Convex tilings of the Sphere

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Introduction

Definition

A triangulation of the sphere is **convex** *if there are never more than six triangles at a vertex.*

Theorem (Thurston)

There is a lattice L in complex Lorentz space $\mathbb{C}(1,9)$ and a group Γ of automorphisms, such that triangulations of non-negative combinatorial curvature are elements of L_+/Γ, where L_+ is the set of lattice points of positive square-norm. The projective action of Γ on complex hyperbolic space $\mathbb{CH}^9$ has quotient of finite volume. The square of the norm of a lattice point is the number of triangles in the triangulation.

Counting triangulations

Theorem (Engel,Smillie)

The weighted number of oriented convex tilings of S^2 with n tiles is $\frac{809}{2^{15}3^{13}5^2}\sigma_9(\frac{n}{2})$
where $\sigma_m(n) = \sum_{d|n} d^m$ *for an integer n and* $\sigma_m(n) = 0$ *otherwise*