Structure-preserving discretization of nonlinear cross-diffusion systems

Monday, 10 June 2024 14:30 (45 minutes)

The main challenges in designing numerical methods for approximating nonlinear cross-diffusion systems is that the diffusion matrix may not be symmetric or positive semidefinite, and that a maximum principle may be not available. In this talk, we present a Local Discontinuous Galerkin method for discretizing nonlinear cross-diffusion systems, which is based on the boundedness-by-entropy framework introduced by A. Jüngel in 2015. Motivated by the underlying entropy structure of the PDE system, nonlinear transformations in terms of the entropy variable allow to enforce positivity of approximate solutions. Moreover, by appropriately introducing auxiliary variables, the problem is reformulated so that nonlinearities do not appear within differential operators or interface terms, leading to nonlinear operators that can be naturally evaluated in parallel. The resulting method has the following desirable properties:

i) it allows arbitrary degrees of approximation in space;

ii) it preserves boundedness of the physical unknowns without requiring postprocessing or slope limiters;iii) nonlinearities do not appear explicitly within differential operators or interface terms, giving the method with a natural parallelizable structure and high efficiency;

iv) it respects a discrete version of the entropy stability estimate of the continuous problem.

This is a joint work with Sergio Gómez and Ansgar Jüngel.

Presenter: PERUGIA, Ilaria (Universität Wien)