Contribution ID: **20** Type: **not specified**

Analysis and numerical approximation of mean field game partial differential inclusions

Thursday, 13 June 2024 11:30 (45 minutes)

Joint work with Yohance A. P. Osborne

Mean field games (MFG) are models for differential games involving large numbers of players, where each player is solving a dynamic optimal control problem that may depend on the overall distribution of players across the state space of the game. In a standard formulation, the Nash equilibria of the game are characterized by the solutions of a coupled system of partial differential equations, involving the Hamilton-Jacobi-Bellman equation for the value function and the Fokker-Planck equation for the density of players over the state space of the game.

However, in many realistic applications, the underlying optimal control problems can lead to systems with nondifferentiable Hamiltonians, such as in minimal time problems, problems with bang-bang controls, etc. This leads to the crucial issue that the PDE system is then not well-defined in the usual sense. From a modelling perspective, this corresponds to nonuniquess of optimal controls, and the question of how players choose among the optimal controls and how this determines the resulting density of players.

In this talk, we show that a suitable generalization of the problem is provided by relaxing the Fokker-Planck equation to a partial differential inclusion (PDI) involving the subdifferential of the Hamiltonian, which expresses mathematically the idea that, in the nondifferentiable case, the structure of the Nash equilibria can become more complicated since players in the same state may be required to make distinct choices among the various optimal controls. Our analytical contributions include theorems on the existence of solutions of the resulting MFG PDI system under very general conditions on the problem data, allowing for both local/nonlocal and nonsmoothing nonlinear couplings, for both the steady-state and the time-dependent cases in the stochastic setting. We also show that the MFG PDI system conserves uniqueness of the solution for monotone couplings, as a generalization of the result of Lasry and Lions. We also give concrete examples of some nontrivial Nash equilibria that can be modelled by our approach.

Regarding the numerical analysis, we also propose and analyse a stabilized finite element method for the PDI system and we present theorems on its well-posedness and its convergence. We also prove that the method is almost quasi-optimal in the sense of near-best approximations for the case of differentiable Hamiltonians, which leads to optimal rates of convergence for solutions with sufficient regularity, and also some theorems illustrating the robustness of the approximation of the value function relative to the density. We present numerical experiments for both steady-state and time-dependent problems.

Presenter: SMEARS, Iain (University College London)