Definition

A *saddle connection* is a geodesic line starting and ending at singularities (not necessarily the same).

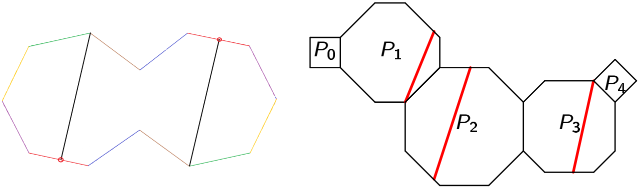
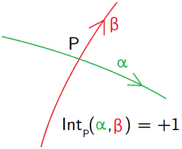


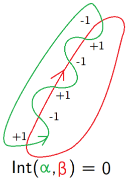
Figure: Examples of saddle connections on the Bouw-Möller surfaces $S_{2,7}$ (*left*) and $S_{5,4}$ (*right*).

Two questions I am interested in

- Study the intersections of saddle connections.
- Characterize the periodic directions on the double n-gon, $n \geq 7$ odd.

Algebraic intersection

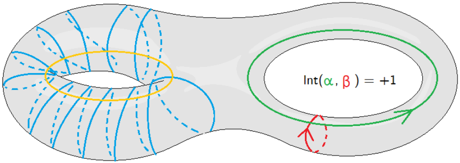




The algebraic intersection is invariant under continuous deformation of the curves.

Question

How many times can two closed curves of a given length intersect ?

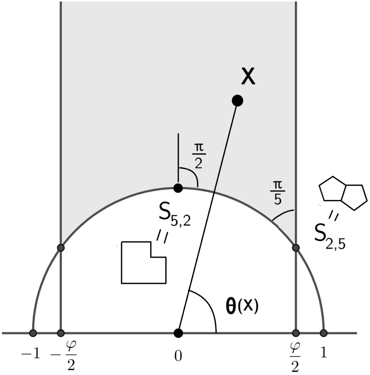


Definition

$$\text{KVol}(X) := \text{Vol}(X) \cdot \sup_{\alpha,\beta \text{ closed curves}} \frac{\text{Int}(\alpha,\beta)}{l(\alpha)l(\beta)}$$

For translation surfaces, we can replace "closed curves" by "union of saddle connections".

With I. Pasquinelli, we generalize a result that I obtained with E. Lanneau and D. Massart and we explicitely calculate KVol on the $SL_2(\mathbb{R})$-orbit of Bouw-Möller surfaces with a single singularity.



A fundamental domain for the $SL_2(\mathbb{R})$-orbit of the double pentagon. In this case

$$KVol(X) = \frac{2\varphi - 1}{(\varphi - 1)^2} \sin \theta(X)$$

where φ is the golden mean.

Theorem (B.-Lanneau-Massart,2022)

KVol in the double pentagon is achieved uniquely by pairs of disctinct sides of the pentagons.