Recurrence and Embeddings in Planar Periodic Billiards

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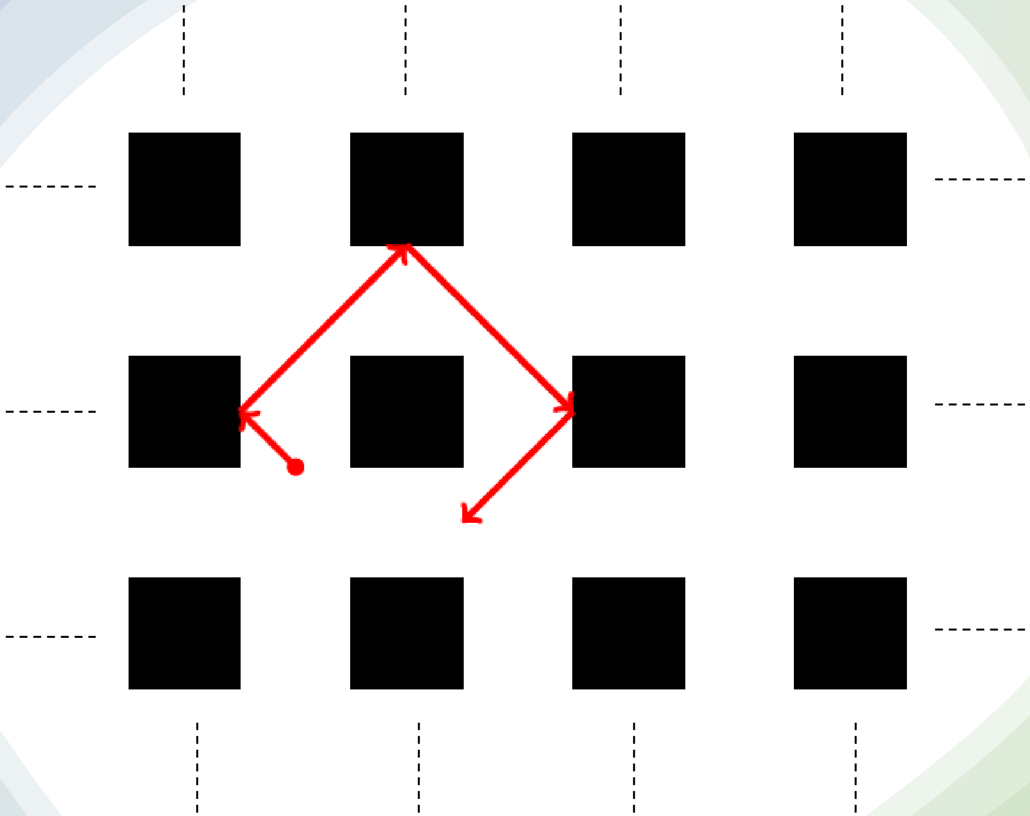
The Wind-Tree Model

While billiards in rational table are quite understood, when we pass to .non-compact table, not much is known

The original Wind-Tree model was introduced by the Ehrenfest way back in 1912. One picks a rectangle, and places it periodically along the - lattice. Only recently, some important results were found. They do no rely on the dimensions of the rectangles, and hold for almost every .direction and any starting point (as long as you don’t run into a corner)

Theorem (Avila, Hubert ‘16): *The billiard flow on the WT is recurrent in .a.e. direction*



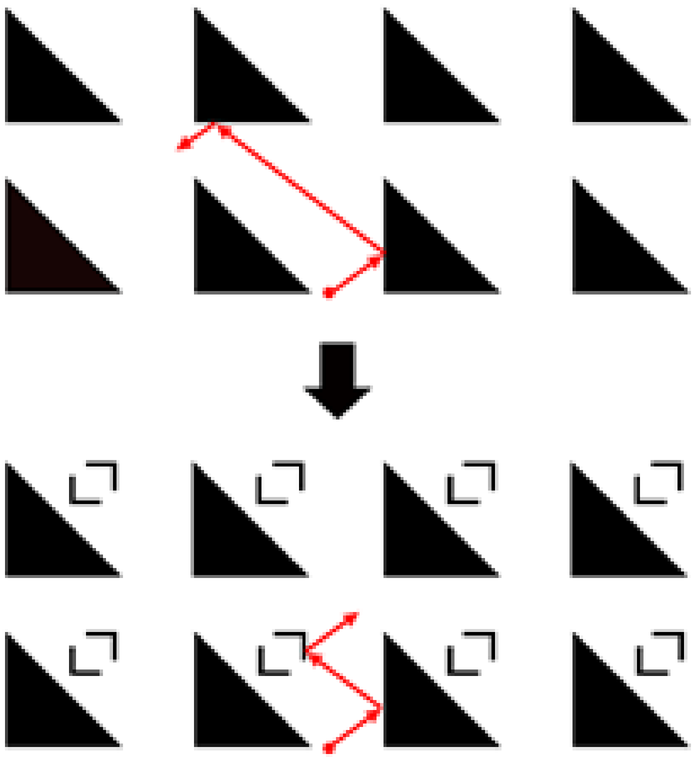
Periodic Rational Billiards

We discuss general planar periodic billiards in the plane. They are parameterized as where is a lattice in , and is a rational configuration which is put periodically on . That is where is a fundamental domain for , and are .polygonal obstacles

.Rationality of is according to the differences of angles of the polygons’ edges

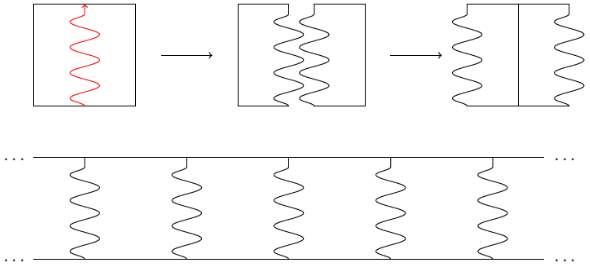
. We add obstacles to to create a new rational configuration , and study

Theorem: *For every one can add obstacles to , so that the billiard flow on is .recurrent for a.e. direction*

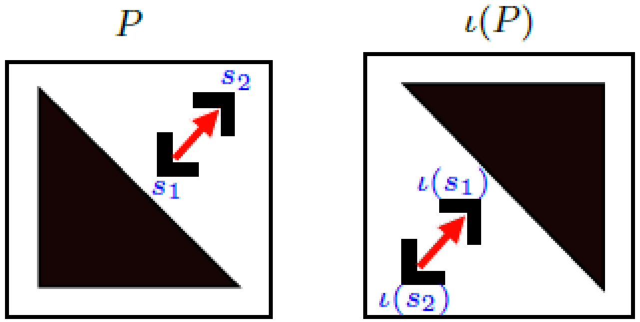


Notes and ideas from the proof

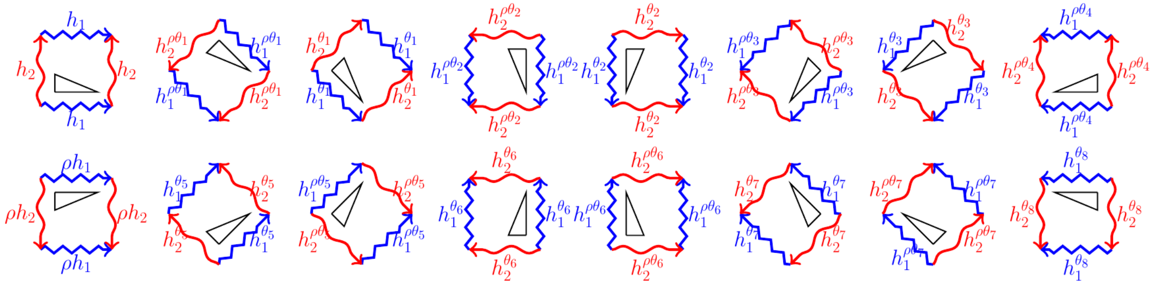
- Each model can be unfolded and described using a pair with a flat surface, . This generalizes the original Wind-Tree unfolding as a -cover.
- The proof uses a geometric criterion by Avila-Hubert. Looking at the orbit closure in the relevant stratum , it involves a splitting of the hodge bundle invariant for the Kontsevich-Zorich cocycle. One needs to have   , and a cylinder with geodesics .
- The embedded obstacles can always be taken as a pair of L-shaped corners, so one can draw a straight line as depicted. The right corners are removeable singularities, so this is indeed a geodesic. This gives the needed cylinder.



Cover defined by a homology class



Closed geodesic, anti-invariant to rotation by



and the cycles describing the unfolding of as a cover