Modular Embeddings of Teichmüller Curves

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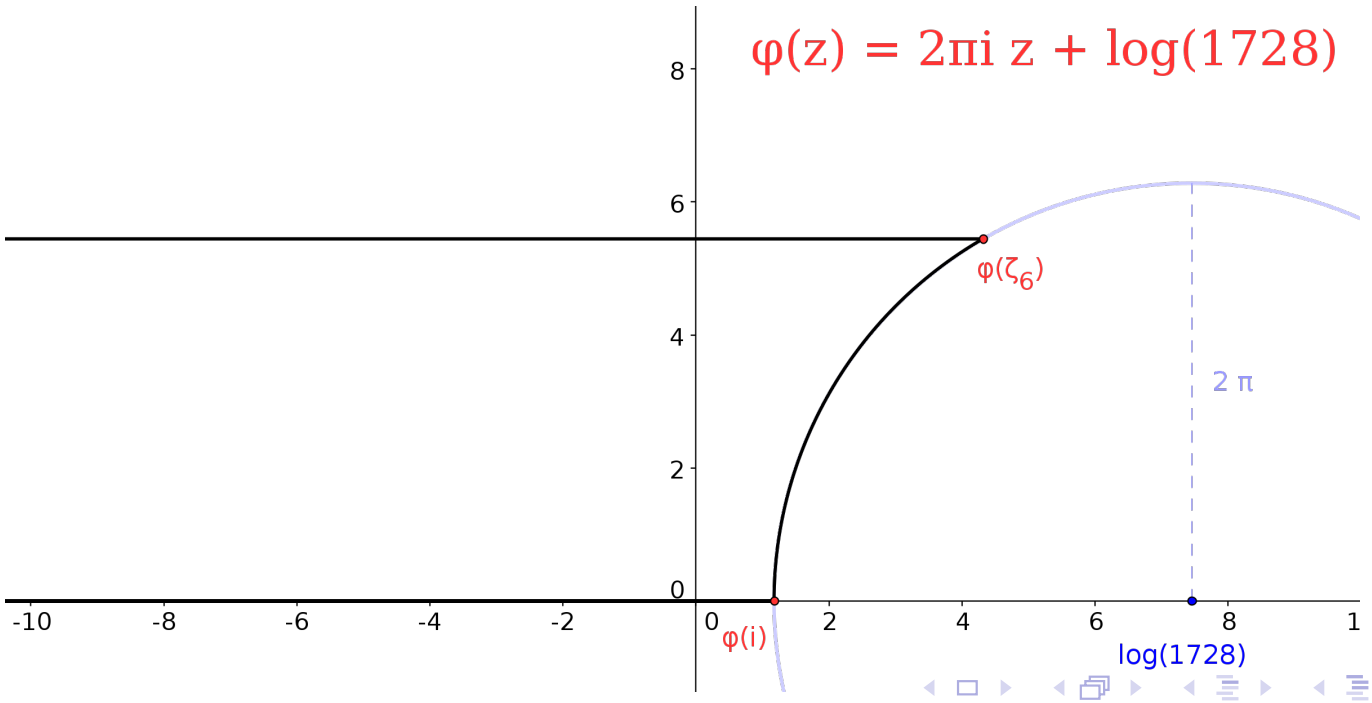

Question

$W := \mathbb{H}/\Gamma$ *Teichmüller curve* with *Veech group* Γ
$X := \mathbb{H} \times \cdots \times \mathbb{H}/\Gamma_K$ *Hilbert modular variety* with *Hilbert modular group* Γ_K for a real number field K

$$\begin{array}{ccc} \mathbb{H} & \longrightarrow & \mathbb{H} \times \cdots \times \mathbb{H} \\ \downarrow & & \downarrow \\ W & \xrightarrow{\phi} & X \end{array}$$

(Γ, ϕ) *Hilbert Modular embedding*

Question: What equations cut out Teichmüller curves on Hilbert modular varieties?

g=2

Consider the *Hilbert modular surface* X_D with discriminant D of order $\mathcal{O}_D \subset K$ for K a real quadratic field.
Let W_D be the union of Teichmüller curves in X_D.

Theorem (Möller, Zagier, 2016)

The function

$$\mathcal{D}\theta(z) := \prod_{(m,m')odd} D_2\theta_{(m,m')}(z)$$

is a modular form of weight $(3,9)$ *for the Hilbert modular group. Its vanishing locus is precisely the Teichmüller curve* W_D.