## Cell Decomposition from $L^{2}$ - Delaunay Triangulations?

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- Idea: Translation surfaces with the same gluing datum of the triangles belong to the same "cell".
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- Idea: Intersect this cell decomposition with the locus of translation surfaces.
$\rightarrow$ Conditions:

$$
\prod_{\ldots} \frac{\sin (\ldots)}{\sin (\ldots)}=1
$$

