Interval Exchange Transformations and Slow Entropy

Yibo Zhai

Department of Mathematics
University of Utah

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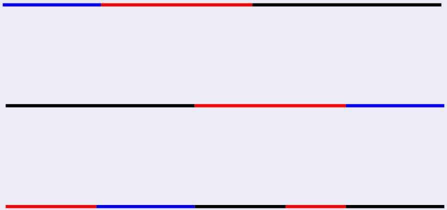
Interval Exchange Transformations

3-IET

Given $\lambda = (\lambda_1, \lambda_2, \lambda_3) \in \mathbb{R}^3_{>0}$, the following graph shows a *3-Interval exchange transformation* T_λ



- For every interval exchange transformation T_λ, $h_{Leb}(T_\lambda) = 0$ with respect to the classical entropy.

Slow Entropy

(ε, n)-Hamming ball for a finite partition $\mathscr{P}$ in system (X, μ, T):

$$B_T^n(x,\varepsilon) = \{y \in X : \overline{d}_T^n(x,y) < \varepsilon\},$$

where

$$\overline{d}_T^n(x,y) = \frac{|\{0 \le i \le n-1 : x_i \ne y_i\}|}{n},$$

$x_i \ne y_i$ means $T^i x$ and $T^i y$ are not in the same atom of $\mathscr{P}$.

Slow Entropy

The slow metric entropy for the partition $\mathscr{P}$ and given scale $a_\chi(n) = n^\chi$ is:

$$h_{\mu,\mathscr{P}}(T) = \lim_{\varepsilon \to 0} \left(\sup \left\{ \chi : \limsup_{n \to \infty} \frac{Q_n}{n^\chi} > 0 \right\} \right),$$

where Q_n is the minimum number of (ε, n)-Hamming balls needed to cover a set of measure at least $1 - \varepsilon$.

Results

Theorem (Ferenczi)

Let (X, μ, T) be an ergodic measure preserving system. Then, (X, μ, T) is isomorphic to the Kronecker system if and only if $h_{\mu, a_\chi}(T) = 0$ for all scales a_χ.

Theorem (Cheng, Ospina, Vinhage, Z.)

There exists a dense set $A \subseteq \mathbb{R}^3_{>0}$, such that if T is a 3-IET determined by A, then $h_{Leb, a_\chi}(T) = 1$ with respect to $a_\chi(n) = n^\chi$.