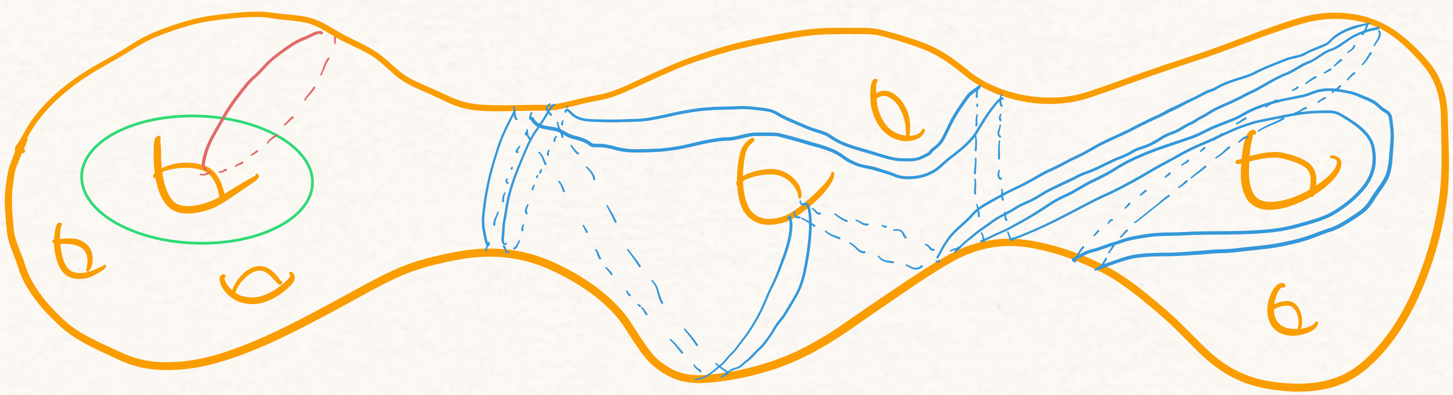
Counting Curves

inside Mapping Class Group Orbits



Viveka Erlandsson

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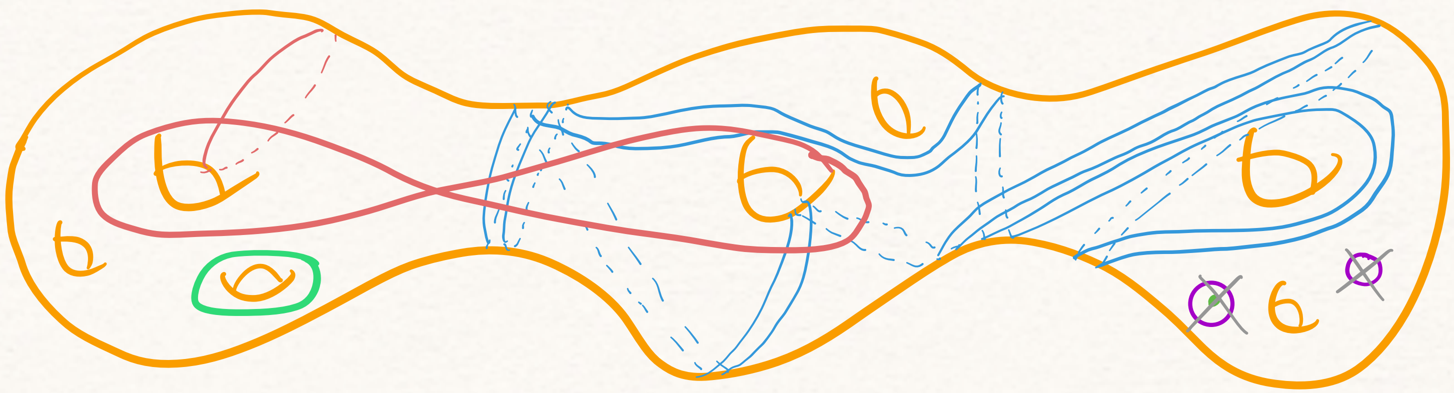
CURVES ON SURFACES

S



S = ORIENTABLE (CLOSED) SURFACE OF GENUS $g \geq 2$
(or genus g w/ r punctures, $2g+r>2$)

⚠ ~~NOT NEC. SIMPLE!~~

<u>γ CURVE</u> : FREE HOMOTOPY CLASS OF IMMERSED LOOP, $S^1 \looparrowright S$
ESSENTIAL (& NON-PERIPHERAL)

- IF S EQUIPPED w/ HYPERBOLIC METRIC X
  CURVE = CLOSED X-GEODESIC

- CURVE = CONJ. CLASS OF (HYPERBOLIC) ELEMENTS IN $\pi_1(S)$

- $\ell_X(\gamma)$ = length of geodesic = length of shortest rep.

Counting curves

For any L, $\#\{\gamma \text{ with } \ell_X(\gamma) \leq L\}$ is finite.

(Bounded by volume of balls in $\mathbb{H}^2$, $\sim \pi L^2$)

As $L \to \infty$, $\#\{\gamma \text{ with } \ell_X(\gamma) \leq L\} \to \infty$

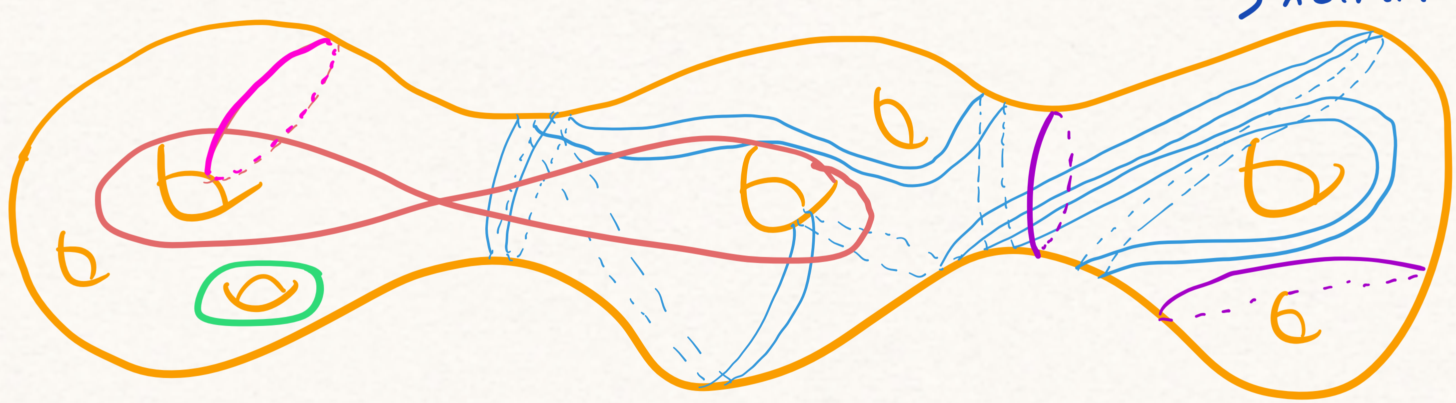
Counting: At what rate?

Huber ('59): For any (closed) hyperbolic X

$$\#\{\gamma \text{ with } \ell_X(\gamma) \leq L\} \sim \frac{e^L}{L}$$

$\sim$ asymptotic

S



COUNTING CURVES

FOR ANY L, $\#\{\gamma \text{ SIMPLE with } \ell_X(\gamma) \leq L\}$ is finite.

AS $L \to \infty$, $\#\{\gamma \text{ SIMPLE with } \ell_X(\gamma) \leq L\} \to \infty$

COUNTING: AT WHAT RATE?

NO SELF-INTERSECTIONS

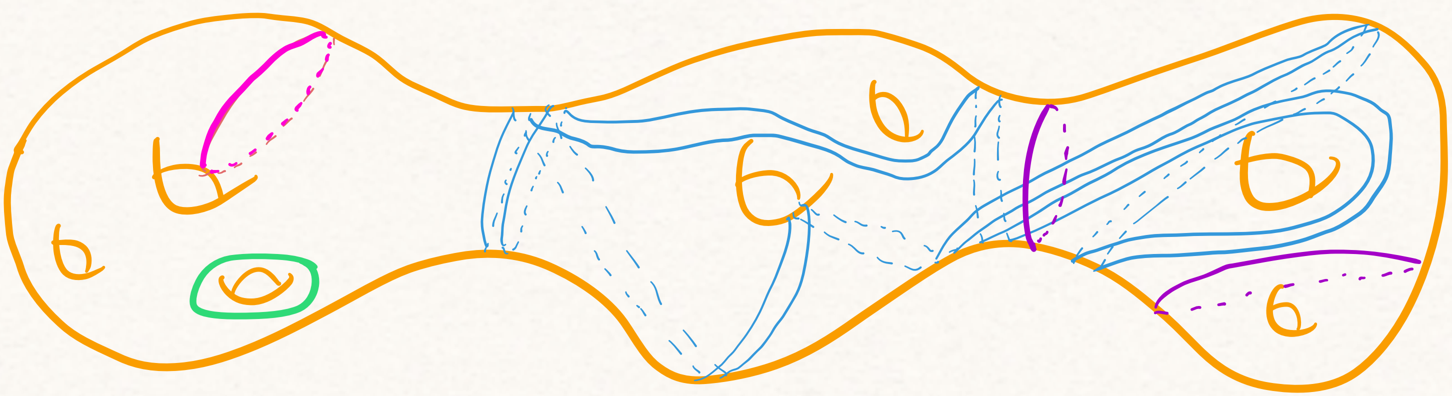
WHAT IF WE ONLY COUNT THE SIMPLE ONES?

MIRZAKHANI ('04) FOR ANY HYPERBOLIC X OF GENUS g

$$\#\{\gamma \text{ SIMPLE with } \ell_X(\gamma) \leq L\} \sim C \cdot L^{6g-6}$$

$C = C(X) > 0$

S



Counting curves

For any L, $\#\{\gamma \text{ SIMPLE with } \ell_X(\gamma) \le L\}$ is finite.

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Counting: At what rate?

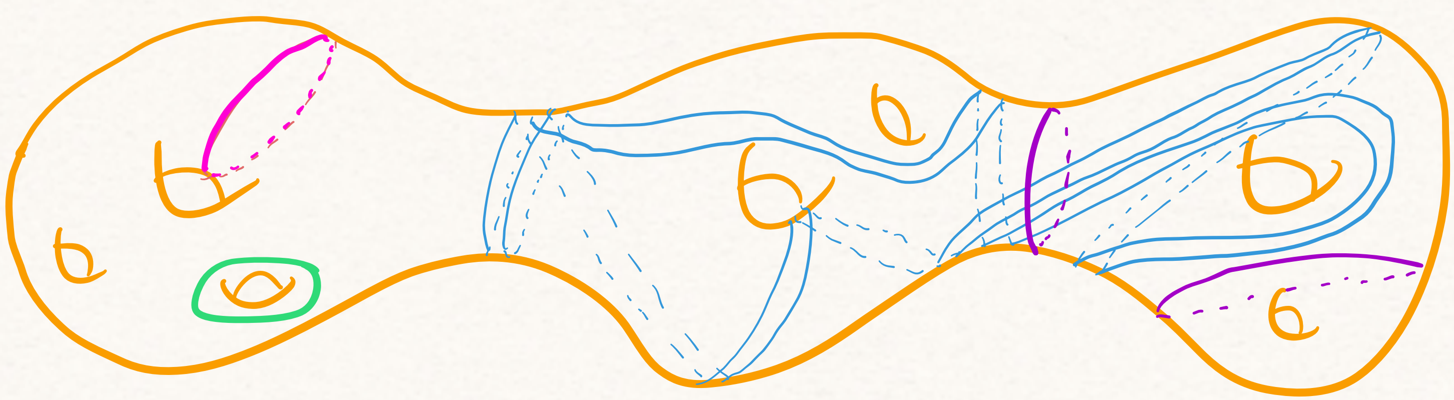
What if we only count the simple ones? ← No self-intersections

Mirzakhani ('04) For any hyperbolic X of genus g

$$\#\{\gamma \text{ SIMPLE OF FIXED TYPE with } \ell_X(\gamma) \le L\} \sim C \cdot L^{6g-6}$$

$C = C(X, \text{type})$

S



CURVES OF FIXED TYPE

- <u>MAPPING CLASS GROUP</u> $\mathrm{Map}(S) = \mathrm{Homeo}^+(S) / \text{homotopy}$
- $\mathrm{Map}(S) \curvearrowright \{\text{curves}\}$
- γ, γ' SIMPLE CURVES ARE OF SAME TYPE, $\gamma \sim \gamma'$

  IF $\gamma' \in \mathrm{Map}(S) \cdot \gamma$

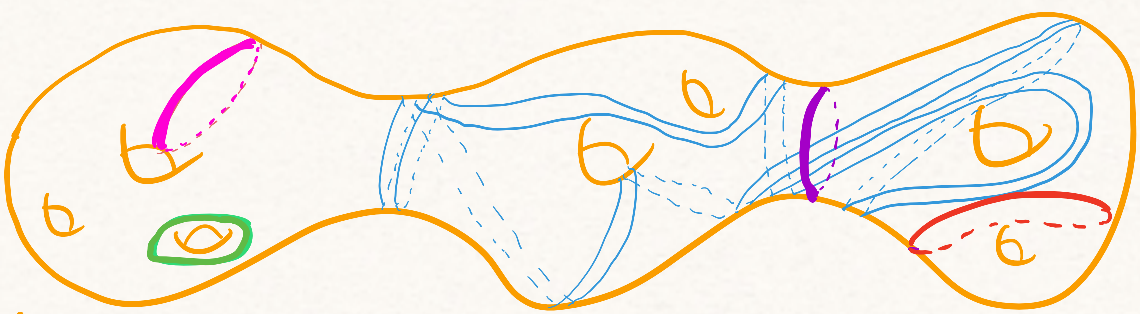
<u>Ex</u>: • IF γ is NON-SEPARATING ($= S \setminus \gamma$ connected)

$$\left.\begin{array}{l} S \setminus \gamma = S_{6,2} \\ S \setminus \gamma = S_{6,2} \end{array}\right\} \text{CLASSIFICATION THM FOR SURFACES} \implies \gamma \sim \gamma$$

$S = S_{7,0}$

genus

\# boundary



CURVES OF FIXED TYPE

• MAPPING CLASS GROUP $\mathrm{Map}(S) = \mathrm{Homeo}^+(S) / \text{homotopy}$

• $\mathrm{Map}(S) \curvearrowright \{\text{curves}\}$

• γ, γ' SIMPLE CURVES ARE OF SAME TYPE, $\gamma \sim \gamma'$
IF $\gamma' \in \mathrm{Map}(S) \cdot \gamma$

Ex: • IF γ is SEPARATING $(= S \setminus \gamma$ two pieces$)$

FINITELY MANY TYPES

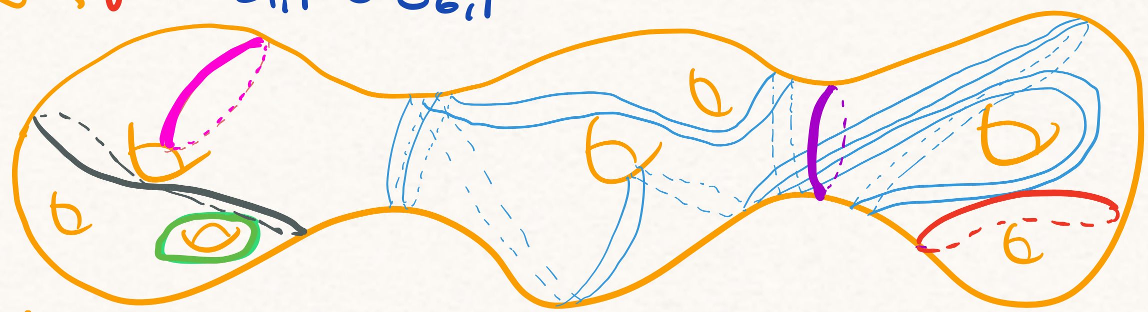
$$\left.\begin{aligned} S \setminus \gamma &= S_{2,1} \cup S_{5,1} \\ S \setminus \gamma &= S_{2,1} \cup S_{5,1} \end{aligned}\right\} \Rightarrow \text{SAME TYPE}$$

$S \setminus \gamma = S_{1,1} \cup S_{6,1} \rightsquigarrow$ DIFFERENT TYPE

$S = S_{7,0}$

genus

$\#$ boundary



CURVES OF FIXED TYPE

- MAPPING CLASS GROUP $\mathrm{Map}(S) = \mathrm{Homeo}^+(S) / \text{homotopy}$
- $\mathrm{Map}(S) \curvearrowright \{\text{curves}\}$
- γ, γ' SIMPLE CURVES ARE OF SAME TYPE, $\gamma \sim \gamma'$

  IF $\gamma' \in \mathrm{Map}(S) \cdot \gamma$

- $\{\text{SIMPLE CURVES}\}$ = UNION OF FINITELY MANY TYPES

THEOREM (MIRZAKHANI '04) X HYPERBOLIC (genus g)

γ_0 A SIMPLE MULTI CURVE $\qquad \gamma_0 = \sum_{i=1}^{n} a_i \gamma_0^i$

$$\#\{\gamma \sim \gamma_0 \text{ with } \ell_X(\gamma) \leq L\} \sim C \cdot L^{6g-6}$$

$C = C(X, \gamma_0)$

CURVES OF FIXED TYPE

- MAPPING CLASS GROUP $\mathrm{Map}(S) = \mathrm{Homeo}^+(S)/\text{homotopy}$
- $\mathrm{Map}(S) \curvearrowright \{\text{curves}\}$
- γ, γ' ~~SIMPLE~~ CURVES ARE OF SAME TYPE, $\gamma \sim \gamma'$

  IF $\gamma' \in \mathrm{Map}(S) \cdot \gamma$

- $\{\text{SIMPLE CURVES}\}$ = UNION OF FINITELY MANY TYPES
- FACT: $\{\text{CURVES } i(\gamma,\gamma) = k\}$ = UNION OF FINITELY MANY TYPES

THEOREM (MIRZAKHANI ~~'04~~ '16) X HYPERBOLIC (genus g)

γ_0 A ~~SIMPLE~~ MULTI CURVE

$$\#\{\gamma \sim \gamma_0 \text{ with } \ell_X(\gamma) \leq L\} \sim C \cdot L^{6g-6}$$

$$C = C(X, \gamma_0)$$

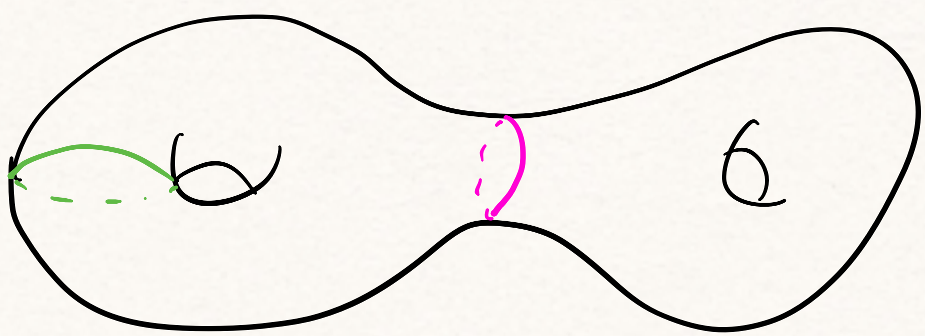
$$\Rightarrow \#\{\gamma \text{ with } i(\gamma,\gamma) \leq k \ \&\ \ell_X(\gamma) \leq L\} \sim \bar{C} \cdot L^{6g-6}, \quad \bar{C} = \bar{C}(X,k)$$

COUNTING ALLOWS YOU TO DO "STATISTICS"

AMONG ALL SIMPLE CURVES OF LENGTH $\leq L$, HOW DOES A RANDOM ONE LOOK LIKE? (AS $L \to \infty$)

E.g:

MIRZAKHANI'S 48-THM: ON A GENUS 2 SURFACE, A RANDOM SIMPLE CURVE IS 48 TIMES MORE LIKELY TO BE NON-SEPARATING THAN SEPARATING



CONVERGENCE OF MEASURES

TO PROVE THE THEOREM (FOR SIMPLE) MIRZAKHANI STUDIED THE MEASURES

$$m_{\gamma_0,L} := \frac{1}{L^{6g-6}} \sum_{\gamma \sim \gamma_0} \delta_{\frac{1}{L}\gamma} \quad \text{ON } \mathcal{ML}(S)$$

MEASURED LAMINATIONS

- HOMEOMORPHIC TO $\mathbb{R}^{6g-6}$
- $\{\text{SIMPLE CURVES}\} \subset \mathcal{ML}(S)$
- $\{\text{SIMPLE MULTI-CURVES}\} \subset \mathcal{ML}(S)$
- $\overline{\{a\gamma \mid a \in \mathbb{R}_+, \gamma \text{ SIMPLE}\}} = \mathcal{ML}(S)$
- HYPERBOLIC LENGTH EXTENDS CONTINUOUSLY
- $\mathrm{Map}(S) \curvearrowright \mathcal{ML}(S)$
- Natural measure m_{Thu} Map(S)-inv. & ergodic

Thm (MIRZAKHANI)

FOR ANY SIMPLE γ_0,

$$m_{\gamma_0,L} \longrightarrow \frac{c(\gamma_0)}{b_g} \cdot m_{Thu} \quad \text{as } L \to \infty$$

LET $U_X = \{\lambda \in \mathcal{ML} \mid \ell_X(\lambda) \leq 1\}$

COR: $m_{\gamma_0,L}(U_X) \longrightarrow \frac{c(\gamma_0)}{b_g} \underbrace{m_{Thu}(U_X)}_{B(X)}$

γ_0 SIMPLE

$$m_{\gamma_0,L}(U_X) = \frac{1}{L^{6g-6}} \# \{\gamma \sim \gamma_0 \mid \ell_X(\tfrac{1}{L}\gamma) \leq 1\}$$

$$\Longrightarrow \quad \frac{1}{L^{6g-6}} \# \{\gamma \sim \gamma_0 \mid \ell_X(\gamma) \leq L\} \longrightarrow \frac{c(\gamma_0)}{b_g} B(X)$$

THE CONSTANT C.

CONVERGENCE OF MEASURES

TO PROVE THE THEOREM FOR GENERAL CURVES WE STUDIED THE MEASURES

$$m_{\gamma_0,L} := \frac{1}{L^{6g-6}} \sum_{\gamma \sim \gamma_0} \delta_{\frac{1}{L}\gamma}$$

ON $C(S)$ GEODESIC CURRENTS

- NICE TOPOLOGICAL SPACE (METRIZABLE, LINEAR CONE)
- $\{\text{CURVES}\} \subset C(S)$
- $\{\text{MULTICURVES}\} \subset C(S)$
- $\overline{\{a\gamma \mid a \in \mathbb{R}_+, \gamma \text{ CURVE}\}} = C(S)$
- HYPERBOLIC LENGTH EXTENDS CONTINUOUSLY
- $\mathrm{Map}(S) \curvearrowright C(S)$
- $\mathcal{ML}(S) \subset C(S)$ CLOSED
- m_{Thu} measure $C(S)$

THM (E.-SOUTO)

FOR ANY CURVE γ_0,

$$m_{\gamma_0,L} \longrightarrow \frac{c(\gamma_0)}{b_g} \cdot m_{\mathrm{Thu}}$$

as $L \to \infty$

COR: WE CAN AGAIN COUNT W.R.T. MANY METRICS

NOTE: THE THM MENTIONS NO METRIC, AND THE ARGUMENT

$$m_{\gamma_0,L}(U_x) = \frac{1}{L^{6g-6}} \#\{\gamma \sim \gamma_0 \mid \ell_x(\tfrac{1}{L}\gamma) \le 1\}$$

$$\Rightarrow \frac{1}{L^{6g-6}} \#\{\gamma \sim \gamma_0 \mid \ell_x(\gamma) \le L\} \to \frac{c(\gamma_0)}{b_g} B(x)$$

ONLY USES: ℓ_x extends, is continuous, & $\ell_x(\frac{1}{L}\gamma) = \frac{1}{L}\ell_x(\gamma)$.

A GENERALIZATION

$F(t\gamma) = tF(\gamma)$

LET $F: \mathcal{C}(\Sigma) \to \mathbb{R}_{\geq 0}$ BE ANY CONTINUOUS, POSITIVE, HOMOGENEOUS FUNCTION. THEN, FOR ANY (MULTI) CURVE γ_0

$$\#\{\gamma \sim \gamma_0 \text{ with } F(\gamma) \leq L\} \sim C \cdot L^{6g-6}$$

$$C = \frac{c(\gamma_0)}{m_g} \underbrace{B(F)}_{m_{Thu}(\{\lambda \in \mathcal{ML} \mid F(\gamma) \leq 1\})}$$

THERE ARE MANY SUCH F!

(Otal) - NEGATIVELY CURVED R. METRIC

(Duchin-Leininger-Rafi) - FLAT METRICS (e.g. TRANSLATION SURFACES)

(E.-Parlier-Souto) - (stable) WORD LENGTH

- ANY (stable) RIEMANNIAN METRIC

(Martinez-Granado + Thurston) - EXTREMAL LENGTH

(Bonahon) - COMBINATORIAL LENGTH: Fix filling σ, $\ell(\gamma) = i(\gamma, \sigma)$

IN FACT, CAN COUNT W.R.T ONE $\Leftrightarrow$ W.R.T. ANY OTHER!

A VERY BRIEF IDEA OF PROOF

① $m_{Thu} := \lim_{L\to\infty} \frac{1}{L^{6g-6}} \sum_{\lambda \in \mathcal{ML}_{\mathbb{Z}}} \delta_{\frac{1}{L}\lambda}$ ($\mathcal{ML}_{\mathbb{Z}}$ = simple multi-curves)

(In charts, just Lebesgue on $\mathbb{R}^n$) $\left(\mathrm{Leb} = \lim_{L\to\infty} \frac{1}{L^n} \sum_{p\in\mathbb{Z}^n} \delta_{\frac{1}{L}p} \right)$

② $m_{\gamma_0,L} = \frac{1}{L^{6g-6}} \sum_{\gamma\sim\gamma_0} \delta_{\frac{1}{L}\gamma}$

$\{m_{\gamma_0,L}\}$ precompact & any limit point m:

$m \leq K\, m_{Thu}$ (for some $K = K(\gamma_0)$)

③ m & m_{Thu} are $\mathrm{Map}(S)$-invariant

MASUR: m_{Thu} ergodic

$\Longrightarrow$ $m = \mathrm{const.} \cdot m_{Thu}$

④ Show "const." is the same for every limit point!

APPLICATIONS: A (SHORT, INCOMPLETE) SURVEY

FIRST: The constants

Recall: $m_{L,\gamma_0} \longrightarrow \frac{c(\gamma_0)}{b_g} \cdot m_{Thu}$

type

metric/length function

$$\#\{\gamma \sim \gamma_0 \mid \ell_X(\gamma) \leq L\} \sim \frac{c(\gamma_0)\, B(X)}{b_g} L^{6g-6}$$

topology

$$B(X) = m_{Thu}\left(\{\lambda \in \mathcal{ML} \mid \ell_X(\lambda) \leq 1\}\right)$$

$$b_g = \sum_{\substack{\gamma \text{ types} \\ \text{of simple} \\ \text{multicurves}}} c(\gamma)$$

* WHAT IF γ_0 IS NOT SIMPLE? ASK MINGKUN!

$$c(\gamma_0) = m_{Thu}\left(\mathrm{Stab}(\gamma_0) \backslash \{\lambda \in \mathcal{ML} \mid \lambda + \gamma_0 \text{ FILL } \& \ i(\gamma_0, \lambda) \leq 1\}\right)$$

WHEN γ_0 SIMPLE*, CAN (IN THEORY) BE COMPUTED!

APPLICATIONS: A (SHORT, INCOMPLETE) SURVEY

FIRST: The constants

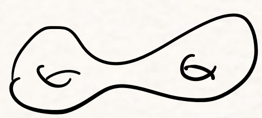
Recall: $$m_{L,\gamma_0} \longrightarrow \frac{c(\gamma_0)}{b_g} \cdot m_{Thu}$$

type   metric/length function

$$\#\{\gamma \sim \gamma_0 \mid \ell_X(\gamma) \leq L\} \sim \frac{c(\gamma_0)\, B(X)}{b_g} L^{6g-6}$$

topology

$$\frac{\#\{\gamma \text{ type } \gamma_1 \text{ WITH } \ell(\gamma) \leq L\}}{\#\{\gamma \text{ type } \gamma_2 \text{ WITH } \ell(\gamma) \leq L\}} \xrightarrow{L \to \infty} \frac{c(\gamma_1)}{c(\gamma_2)}$$

E.g. For $S=$ 

$$\frac{\#\{\gamma \text{ NON-SEPARATING, } \ell(\gamma) \leq L\}}{\#\{\gamma \text{ SEPARATING, } \ell(\gamma) \leq L\}} \longrightarrow 48$$

OR: TAKE A SIMPLE CURVE AT RANDOM ($\ell \leq L$), THEN PROBABILITY IT IS SEPARATING IS $1/49$.

APPLICATIONS: A (SHORT, INCOMPLETE) SURVEY

FIRST: The constants

Recall:

$$m_{L,\gamma_0} \longrightarrow \frac{c(\gamma_0)}{b_g} \cdot m_{Thu}$$

$$\#\{\gamma \sim \gamma_0 \mid \ell_X(\gamma) \le L\} \sim \frac{c(\gamma_0)\, B(X)}{b_g} L^{6g-6}$$

type

metric/length function

topology

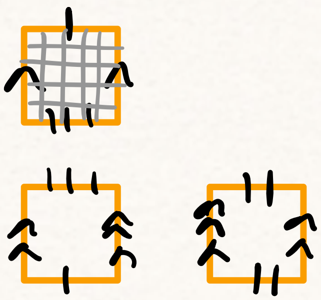
$$\frac{\#\{\gamma \text{ type } \gamma_1 \text{ WITH } \ell(\gamma) \le L\}}{\#\{\gamma \text{ type } \gamma_2 \text{ WITH } \ell(\gamma) \le L\}} \xrightarrow{L\to\infty} \frac{c(\gamma_1)}{c(\gamma_2)}$$

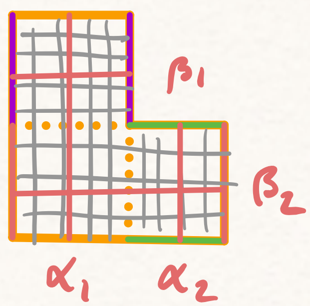
IN FACT: [DELECROIX - GOUJARD - ZOGRAF - ZORICH]

$$\frac{\#\{\gamma \text{ SEPARATING ON } S_g\}}{\#\{\gamma \text{ NON-SEPARATING ON } S_g\}} \longrightarrow 0 \text{ as } g\to\infty \quad \text{(EXPONENTIALLY)}$$

COUNTING SQUARE TILED SURFACES (STS)

[FIRST DONE BY DELECROIX-GOUJARD-ZOGRAF-ZORICH & ARANA-HERRERA]





STS STRUCTURES ON S_g 

$\rightsquigarrow$ (γ^v, γ^h) PAIR OF SIMPLE MULTICURVES

$\gamma^v = 2\alpha_1 + \alpha_2$, $\gamma^h = \beta_1 + 2\beta_2$

Square^S_τ = HOMOTOPY CLASSES; $\mathrm{Square}^S = \mathrm{Square}^S_\tau / \mathrm{Map}(S)$

$\mathrm{Square}^S(L)$ = THOSE WITH AREA (# SQUARES) $\leq L$

$\mathrm{Square}^{S,\gamma_0^v}(L)$ = THOSE WITH VERTICAL OF TYPE γ_0^v

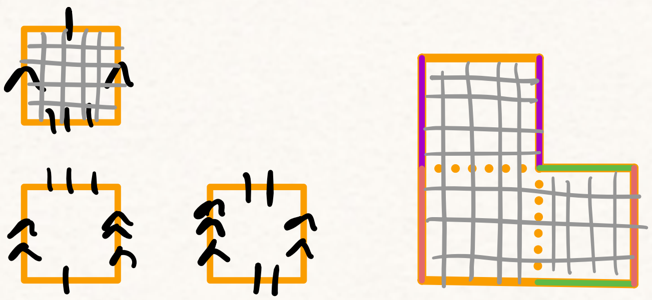
BIJECTION: $\mathrm{Square}^S_\tau \longleftrightarrow \{(\alpha,\beta)$ S.M.C. WITH $\alpha+\beta$ FILLING$\}$

PRESERVES $\mathrm{Map}(S)$-ORBITS

$$\mathrm{Square}^{S,\gamma_0^v} = {}_{\mathrm{Map}}\backslash \{(\gamma^v,\gamma^h) \mid \gamma^v+\gamma^h \text{ FILL}, \gamma^v \text{ TYPE } \gamma_0^v\} = \underbrace{{}_{\mathrm{Stab}(\gamma_0)}\backslash \{\gamma \mid \gamma+\gamma_0^v \text{ FILL}\}}_{m_{Thu}(\cdot) = c(\gamma_0^v)}$$

Counting Square Tiled Surfaces (STS)

[First done by Delecroix-Goujard-Zograf-Zorich & Arana-Herrera]



STS structures on S_g 

$\rightsquigarrow$ (γ^v, γ^h) pair of simple multicurves

Square^S_τ = homotopy classes; $\mathrm{Square}^S = \mathrm{Square}^S_\tau / \mathrm{Map}(S)$

$\mathrm{Square}^S(L)$ = those with area (# squares) $\leq L$

$\mathrm{Square}^{S,\gamma_0^v}(L)$ = those with vertical of type γ_0^v

THM: $$\left|\mathrm{Square}^{S,\gamma_0^v}(L)\right| \sim c(\gamma_0) \cdot L^{6g-6+2r}$$

$$\left|\mathrm{Square}^{S}(L)\right| \sim b_g \cdot L^{6g-6+2r}$$

ETC...

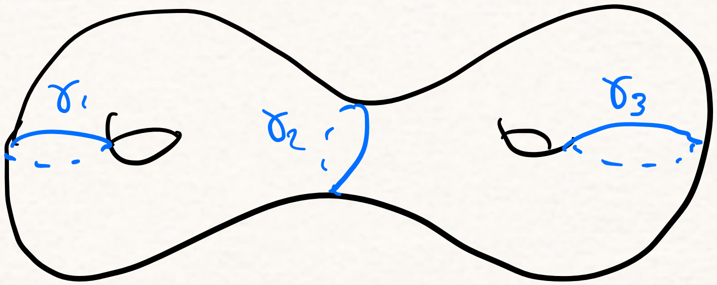
EXPECTED NUMBER OF SELF-INTERSECTIONS

[JOINT WORK IN PROGRESS WITH J. SOUTO]

Q: PICK n (SIMPLE OR NOT) CURVES $\gamma_1, \ldots, \gamma_n$ AT RANDOM (OF THOSE OF LENGTH $\leq L$)
WHAT IS THIER EXPECTED # OF INTERSECTIONS
(AS $L \to \infty$)?

A: $K(X) \cdot B(X)^{n-2} \cdot L^2$
FOR AN EXPLICT CONSTANT K DEPENDING ON METRIC X.

LENGTH DISTRIBUTION OF MULTI-CURVES



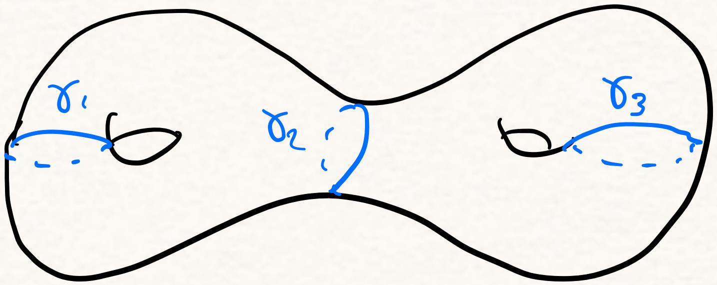
$\Gamma_0 = \gamma_1 + \gamma_2 + \cdots + \gamma_n$

$\vec{L} = (L_1, L_2, \ldots, L_n) \in \mathbb{R}^n_+$

WHAT IS THE EXPECTED LENGTH DISTRIBUTION OF THE COMPONENTS OF Γ OF TYPE Γ_0?

MIRZAKHANI: For Γ_0 A PAIR OF PANTS DECOMPOSITION ($\Gamma_0 = \gamma_1 + \gamma_2 + \cdots + \gamma_{3g-3}$) THE PROBABILITY THAT Γ OF TYPE Γ_0 HAS A COMPONENT WITH LENGTH $\leq 10\%$ OF $\dfrac{\text{total length}}{3g-3}$ IS $\leq 10\%$

LENGTH DISTRIBUTION OF MULTI-CURVES



$\Gamma_0 = \gamma_1 + \gamma_2 + \dots + \gamma_n$

$\vec{L} = (L_1, L_2, \dots, L_n) \in \mathbb{R}^n_+$

WHAT IS THE EXPECTED LENGTH DISTRIBUTION OF THE COMPONENTS OF Γ OF TYPE Γ_0?

$\gamma \in \mathcal{ML}_{\mathbb{Z}}$, $\vec{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_k) \in \mathcal{ML}^k$, $\mathbb{L}_X(\vec{\gamma}) \in \Delta^k \times \mathbb{R}_{\geq 0}$

$\left(\frac{1}{\text{total length}} \cdot (\ell_X(\gamma_1), \dots, \ell_X(\gamma_k)), \text{total length} \right)$

DEFINE MEASURE ON $\Delta^k \times \mathbb{R}_{\geq 0}$

$$\underline{m}(\vec{\gamma}_0, L, X) = \frac{1}{\mathcal{M}_{X,\gamma_0}(L)} \sum_{\vec{\gamma} \sim \vec{\gamma}_0} \delta_{\frac{1}{L} \mathbb{L}_X(\vec{\gamma})}$$

MIRZAKHANI: $\underline{m}(\vec{P}_0, L, X)$ converges to an explicit measure.

LIU, ARANA-HERRERA: SAME FOR ANY SIMPLE MULTICURVE

LENGTH DISTRIBUTION OF MULTI-CURVES

WORK IN PROGRESS W/ JUAN SOUTO:

DO THE SAME FOR NON-SIMPLE!

$\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_k$ ANY MULTI-CURVE

$\vec{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_k) \in C(S) \times \cdots \times C(S) = C(S)^k$

$$\underline{m}(\vec{\gamma}_0, L, X) = \frac{1}{\# M_{X,\gamma_0}(L)} \sum_{\gamma \sim \gamma_0} \delta_{\frac{1}{L} \ell_X(\vec{\gamma})}$$

E.-SOUTO: Again, convergence ("explicit" measure = in progress...)

IDEA: - STUDY THE MEASURES $\mu(\vec{\gamma}_0, L) := \frac{1}{L^{6g-6}} \sum_{\vec{\gamma} \sim \vec{\gamma}_0} \delta_{\frac{1}{L}\vec{\gamma}}$ ON $C(S)^k$

- SHOW CONVERGENCE

$\Longrightarrow$ HYPERBOLIC METRIC (X) CAN BE REPLACED WITH ANY "NICE" F

E.-SOUTO: FOR ANY NICE LENGTH FUNCTION, $\underline{m}(\vec{\gamma}_0, L, F)$ CONVERGES AND THE LIMITING MEASURE IS INDEPENDENT OF F.

LENGTH DISTRIBUTION OF MULTI-CURVES

WHAT HAPPENS AS $g \to \infty$?

DELECROIX - MINGKUN:

E.g.: IF Γ IS A SIMPLE MULTICURVE PICKED AT RANDOM ON S_g, ITS 3 LONGEST COMPONENTS WILL HAVE 76%, 17%, & 5% (RESP.) OF ITS TOTAL LENGTH (as $g \to \infty$)

(LENGTH = HYPERBOLIC)

THANK YOU!