

Designing conservative and accurately dissipative numerical integrators in time

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Numerical methods for the simulation of transient systems with structure-preserving properties are known to exhibit greater accuracy and physical reliability, in particular over long durations. These schemes are often built on powerful geometric ideas for broad classes of problems, such as Hamiltonian or reversible systems. However, there remain difficulties in devising higher-order- in-time structure-preserving discretizations for nonlinear problems, and in conserving non-polynomial invariants.

In this work we propose a new, general framework for the construction of structure-preserving timesteppers via finite elements in time and the systematic introduction of auxiliary variables. The framework reduces to Gauss methods where those are structure-preserving, but extends to generate arbitrary-order structure-preserving schemes for nonlinear problems, and allows for the construction of schemes that conserve multiple higher-order invariants. We demonstrate the ideas by devising novel schemes that exactly conserve all known invariants of the Kepler and Kovalevskaya problems, high-order energy-conserving and entropy-dissipating schemes for the compressible Navier–Stokes equations, and multi-conservative schemes for the Benjamin-Bona-Mahony equation.

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