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Walks, difference equations and elliptic surfaces

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A walk in the quarter plane is a path in the lattice $\mathbb{Z}^2$ with a prescribed set of directions that is confined in the quarter plane. In the recent years, the enumeration of such walks has attracted the attention of many authors in combinatorics and probability. The complexity of their enumeration is encoded in the algebraic nature of their associated generating series. The main questions are: are these series algebraic, holonomic (solutions of linear differential equations) or differentially algebraic (solutions of algebraic differential equations)?

In this talk, we will show how the algebraic nature of the generating series can be approached via the study of a discrete functional equation over a curve E of genus zero or one over a function field and the Galois theory of difference equations. In the genus zero case, the functional equation corresponds to a so called q-difference equation and the generating series is differentially transcendental. In genus one, the dynamic of the functional equation is the addition by a given point P of the elliptic curve E. If the point P is torsion then the generating series is holonomic. When P is non torsion, the nature of the generating series is captured by the linear dependence relations of certain prescribed points in the Mordell-Weil lattice of an elliptic surface.

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