Counting Saddle Connections on flat Cone Spheres

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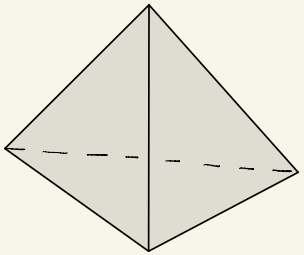
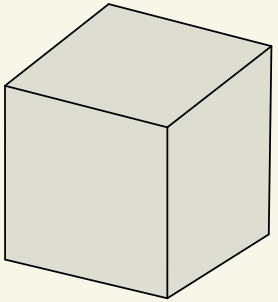
- X : $(\mathbb{S}^2, \Sigma)$

  - flat structure on $\mathbb{S}^2 \setminus \Sigma$
  - cone structure at $p \in \Sigma$
  - cone angle at p is less than 2π





- $N(X, L)$

  $= \#\{$Saddle connections on X with lengths less than $L\}$

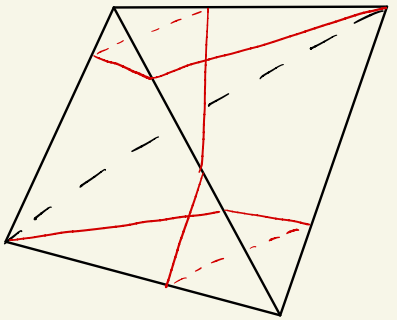
- **Main question**

  Upper bounds / growth rate of $N(X, L)$?

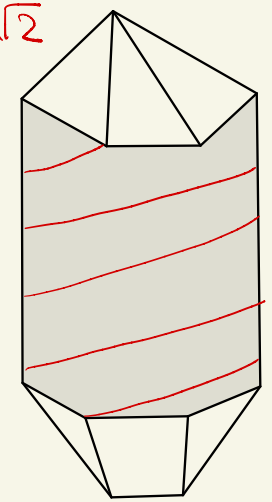
- We showed that if there is no embedded cylinder on X,

$$\text{Length of } \gamma \text{ on } X \asymp \sqrt{L(\gamma, \gamma)}$$

for $\forall$ saddle connection γ on X.



"$\ell(\gamma) \approx \sqrt{2}$"



"$A \asymp B$" means that $\exists\, a>1$ and $b>0$ s.t. $\frac{1}{a}A - b \le B \le aA + b$

- Estimate $N(X, L)$

by $n(X, k) = \#\{$ saddle connections on X with self-intersections less than $k\}$

- Understand $\int_{\Omega(\underline{k})} N(X, L)\, d\mu_{Thu} = \cdots$

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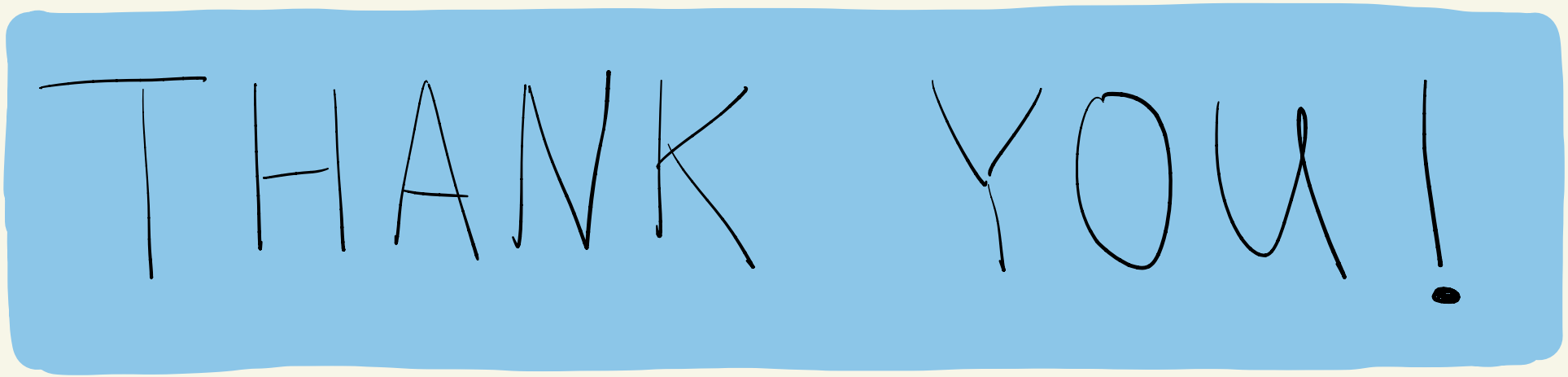
THANK YOU!