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Polytopal methods on Riemannian manifolds

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Discretization methods based on differential complexes have many advantageous properties in terms of stability, framework for analysing the discrete formulation, and preservation of important quantities such as the mass, helicity, or the pressure robustness in fluid dynamics.

The premise of this kind of approach appeared early on with elements based on the compatibility between the geometry and the differential operator, such as the Nédélec or the Raviart-Thomas elements relating the curl operator to the circulation along edges, and the divergence operator to the flux across surfaces. The connection between the usual differential operators (gradient, curl, and divergence) and the geometry can be better seen through framework of the exterior calculus, where these operators are unified as the exterior derivative applied to differential forms of different degrees. The associated finite element spaces also have a natural description which has been developed into the Finite Element Exterior Calculus (FEEC) framework, leading to an intrinsic definition common to each space and operator in any dimension.

Several other methods that replicate the complexes structuring systems of differential equations at the discrete level were then developed to use smoother spaces, different complexes, or more general meshes such as the Discrete De Rham (DDR) method.

Although the notions involving exterior calculus are manifestly independent of the underlying metric, most discrete methods must ultimately assume a trivial space for the notion of simplicial/polytopal mesh, and for the notion of polynomial. In this talk, we will present the generalisation of the Exterior Calculus Discrete De Rham (ECDDR) method to general Riamannian manifolds. This construction uses a much more lenient notion of mesh, allowing to consider manifolds described by several charts, and to use potentially any shape for the elements. In particular, it is possible to use curved elements even when working on a flat space. The basis functions are intrinsically defined element-wise. They are based on polynomial spaces of arbitrary order and are adapted to the chart and the metric. We will then present a numerical application of this method to the Maxwell equations on a surface.

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