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The Exterior Calculus Discrete De Rham complex

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In its standard presentation, the de Rham complex organises the gradient, curl and divergence operator into a sequence that embeds the well-known calculus relations: the image of one operator (e.g. gradient) is included in the kernel of the following one (e.g. curl). The de Rham theorem states that the gaps between these images and kernels, embedded into the cohomology of the complex, is related to the topology of the domain. The importance of this complex and its properties in the stability analysis of models of partial differential equations (such as the Stokes/Navier-Stokes equations, magnetostatic equations, etc.) has been understood for decades. Reproducing the properties of this complex at the discrete level is essential for the design of stable schemes for these models, and is related, e.g., to the design of inf-sup stable methods for saddle point problems.

In the last two decades, the Finite Element Exterior Calculus (FEEC) framework has been set up to devise versions of the de Rham complex through the exterior calculus framework, which allows to treat all operators (gradient, curl, divergence) in a unified way as exterior derivatives of differential forms of certain degrees. These discrete complexes are however restricted to particular meshes (mostly made of tetrahedra and hexahedra), which do not easily lends themselves to standard scientific calculus techniques like local mesh refinement or mesh agglomeration (appearing, e.g., in multi-grid methods).

In this talk, we will present the Exterior Calculus Discrete De Rham (ECDDR) method. This is a discrete version of the de Rham complex of differential forms, that can be applied on polytopal meshes (made of generic polygons in 2D, generic polyhedra in 3D). As many polytopal methods, its design is based on adopting a higher and systematic view, which not only relaxes the conditions on the meshes, but can also lead to leaner methods than standard Finite Element methods. The design of ECDDR relies on the Stokes formula, which identifies the relevant degrees of freedom, as well as provides expressions for the discrete differential forms and potential reconstructions. We will show that the algebraic properties of the de Rham complex are preserved at the discrete level (including its cohomology), and we will explain how the tools in the ECDDR can be used to design numerical schemes.

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