

$$\rho = \sum_{k=1}^{\kappa} K_k \, \tau \, K_k^\dagger$$

set of (N+1)x2-dimensional Krausoperators



noise acts on separate subspaces $\mathscr{H}_A$ and $\mathscr{H}_B$, with projectors $\Pi^A = |0\rangle\langle 0|$ and $\Pi^B = \mathbb{1}_{N+1} - \Pi^A$

$$K_k = \sum_{X=A,B} \Pi^X \, K_k \, \overline{\Pi}^X \quad \text{(subspace-confined leakage)}$$

$$\sum_{k=1}^{\kappa} K_k^\dagger \, \Pi^X \, K_k = \overline{\Pi}^X \text{ for } X = A, B \text{ (completeness).}$$

Problem statement

To summarise, we seek τ's recovered qubit state $\sigma_{\rm min}$ that fulfils the following conditions:

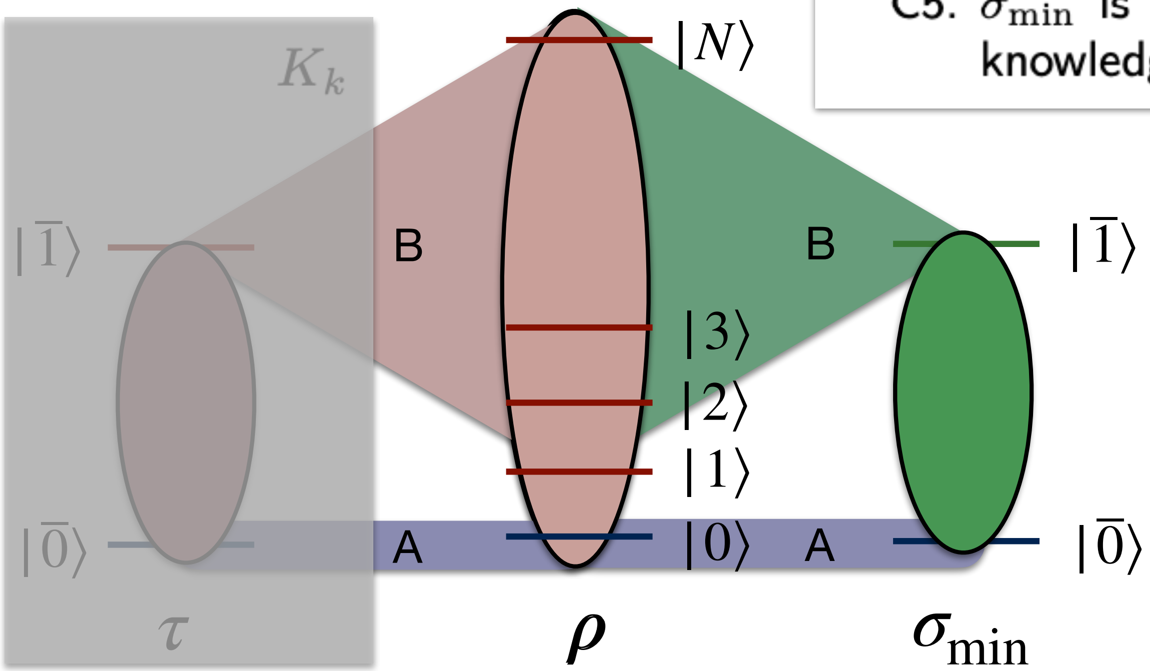
C1. $\sigma_{\rm min}$ is a density matrix in the qubit space, spanned by $\{|\bar{0}\rangle, |\bar{1}\rangle\}$,

C2. $\sigma_{\rm min}$ is constructed using the … Kraus operators act on separa… B, see Eq. (3), but without re… set $\{K_k\}$,

C3. $\sigma_{\rm min}$ has τ's subspace probabilities $\text{tr}[\sigma_{\rm min}\,\Pi^X] = p_X = \text{tr}[\rho\,\Pi^X]$ for $X = A, B$,

C4. $\sigma_{\rm min}$ has the smallest possible entropy that is consistent with $\mathcal{S}(\tau) \le \mathcal{S}(\sigma_{\rm min})$,

C5. $\sigma_{\rm min}$ is obtained from the noisy state ρ without knowledge of the initial state τ.



condition C4: $S(\tau) \le S(\sigma_{\rm min})$

$S(\tau) \le S(\rho)$

we also expect: $S(\sigma_{\rm min}) \le S(\rho)$

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Recovery map

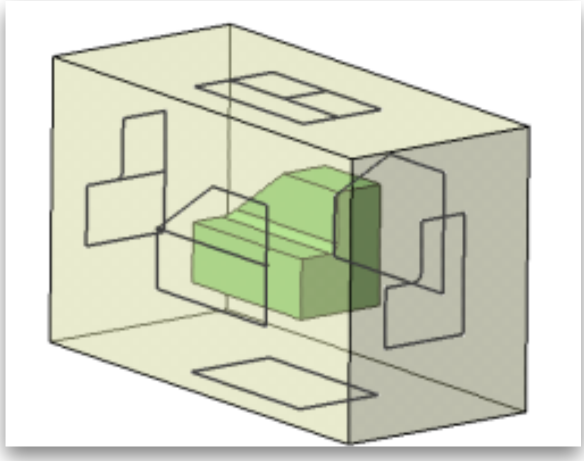
To reduce dimension, need projection

$$M^\dagger \rho M = \sigma_M$$

candidate qubit matrix

any (N+1)x2-dimensional matrix

note: coefficients of M
may depend on ρ (nonlinear map)



https://www.technia.us/blog/3rd-angle-projection/

Expand in qubit basis

$$M^\dagger = e^{-i\varphi_M} |\bar{0}\rangle\langle 0| + g_M^2 \, |\bar{1}\rangle\langle\psi_M|$$

can only map to $\langle 0 |$

can only map to one, arbitrary direction $\langle \psi_M |$ in subspace $\mathscr{H}_B$

with weight $g_M^2 = \sqrt{\frac{p_B}{\langle\psi_M|\rho|\psi_M\rangle}}$ so that $\langle \bar{1} | \sigma_M | \bar{1} \rangle = p_B$

Obtain qubit state

$$\sigma_M = p_A \overline{\Pi}^A + p_B \overline{\Pi}^B + c_M \, |\bar{0}\rangle\langle\bar{1}| + h.c.$$

with coherence $c_M = \sqrt{\frac{p_B}{\langle\psi_M|\rho|\psi_M\rangle}} e^{-i\varphi_M} \langle 0|\rho|\psi_M\rangle$

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Minimising entropy

Have qubit state

$$\sigma_M = p_A \overline{\Pi}^A + p_B \overline{\Pi}^B + c_M\, |\bar{0}\rangle\langle\bar{1}| + h.c.$$

with coherence $c_M = \sqrt{\frac{p_B}{\langle\psi_M|\rho|\psi_M\rangle}}\, e^{-i\varphi_M} \langle 0|\rho|\psi_M\rangle$

Entropy

$$S(\sigma_M) = -\lambda_1 \log_2 \lambda_1 - \lambda_2 \log_2 \lambda_2$$

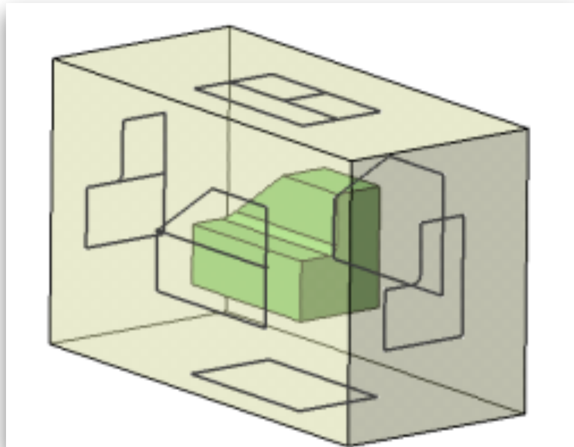
where eigenvalues are $\lambda_1 + \lambda_2 = 1$

$$\lambda_1 \lambda_2 = p_A\, p_B - |c_M|^2$$

depends on |coherence|, which depends on $|\psi_M\rangle$

! Find candidate state with minimum entropy

$$\sigma_{\min} := \arg\min_{\sigma_M}(S(\sigma_M)) = p_A \overline{\Pi}^A + p_B \overline{\Pi}^B + |c_{\min}|\, |\bar{0}\rangle\langle\bar{1}| + h.c.,$$



https://www.technia.us/blog/3rd-angle-projection/

Minimising entropy

Answer

$$|c_{\min}| = \sqrt{|r|^2 \sum_{n=1}^{N} \frac{\langle n|\chi\rangle\langle\chi|n\rangle}{\langle n|\rho^B_{\text{diag}}|n\rangle}}.$$

where $\rho^B_{\text{diag}} = U_{\text{diag}}\, \rho^B\, U^\dagger_{\text{diag}}$

with $\rho^B = \Pi^B \rho\, \Pi^B / p_B$

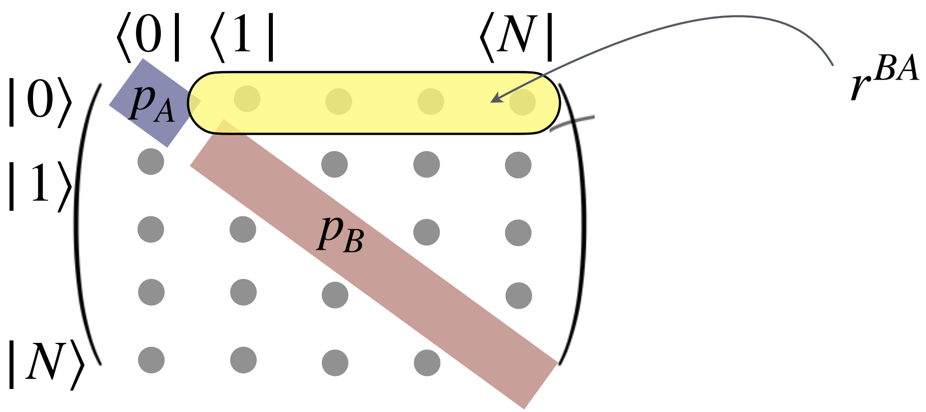
is the diagonal, trace one, density matrix in $\mathscr{H}_B$

and $|\chi\rangle = \frac{1}{|r|} U_{\text{diag}}\, r^{BA}\, |0\rangle$

with $r^{BA} = \Pi^B \rho\, \Pi^A$

and $|r|^2 = \text{tr}[\rho\, \Pi^B \rho\, \Pi^A]$

is a normalised state in $\mathscr{H}_B$



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To summarise, we seek τ's recovered qubit state $\sigma_{\min}$ that fulfils the following conditions:

C1. $\sigma_{\min}$ is a density matrix in the qubit space, spanned by $\{|\bar{0}\rangle, |\bar{1}\rangle\}$, ✓

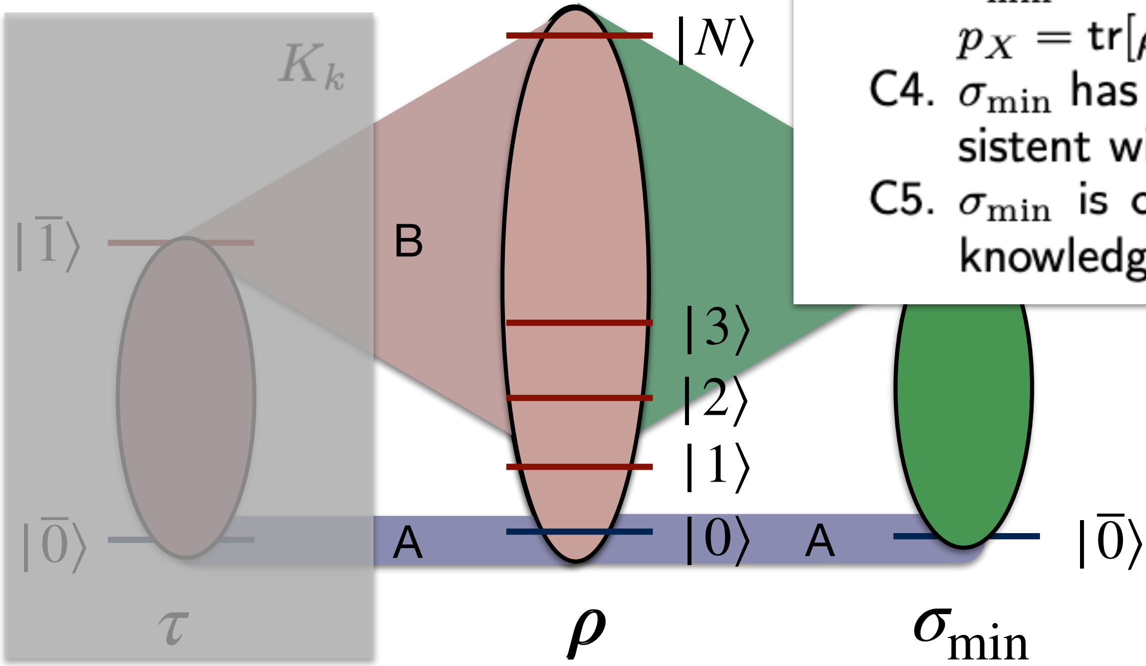
C2. $\sigma_{\min}$ is constructed using the knowledge that the Kraus operators act on separate subspaces A and B, see Eq. (3), but without reference to a specific set $\{K_k\}$, ✓

$$M^\dagger \rho M = \sigma_M$$

C3. $\sigma_{\min}$ has τ's subspace probabilities $\mathrm{tr}[\sigma_{\min} \Pi^X] = p_X = \mathrm{tr}[\rho \Pi^X]$ for $X = A, B$, ✓

C4. $\sigma_{\min}$ has the smallest possible entropy ✓ that is consistent with $S(\tau) \leq S(\sigma_{\min})$, ?

C5. $\sigma_{\min}$ is obtained from the noisy state ρ without knowledge of the initial state τ. ✓



condition C4: $S(\tau) \leq S(\sigma_{\min})$?

$$S(\tau) \leq S(\rho)$$

we also expect: $S(\sigma_{\min}) \leq S(\rho)$?

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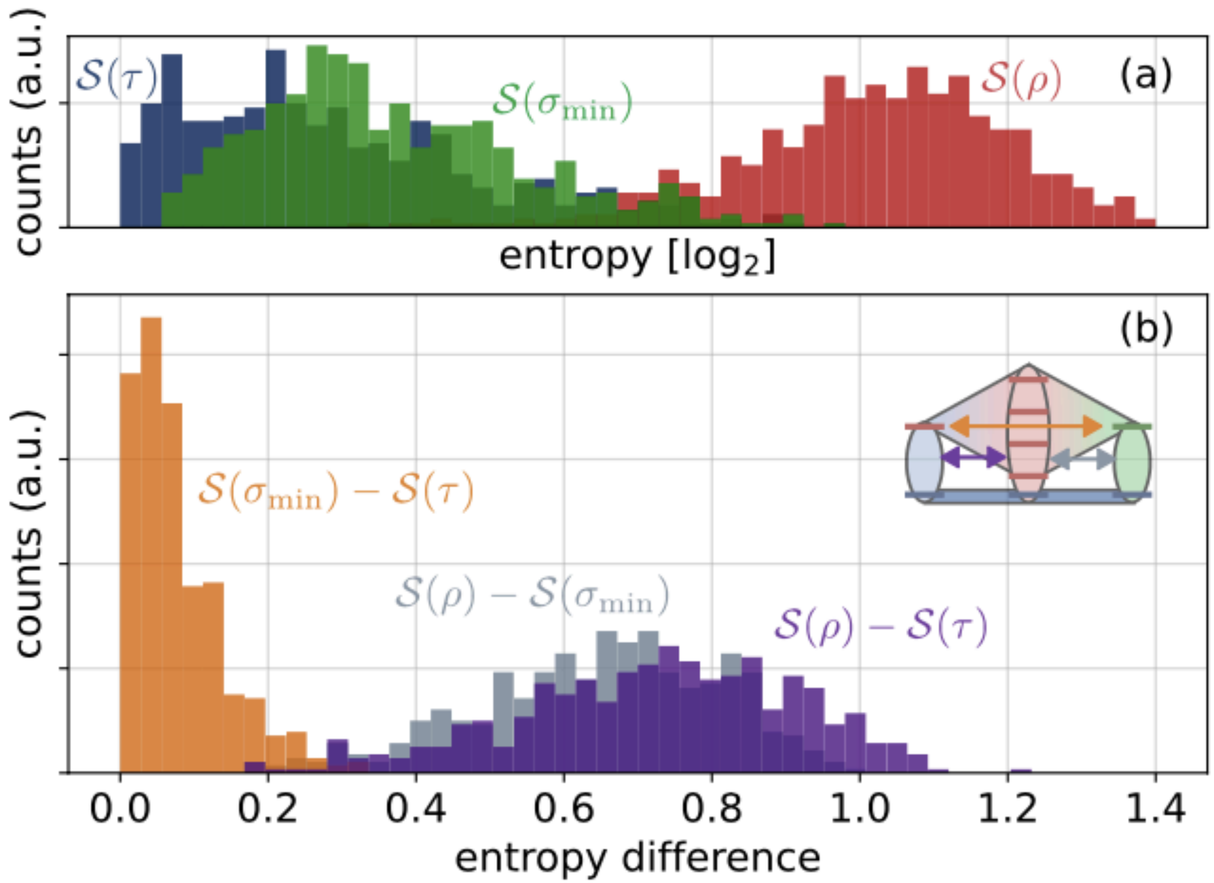
Example: Random samples

generate $L = 500$ random initial qubit states τ

generate L random sets of subspace-confining Krausoperators $\{K_k\}_{k=1}^{\kappa=6}$

construct L noisy states $\rho = \sum_k K_k \, \tau \, K_k^\dagger$

apply recovery method to each ρ to obtain L recovered qubit states $\sigma_{\min}$



condition C4: $S(\tau) \leq S(\sigma_{\min})$ ✔

$S(\tau) \leq S(\rho)$

we also expect: $S(\sigma_{\min}) \leq S(\rho)$ ✔

Example: Random samples

generate $L = 500$ random initial qubit states τ

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construct L noisy states $\rho = \sum_k K_k \, \tau \, K_k^\dagger$

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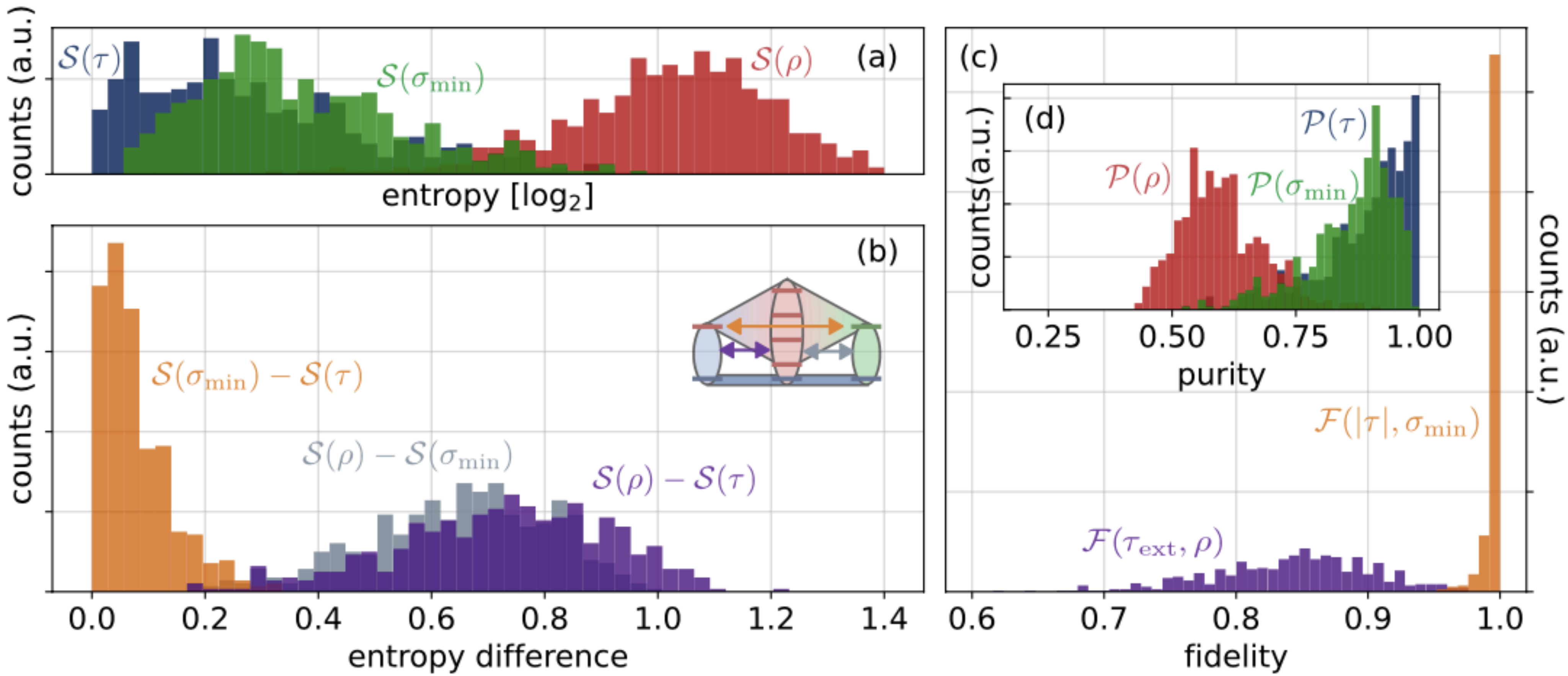

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Reading the demon's mind

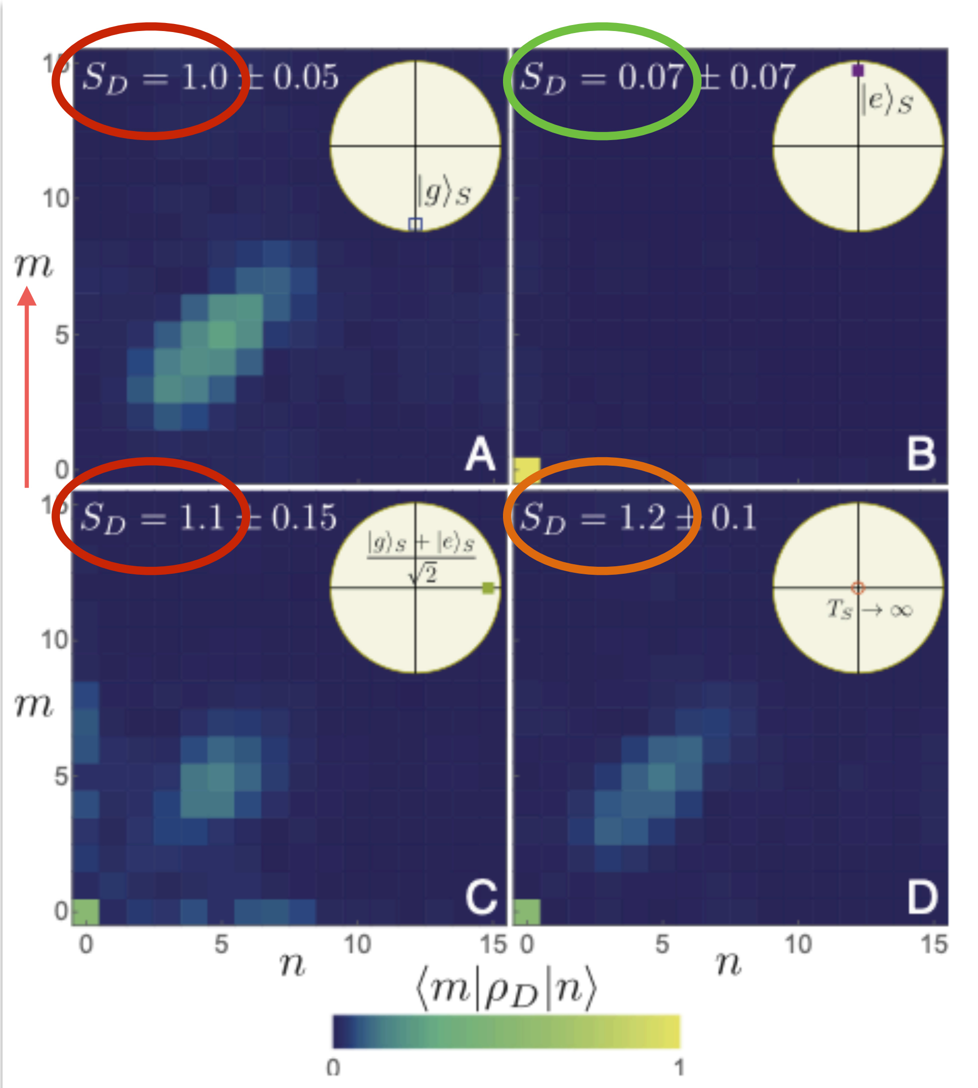
Tomography of the demon (cavity) state ρ

expected:

$|\alpha\rangle \qquad S = 0$

expected:

$|0\rangle \qquad S = 0$



expected:

$\frac{|0\rangle + |\alpha\rangle}{\sqrt{2}} \qquad S = 0$

expected:

$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|\alpha\rangle\langle\alpha|$

$S = \log_2 2 = 1$

Cottet, …, Huard, PNAS (2017)

converting to entropy to base-2

Janet Anders janet@qipc.org

Reading the demon’s mind

Tomography of the demon (cavity) state ρ

expected:

$|\alpha\rangle \qquad S = 0$

actual, noisy:

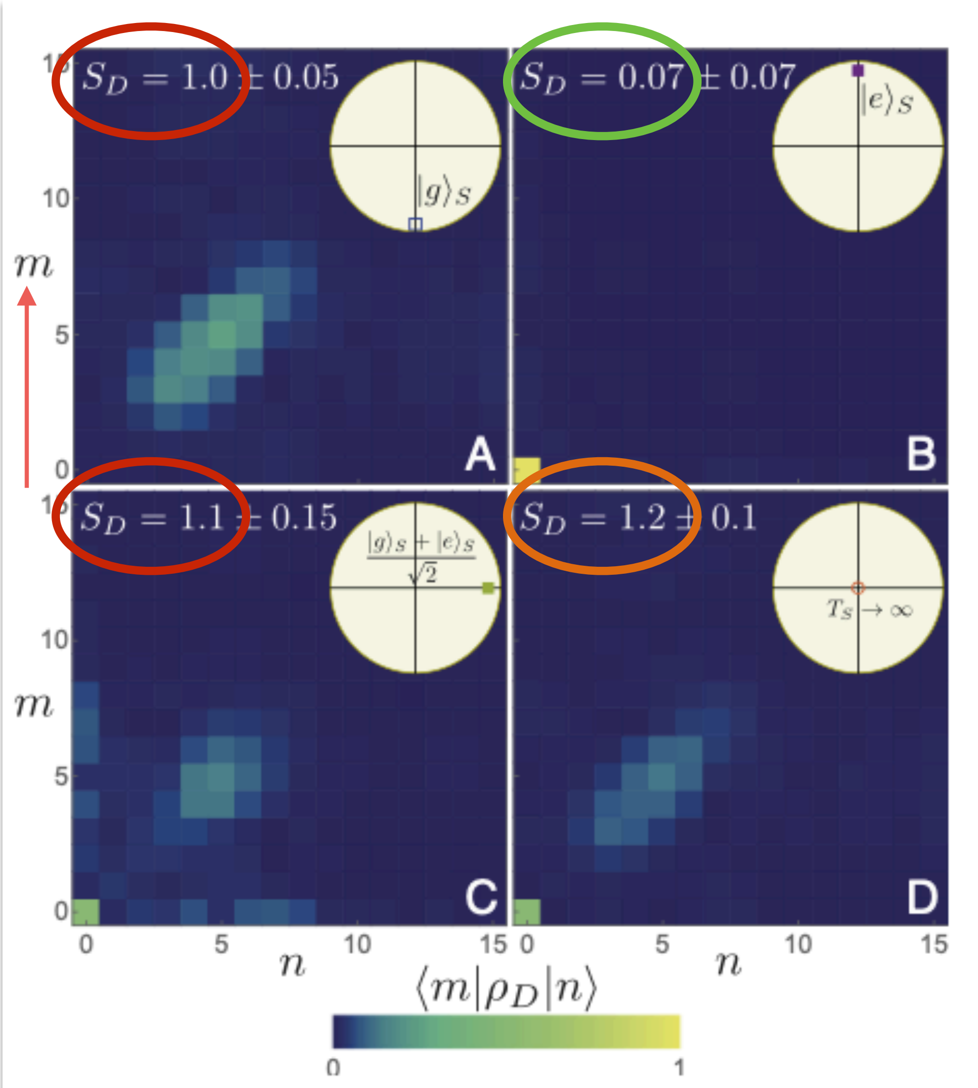
$S_D = S(\rho) = 1.34$

expected:

$|0\rangle \qquad S = 0$

actual, noisy:

$S_D = S(\rho) = 0.08$



expected:

$\frac{|0\rangle + |\alpha\rangle}{\sqrt{2}} \qquad S = 0$

actual, noisy:

$S_D = S(\rho) = 1.50$

expected:

$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|\alpha\rangle\langle\alpha|$

$S = \log_2 2 = 1$

actual, noisy:

$S_D = S(\rho) = 1.73$

Cottet, …, Huard, PNAS (2017)

converting to entropy to base-2

Reading the demon's mind

Tomography of the demon (cavity) state ρ

expected:

$|\alpha\rangle \qquad S = 0$

actual, noisy:

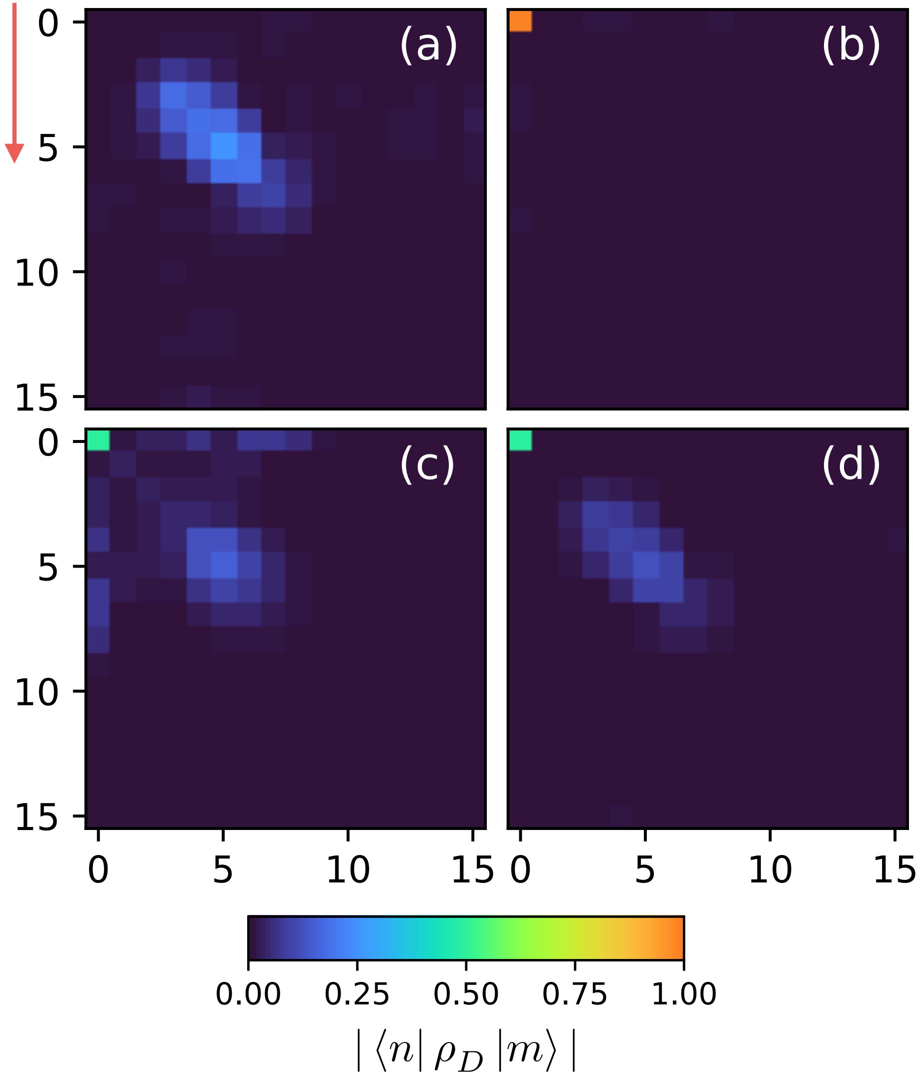
$S_D = S(\rho) = 1.34$

expected:

$|0\rangle \qquad S = 0$

actual, noisy:

$S_D = S(\rho) = 0.08$



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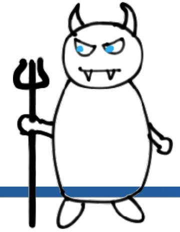
entropy to base-2

Reading the demon's mind



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Tomography of the demon (cavity) state ρ

expected:

$|\alpha\rangle \qquad S = 0$

actual, noisy:

$S_D = S(\rho) = 1.34$

$S(\sigma_{\min}) = 0.00$

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actual, noisy:

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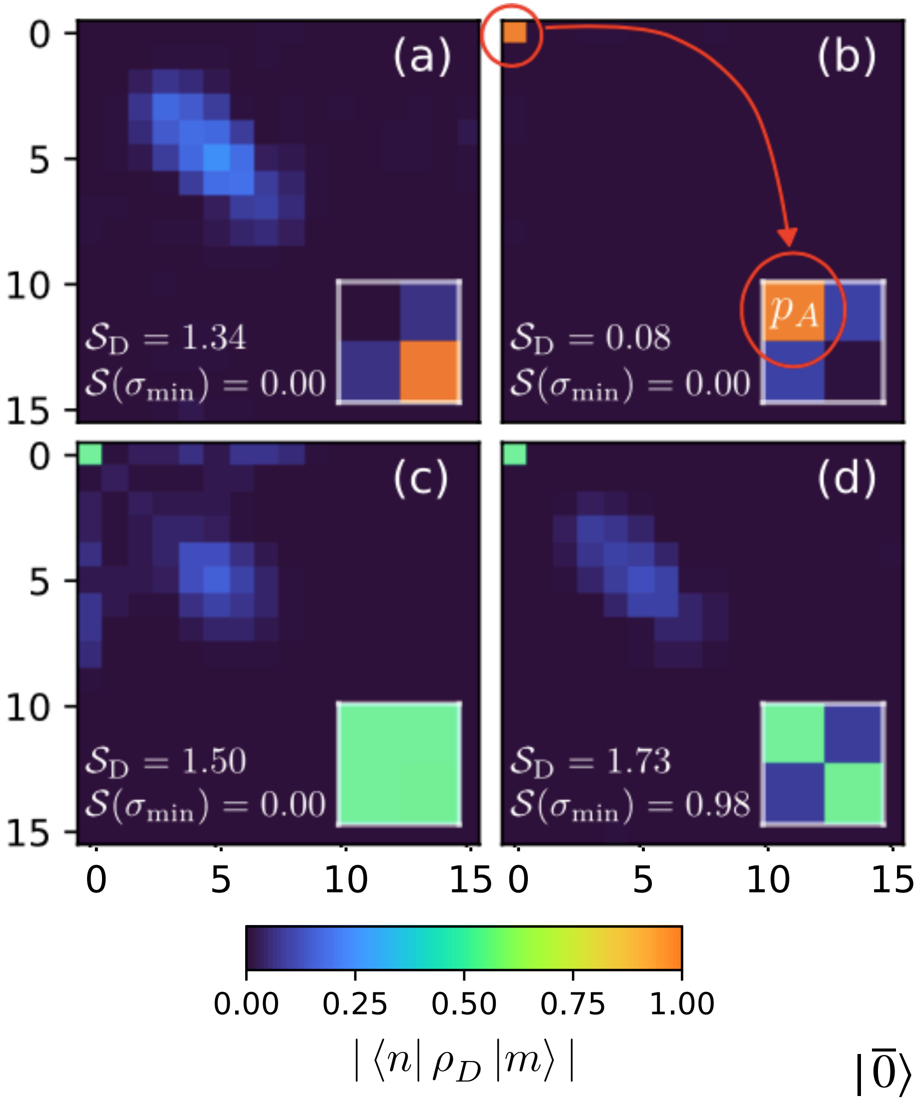


entropy to base-2

expected:

$|0\rangle \qquad S = 0$

actual, noisy:

$S_D = S(\rho) = 0.08$

$S(\sigma_{\min}) = 0.00$

expected:

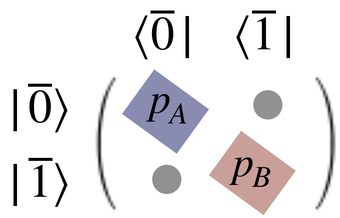
$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|\alpha\rangle\langle\alpha|$

$S = \log_2 2 = 1$

actual, noisy:

$S_D = S(\rho) = 1.73$

$S(\sigma_{\min}) = 0.98$



Janet Anders janet@qipc.org

Outline

- Physical encoding and noise
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Quantum Work distributions
Open quantum system equilibrium

Janet Anders janet@qipc.org

Work in the quantum regime

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FAST TRACK COMMUNICATION

Time-reversal symmetric work distributions for closed quantum dynamics in the histories framework

Harry J D Miller and Janet Anders
CEMPS, Physics and Astronomy, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

Harry Miller
Exeter -> Manchester

HJD Miller, J Anders,
New J. Phys. 19, 062001 (2017)

Einstein's enquiry to Bohr if 'the Moon does not exist if nobody is looking at it' questions the indeterminate nature of a quantum state when it is not measured [1].

Work in the quantum regime

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New Journal of Physics
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FAST TRACK COMMUNICATION

Time-reversal symmetric work distributions for closed quantum dynami

Harry J D M
CEMPS, Physi

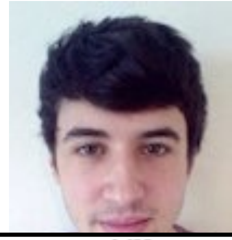

Table 1. Comparison between the work *quasi-probability distributions* $p(w)$ given in equation (17) and $p^{\text{MH}}(w)$ given in equation (40) and the work probability distributions $p^{\text{tpm}}(w)$ given in equation (7) and $\tilde{p}(w)$ given in equation (29).

(i)	(ii)	(iii) operator	(iv) steps	(v) energy conservation	(vi) time-reversal symmetric	(vii) JE satisfied
$p^{\text{tpm}}(w)$	+	H	2	$\forall\rho, [H_H(\tau), H(0)] = 0$ or $\forall H_H(\tau), [\rho, H(0)] = 0$	$\forall\rho, [H_H(\tau), H(0)] = 0$ or $\forall H_H(\tau), [\rho, H(0)] = 0$	$\forall H(t)$
$p(w)$	−	X	$K \to \infty$	$\forall\rho, \forall H(t)$	$\forall\rho, \forall H(t)$	$[H_H(\tau), H(0)] = 0$
$\tilde{p}(w)$	+	X	1	$\forall\rho, H_H(\tau) = H(\tau)$	$\text{Tr}\ [\sum_{\vec{n}\in w}[C_{\vec{n}}, C_{\vec{n}}^{\dagger}]\ \rho] = 0, \forall w$	$H_H(\tau) = H(\tau)$
$p^{\text{MH}}(w)$	−	H	2	$\forall\rho, \forall H(t)$	$\forall\rho, \forall H(t)$	$\forall H(t)$

Note. The columns refer to: (i) symbol of work distribution, (ii) work distribution is positive in general (+) or can be negative for some states and evolutions (−), (iii) operator used to establish the work distribution: power operator X or Hamiltonian H, (iv) number of points in time considered in the construction of the work distribution for $\Delta t \to 0$. Columns (v), (vi) and (vii) list conditions under which the work distributions satisfy energy conservation in the limit $\Delta t \to 0$, time-reversal symmetry, and the Jarzynski equality (JE) assuming an initial thermal state and the limit $\Delta t \to 0$. All four work distributions become equal in the classical limit where ρ, $H_H(0)$ and $H_H(\tau)$ commute.

Fluctuating **work** performed on an undisturbed quantum system is described by a **quasi-probability** distribution rather than a probability distribution.

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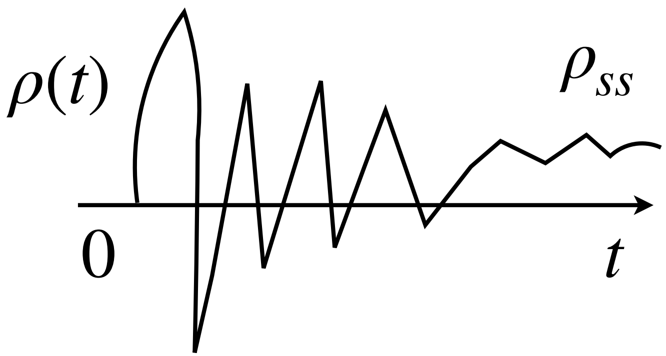
Quantum Work distributions

Open quantum system equilibrium

Janet Anders janet@qipc.org

Steady state



equilibration

Gibbs state

$\tau \propto e^{-\beta H_S}$

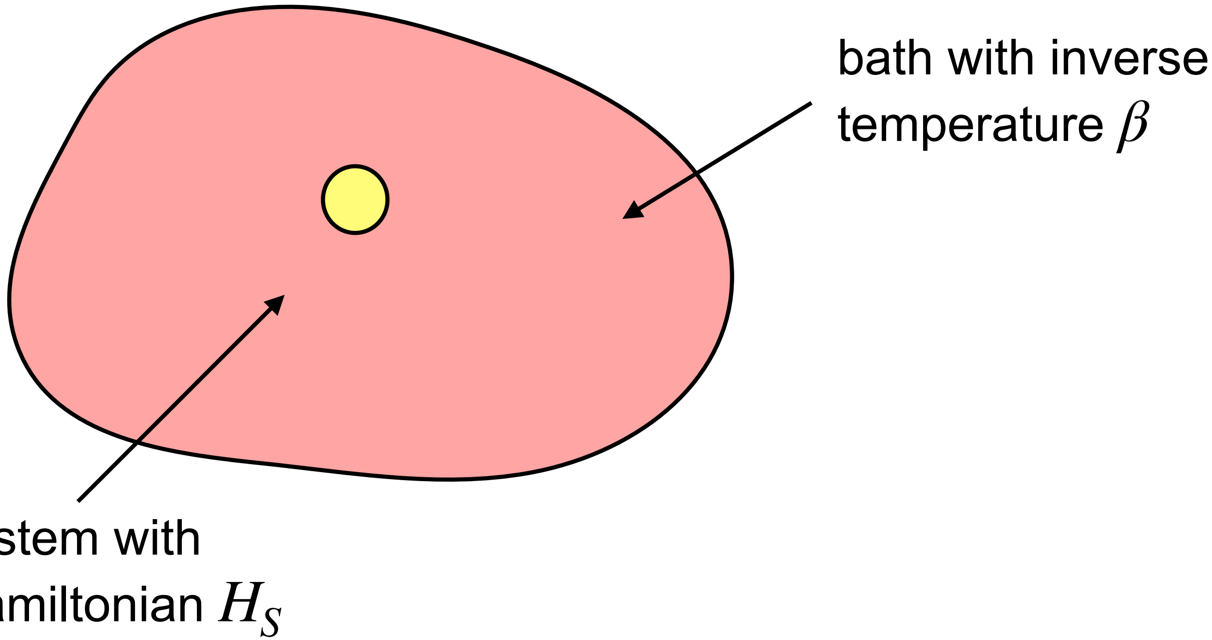
James (Jim) Cresser

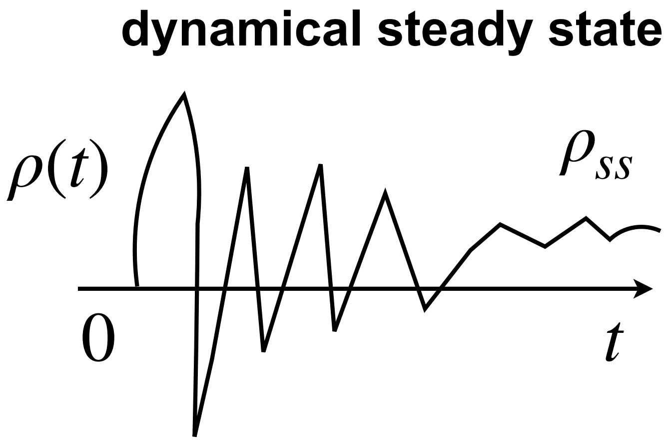
Anton Trushechkin

Marco Merkli

[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

Steady state





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[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

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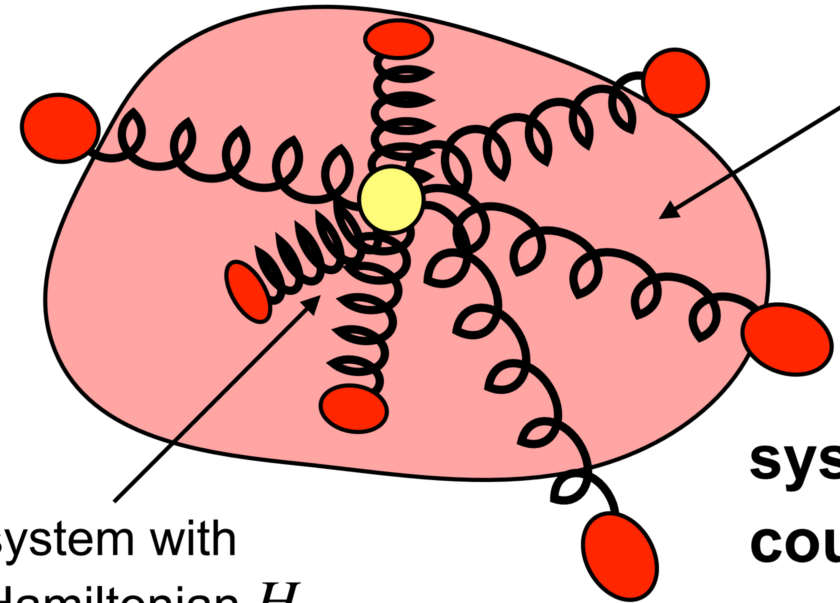
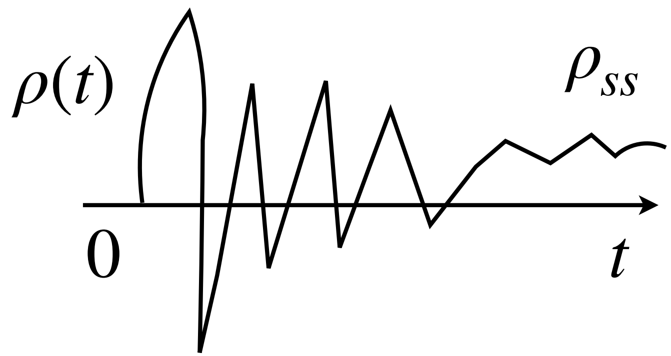


bath with inverse temperature β

system with Hamiltonian H_S

system-bath coupling λ

dynamical steady state



$$H_{tot} = H_S + H_B + \lambda\, V_{int}$$

equilibration

mean force Gibbs state (MFGS)

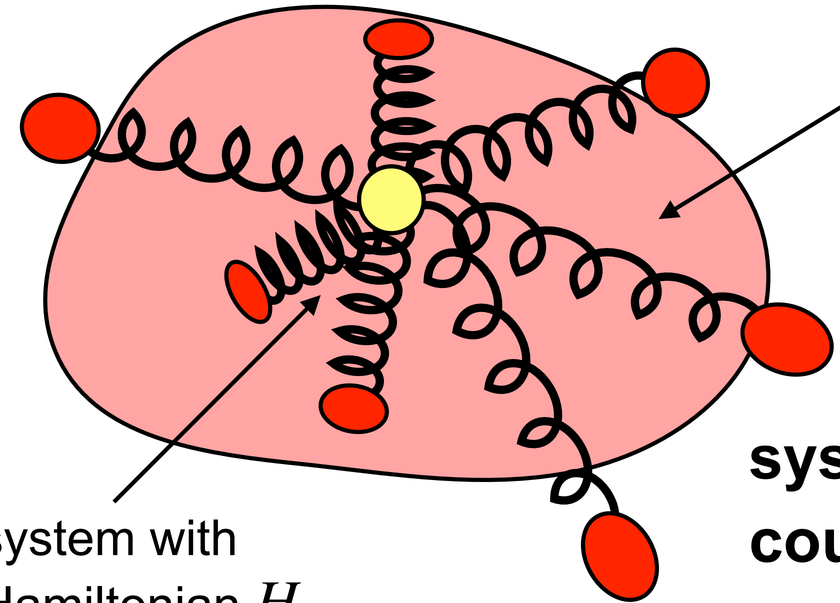
$$\tau_{MF} \propto \mathrm{tr}_B[e^{-\beta H_{tot}}]$$

for $\lambda \to 0$

$$\approx \mathrm{tr}_B[e^{-\beta H_S}\, e^{-\beta H_B}] \propto e^{-\beta H_S}$$

[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

Steady state



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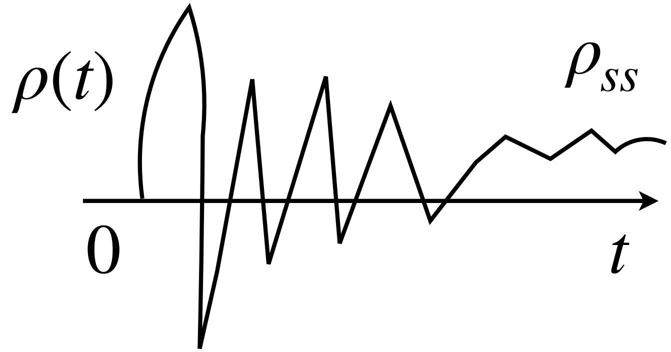


$$H_{tot} = H_S + H_B + \lambda\, V_{int}$$

equilibration

mean force Gibbs state (MFGS)

$$\tau_{MF} \propto \mathrm{tr}_B[e^{-\beta H_{tot}}]$$

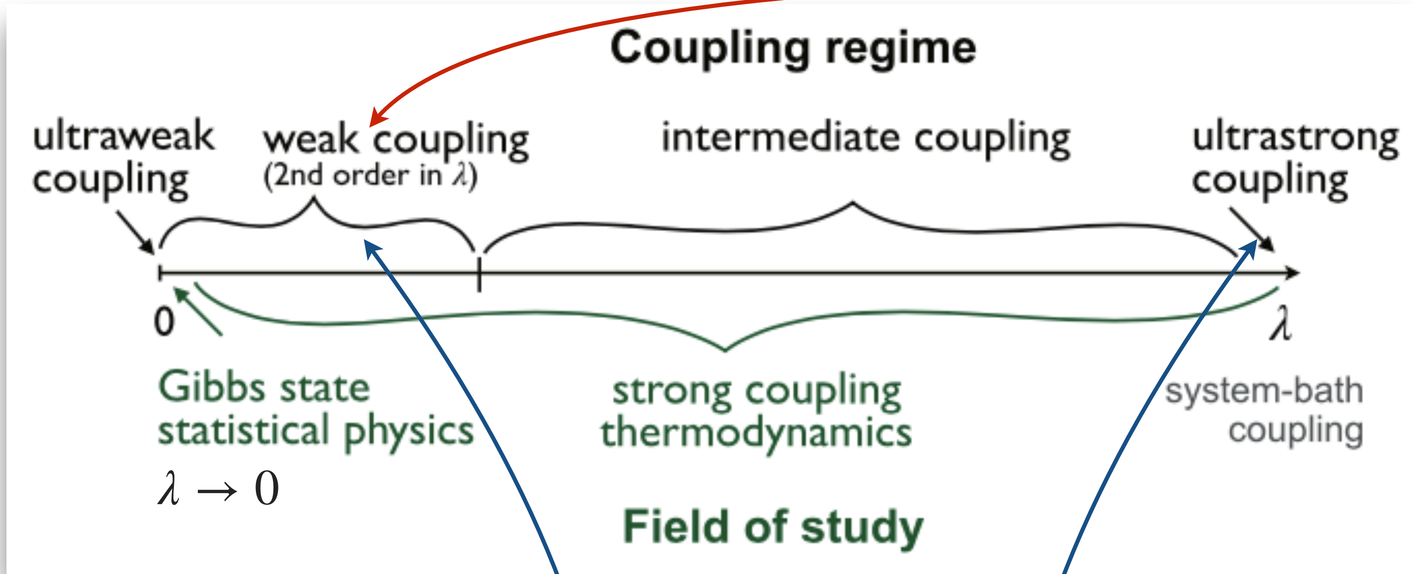
can one give the MFGS in terms of system operators alone?

and is it the steady state?

[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

Coupling regimes

most master equations



$$\frac{1}{\lambda} \to 0$$

$$H_{tot} = H_S + H_B + \lambda\, V_{int}$$

general **analyt. expression** for **quantum** mean force state

[1] Cresser, Anders, **PRL** 127, 250601 (2021)
[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

Weak coupling limit $(\lambda \ll 1)$

$$\tau_{MF} \propto \mathrm{tr}_R \left[e^{-\beta\left(H'_S + \int_0^\infty d\omega\, \omega \left(b_\omega^\dagger b_\omega + \frac{1}{2}\right) + \lambda X_S \otimes \int_0^\infty d\omega \sqrt{J(\omega)}(b_\omega + b_\omega^\dagger)\right)} \right]$$

general system

bosonic bath

coupling: system operator X_S to bath osci positions

corrections to second order in λ

$$\tau_{MF}^{(2)} = \tau_S$$

standard Gibbs state

[1] Cresser, Anders, **PRL** 127, 250601 (2021)

Weak coupling limit $(\lambda \ll 1)$

$$\tau_{MF} \propto \mathrm{tr}_R \left[e^{-\beta\left(H'_S + \int_0^\infty d\omega\, \omega \left(b^\dagger_\omega b_\omega + \frac{1}{2}\right) + \lambda X_S \otimes \int_0^\infty d\omega \sqrt{J(\omega)}(b_\omega + b^\dagger_\omega)\right)} \right]$$

general system — bosonic bath — coupling: system operator X_S to bath osci positions

corrections to second order in λ

$$\tau^{(2)}_{MF} = \tau_S + \lambda^2 \beta \sum_n \tau_S \left(X_n X_n^\dagger - \mathrm{tr}_S \left[\tau_S\, X_n X_n^\dagger\right]\right) \mathscr{D}_\beta(\omega_n)$$
$$+ \lambda^2 \sum_n \left[X_n^\dagger, \tau_S X_n\right] \frac{\partial \mathscr{D}_\beta(\omega_n)}{\partial \omega_n} + \lambda^2 \sum_{m \neq n} \left(\left[X_m, X_n^\dagger \tau_S\right] + h.c. \right) \frac{\mathscr{D}_\beta(\omega_n)}{\omega_m - \omega_n}$$

standard Gibbs state

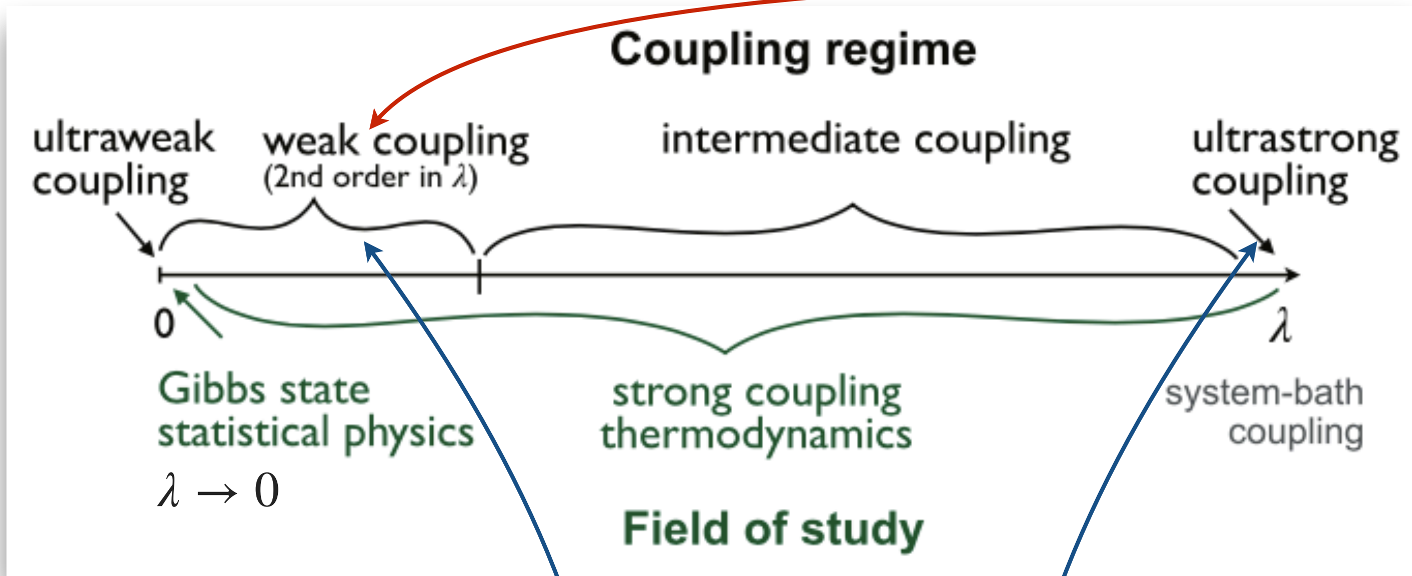
valid for small λ, i.e.: $$|\lambda| \ll \frac{1}{\sqrt{|\beta \sum_n \mathrm{Tr}_S[\tau_S X_n X_n^\dagger] \mathcal{D}_\beta(\omega_n)|}}$$

➠ validity range depends on temperature (higher temperature = easier to fulfil)

[1] Cresser, Anders, **PRL** 127, 250601 (2021)

Coupling regimes

most master equations



$$\frac{1}{\lambda} \to 0$$

$$H_{tot} = H_S + H_B + \lambda\, V_{int}$$

general **analyt. expression** for **quantum** mean force state

[1] Cresser, Anders, **PRL** 127, 250601 (2021)
[2] Trushechkin, Merkli, Cresser, Anders, **AVS Quantum Sci.** 4, 012301 (2022)

Ultrastrong coupling limit

$$\frac{1}{\lambda} \to 0$$

$$\tau_{MF} \propto \mathrm{tr}_R \left[e^{-\beta\left(H'_S + \int_0^\infty d\omega\, \omega \left(b^\dagger_\omega b_\omega + \frac{1}{2}\right) + \lambda X_S \otimes \int_0^\infty d\omega \sqrt{J(\omega)}(b_\omega + b^\dagger_\omega)\right)} \right]$$

general system

bosonic bath

coupling: system operator X_S to bath osci positions

eigenstates of X_S : $X_S |x_n\rangle = x_n |x_n\rangle$

projectors: $P_n = |x_n\rangle\langle x_n|$

conjectured steady state:

$$\rho_{ss} \stackrel{?}{=} \sum_n P_n \tau_S P_n$$

Goyal, Kawai, Phys. Rev. Res. 1, 033018 (2019)

[1] Cresser, Anders, **PRL** 127, 250601 (2021)

Ultrastrong coupling limit $\frac{1}{\lambda} \to 0$

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$$\tau_{MF} \propto \mathrm{tr}_R \left[e^{-\beta\left(H'_S + \int_0^\infty d\omega\, \omega \left(b^\dagger_\omega b_\omega + \frac{1}{2}\right) + \lambda X_S \otimes \int_0^\infty d\omega \sqrt{J(\omega)}(b_\omega + b^\dagger_\omega)\right)} \right]$$

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projectors: $P_n = |x_n\rangle\langle x_n|$

quantum ultrastrong MF state

$$\tau_{MF} = \frac{e^{-\beta \sum_n P_n H_S P_n}}{tr[e^{-\beta \sum_n P_n H_S P_n}]}$$

$\neq$

conjectured steady state:

$$\rho_{ss} \stackrel{?}{=} \sum_n P_n \tau_S P_n$$

[1] Cresser, Anders, **PRL** 127, 250601 (2021)

Goyal, Kawai, Phys. Rev. Res. 1, 033018 (2019)

Ultrastrong coupling limit

$$\frac{1}{\lambda} \to 0$$

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$$\tau_{MF} \propto \mathrm{tr}_R \left[e^{-\beta\left(H'_S + \int_0^\infty d\omega\, \omega \left(b^\dagger_\omega b_\omega + \frac{1}{2}\right) + \lambda X_S \otimes \int_0^\infty d\omega \sqrt{J(\omega)}(b_\omega + b^\dagger_\omega)\right)} \right]$$

general system

bosonic bath

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eigenstates of X_S : $X_S |x_n\rangle = x_n |x_n\rangle$

projectors: $P_n = |x_n\rangle\langle x_n|$

quantum
ultrastrong MF state

$$\tau_{MF} = \frac{e^{-\beta \sum_n P_n H_S P_n}}{tr[e^{-\beta \sum_n P_n H_S P_n}]}$$

Trushechkin, arXiv:2109.01888 (2021)

Now proven to be the steady state of an **ultrastrong coupling master equation**

[1] Cresser, Anders, **PRL** 127, 250601 (2021)

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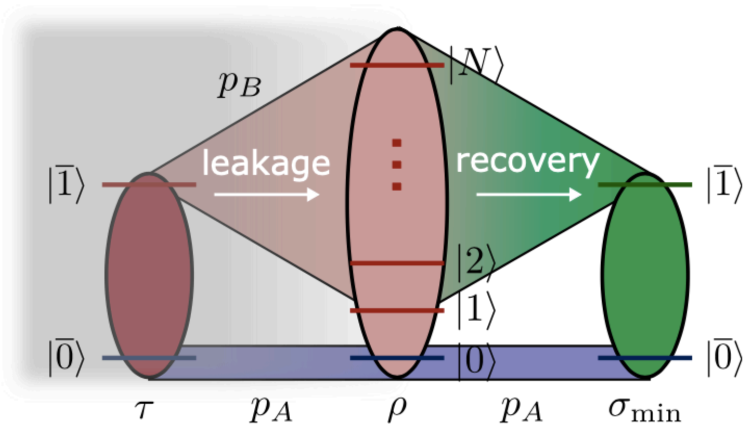
- Summary

Summary

Formalised common problem in quantum experiments and technology: noise is always there, but often has subspace structure.

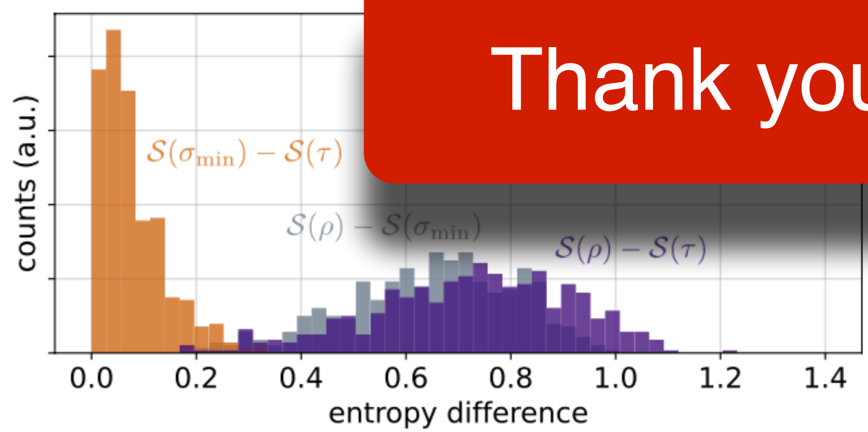
$$K_k = \sum_{X=A,B} \Pi^X K_k \Pi^X \quad \text{(subspace-confined leakage)}$$



Solved recovery problem.

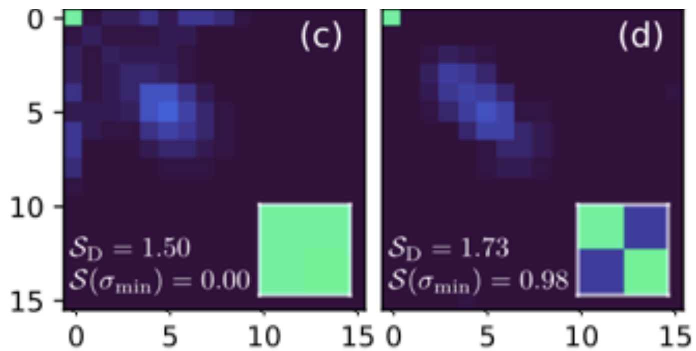
Have provided a formula of how to compute the |coherence| for the absolute **best guess qubit state $\sigma_{\min}$ compatible with a given noisy state ρ.**

Thank you!



Provided numerical illustration of recovery method on **random samples.**

Proof of two entropy bounds.



Uncovered "buried" **experimental** Maxwell demon **qubit states.**

our paper will be on arxiv this March - talk to me for more information

Janet Anders janet@qipc.org