Entropic fluctuations in quantum two-time measurement framework

joint work in progress with T.Benoist, L.Bruneau, V.Jakšić, C.-A.Pillet

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Classical Non Equilibrium Statistical Mechanics

Classical System $(\mathcal{M}, \phi_t, \omega)$ dynamical system
ω reference state= probability measure on $\mathcal{M}$
$\omega_t = \omega \circ \phi_{-t}$

Relative entropy: $Ent(\nu|\mu) = -\nu(\log \frac{d\nu}{d\mu})$ (if ν and μ are absolutely continuous)

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$\Sigma_t := Ent(\omega_t|\omega) - Ent(\omega|\omega) = -\int_0^t \omega_s(\sigma)\mathrm{d}s$
Σ_t entropy production
σ entropy production observable

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Σ_t can be viewed as a random variable on $(\mathcal{M}, \omega)$
We denote its law by $\mathbb{P}_{\Sigma_t,\omega}$

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Statistical refinement of thermodynamics second law

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If $\mathbb{P}_{\Sigma_t,\omega}$ is the law of the random variable Σ_t on $(\mathcal{M}, \omega)$ and $\bar{\mathbb{P}}_{\Sigma_t,\omega}(s) = \mathbb{P}_{\Sigma_t,\omega}(-s)$.
If ω if equilibrium state at temperature β, or a multi-thermal state and the system is T.R.I.

$$\frac{\mathrm{d}\bar{\mathbb{P}}_{\Sigma_t,\omega}}{\mathrm{d}\mathbb{P}_{\Sigma_t,\omega}}(s) = \mathrm{e}^{-s}$$

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$$\frac{\mathrm{d}\bar{\mathbb{P}}_{\Sigma_t,\omega}}{\mathrm{d}\mathbb{P}_{\Sigma_t,\omega}}(s) = \mathrm{e}^{-s}$$

This is called classical transient (ES) fluctuation relation

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$$\frac{\mathrm{d}\overline{\mathbb{P}}_{\Sigma_t,\omega}}{\mathrm{d}\mathbb{P}_{\Sigma_t,\omega}}(s) = \mathrm{e}^{-s}$$

equivalent to

$$\mathcal{F}^{cl}_{\omega,t}(\alpha) = \mathcal{F}^{cl}_{\omega,t}(1-\alpha)$$

with

$$\mathcal{F}^{cl}_{\omega,t}(\alpha) = \int e^{-\alpha s}\mathrm{d}\mathbb{P}_{\Sigma_t,\omega}(s) = \omega(e^{-\alpha\Sigma_t})$$

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The related object cumulant generating function

$$e^{cl}_{\omega,t}(\alpha) = \frac{1}{t}\log \mathcal{F}^{cl}_{\omega,t}(\alpha)$$

is a well known object in large deviation theory.
Large time fluctuations can be described (via Gärtner-Ellis theorem) using properties of the limit

$$e^{cl}_{\omega,+}(\alpha) := \lim_{t\to\infty} e^{cl}_{\omega,t}(\alpha)$$

Classical steady(GC) fluctuation relation

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- for $t \to \infty$ the following is expected to hold under strong ergodic hypothesis

$$e^{cl}_{\omega_{NESS},+}(\alpha) = e^{cl}_{\omega,+}(\alpha)$$

proven in some (paradigmatic) models ; in [JPR11] advocated as Principle of Regular Entropic Fluctuation

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- *for t finite* $\mathcal{F}^{cl}_{T,t}(\alpha) \neq \mathcal{F}^{cl}_{\omega,t}(\alpha)$ unless $\omega = \omega_T$
- for $t \to \infty$ one has in many cases (for example on compact spaces)

$$e^{cl}_{\omega_T,+}(\alpha) := \lim_{t\to\infty} e^{cl}_{T,t}(\alpha) = e^{cl}_{\omega,+}(\alpha)$$

  Principle of Regular Entropic Fluctuation equivalent to a limit exchange property

Quantization : transient fluctuation relation

Quantum case ?? Transient case

Attempt 1: "Naive quantization"

Mathematical setting: Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots \mathcal{R}_M$
coupled directly or through a small system $\mathcal{S}$, $\dim \mathcal{H}_\mathcal{S} = N$

Notation

Reservoirs $\mathcal{R}_1, \mathcal{R}_2, \dots \mathcal{R}_M$
$(\mathcal{O}_j, \tau_{\mathcal{R}_j,t}, \omega_{\beta_j})$, where ω_{β_j} is β_j KMS state for τ^j_t

Small system $\mathcal{S}$, $\dim \mathcal{H}_\mathcal{S} = N$
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with generator δ_ω

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Perturbed dynamics: $\tau_t(A)$ with generator $\delta_\omega = \delta_\omega + i[-, V]$

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—attempted in work related litterature [Bochkov-Kuzovlev '70s-'80s])
—attempted in the '90, called "naive quantization"

Underlying idea in simplified setting : $\Sigma_t = -\beta H_{0,t} + \beta H_0 = S_t - S$ and consider its spectral measure on $\mathcal{H}$ and consider the spectral measure μ_{Σ_t}
From previous lecture:

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega) - Ent(\omega|\omega)$$

In the GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ associated to ω;

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Remark: Σ_t is constructed as difference at time t and at time 0 of a given observable.

Quantization: transient fluctuation relation

Attempt 2:

Mathematical point of view: Phase space contraction

What is relevant is

$$\Sigma_t \text{ corresponds to } Ent(\omega_t|\omega)$$

In a GNS representation

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega_\omega, e^{-\alpha \log \Delta_{\omega_{-t}|\omega}} \Omega_\omega) = (\Omega_\omega, \Delta_{\omega_{-t}|\omega}^{-\alpha} \Omega_\omega)$$

In terms of cocycle

$$\mathcal{F}_{\omega,t}(\alpha) = \omega([D_{\omega_{-t}} : D_\omega]^\alpha)$$

leads to *fluctuation relation*!!!!!

Let $\mathbb{P}_{\omega,t}$ be such that

$$\mathcal{F}_{\omega,t}(\alpha) = \int e^{-\alpha s} \mathrm{d}\mathbb{P}_{\omega,t}.$$

What does $\mathbb{P}_{\omega,t}$ means?

Quantization: transient fluctuation relation

Attempt 2:

Physical point of view [Kurchan'00] Measurement has been neglected. Associate to S the *two-time measurement statistics* $\mathbb{P}_{\omega,t}$ defined as difference between two measurements

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The success of TTM come with a price: unexpected phenomena (with no classical countepart) due to the invasive role of measurement [Benoist-P.Raquépas19, Benoist-P.Pautrat 20] and *now*

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Two-time measurement statistics

Full (Counting) Statistics [Lesovik, Levitov '93][Levitov, Lee,Lesovik '96]

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Given an observable A: $A = \sum_j a_j P_{a_j}$ where $a_j \in \sigma(A)$ P_{a_j} associated spectral projections

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Protocol:
- $t = 0$, we measure A (outcome a_j)
- evolve for time t
- measure again at time t (outcome a_k)

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$$\mathbb{P}_{A,t}(\phi) = \sum_{a_k - a_j = \phi} \mathrm{tr}(\rho P_{a_j})\mathrm{tr}(e^{-\mathrm{i}tH}\rho_{am}e^{\mathrm{i}tH}P_{a_k})$$

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Two-time measurement statistics -thermodynamic limit

This defines $\mathcal{F}^{(n)}_{\omega,t}(\alpha) = \int e^{-\alpha s} \mathrm{d}\mathbb{P}^{(n)}_{S,t}(s)$.

Theorem

It is possible to rewrite $\mathcal{F}^{(n)}_{\omega,t}(\alpha)$ in term of algebraic objects that survive the limit $n \to \infty$. Under standard hypothesis, $\lim_{(n)\to\infty} \mathcal{F}^{(n)}_{\omega,t}(\alpha) =: \mathcal{F}_{\omega,t}(\alpha)$ is well defined and correspond to the same formal expression.

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And that expression is same as before!!!

$$\mathcal{F}_{\omega,t}(\alpha) = (\Omega_\omega, e^{-\alpha \log \Delta_{\omega_{-t}|\omega}}\Omega_\omega) = (\Omega_\omega, \Delta^{-\alpha}_{\omega_{-t}|\omega}\Omega_\omega)$$

In other words
$\mathcal{F}_{\omega,t}(\alpha) = \mathcal{F}^{\mathrm{ttm}}_{\omega,t}(\alpha)$ obtained though two-time measurement protocol
$\mathcal{F}_{\omega,t}(\alpha) = \mathcal{F}^{\mathrm{psc}}_{\omega,t}(\alpha)$ obtained in the spirit of quantum phase phase contraction

Quantum steady entropic functional: our proposal

How to quantize steady FR? Difficulties:

- A measurement would destroy the steady state
- steady state exists only in the thermodynamic limit
- in the thermodynamic limit, non-normality of the steady state (to the initial state)

Quantum steady entropic functional: our proposal

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Two time measurement framework (start with finite dimensional approximation $\dim \mathcal{H} = n$)
– start with ω initial state as in the transient case
– perform the first measurement at an unknown very large time T
– let the system evolve for time t
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This defines

$$\mathbb{P}^{(n)}_{T,t}, \quad \mathcal{F}^{(n)}_{\omega_T,t}(\alpha) = \int e^{-t\alpha s} \mathrm{d}\mathbb{P}^{(n)}_{T,t}(s)$$

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Proposition (Thermodynamic limit
(T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23))

It is possible to rewrite $\mathcal{F}^{(n)}_{\omega_T,t}(\alpha)$ in term of algebraic objects that survive the limit. Under standard hypothesis,
$\mathcal{F}_{\omega_T,t}(\alpha) := \lim_{n\to\infty} \mathcal{F}^{(n)}_{\omega_T,t}(\alpha)$ is well defined correspond to the same formal expression.

$\mathcal{F}_{\omega_T,t}(\alpha) = \lim_{R\to} \int_0^R \omega_T(\zeta_\theta[D_{\omega_{-t}} : D_\omega]^\alpha)\mathrm{d}\theta$
with $\zeta_\theta(-)$ modular dynamic associated to ω (free dynamics up to a temperature time scale factor)

Recall:

$$\mathcal{F}_{\omega,t}(\alpha) = \omega([D_{\omega_{-t}} : D_\omega]^\alpha)$$

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Definition

$$\mathcal{F}_{\omega_{NESS},t}(\alpha) := \lim_{T\to\infty} \mathcal{F}_{\omega_T,t}(\alpha)$$

Quantum steady entropic functional: our proposal

Theorem (direct coupling, no S, T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are coupled directly (no S); assume the dynamics $\tau^0_{\mathcal{R},t}$ is ergodic. Then

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Consequences:

- $\mathcal{F}_{\omega_{NESS},t}(\alpha) = \mathcal{F}_{\omega,t}(\alpha)$
- If the symmetry true for $\mathcal{F}_{\omega,t}(\alpha)$, then for the steady functional $\mathcal{F}_{\omega_{NESS},t}(\alpha)$ also satisfy the symmetry AT FINITE TIME t;
- if $e_{\omega_{NESS},+}(\alpha), e_{\omega,+}(\alpha)$ exist, they are equal.

Theorem (coupling through $\mathcal{S}$, T.Benoist, L.Bruneau, V.Jakšić, A.P., C.-A.Pillet '23)

Let assume the reservoirs are through a small system $\mathcal{S}$; assume the dynamics $\tau^0_{\mathcal{R},t}$ is ergodic. Then

$$e_{\omega_{NESS},+}(\alpha) = e_{\omega,+}(\alpha)$$

Remark

In both theorems:

1. *No additional hypothesis on the perturbed dynamics; strong coupling allowed.*
2. *Underlying mechanism: invasive measurement role; memory erasing effect of first measurement; stability of the fluctuation*
3. *General proof, with algebraic methods (no need for resonance analysis model by model)*
4. *We are able to cope with NESS in the thermodynamic limit.*

Proof in abstract terms

Consider $(\mathcal{O}, \tau_t, \omega)$ with ω a τ_t invariant.
$(\mathcal{O}, \tau_t, \omega)$ is ergodic iff for any ω normal state ν and any $A \in \mathcal{O}$

$$\lim_{T\to\infty} \int_0^T \nu(\tau_t(A))\mathrm{d}t$$

Apply this property to

$$\mathcal{F}_{\omega_T,t}(\alpha) = \lim_{R\to\infty} \int_0^R \omega_T(\zeta_\theta[D_{\omega_{-t}} : D_\omega]^\alpha)\mathrm{d}\theta$$

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$$\mathcal{F}_{\omega_T,t}(\alpha) = \lim_{A\to\infty}\frac{1}{A}\int_0^A(\Omega_T, e^{i\theta L_0}\Delta^\alpha_{\omega_t|\omega}e^{-i\theta L_0}\Omega_T)\mathrm{d}\theta$$
$$= (\Omega, e^{iTL_\infty}1_{\{0\}}(-\beta L_0)e^{-itL_\alpha}\Omega)$$

Direct coupling: If the dynamics on $\mathcal{R}$ is ergodic, 0 is a simple eigenvalue for L_0 and $1_{\{0\}}(L_0) = |\Omega\rangle\langle\Omega|$

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Coupling through a small system $\mathcal{S}$: If the dynamics on $\mathcal{R}$ is ergodic, 0 is a simple eigenvalue for L_0 and $\ker(L) = \ker(L_\mathcal{S}) \otimes \Omega_\mathcal{R}$; equality is attained in in the long time limit $t \to \infty$

Summing up so far

- we have introduced classical entropic functional and we have review the quantization in the transient case
- we have introduced a proposal for quantum steady (GC) entropic functional $\mathcal{F}_{\omega_{NESS},t}(\alpha)$
- we have shown a "memory erasing effect"/ stability due to the first measurement ; this can be interpreted as stability in the quantum case of the fluctuations
- we have shown $e_{\omega_{NESS},+}(\alpha) = e_{\omega,+}(\alpha)$ under very minimal ergodicity hypothesis
- direct measurement on infinitely extended or (very large) reservoir is an idealization. Are we able to write similar (less general) results ins the framework of an indirect measurement using resonance theory ? YES, next subject.

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