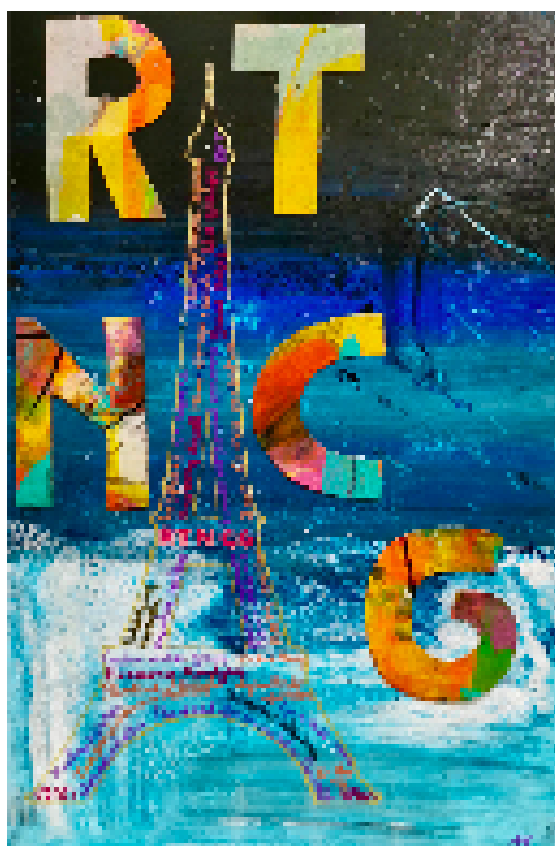


# Intertwining operators and geometry



## Rapport sur les contributions

ID de Contribution: 1

Type: **Non spécifié**

## **Hidden symmetry and spectral analysis on locally pseudo-Riemannian symmetry spaces**

*lundi 20 janvier 2025 10:00 (50 minutes)*

I plan to discuss an application of branching laws in spectral analysis on standard locally symmetric spaces, extending beyond the classical Riemannian setting. Recent advancements have overcome challenges in global analysis with indefinite metrics, thanks to developments in the branching theory of infinite-dimensional representations of reductive groups, which are based on geometries with hidden spherical symmetries. During the lecture, I aim to highlight important, non-technical concepts.

**Orateur:** KOBAYASHI, Toshiyuki (The University of Tokyo)

ID de Contribution: 2

Type: **Non spécifié**

## Generating operators and symmetry breaking

*lundi 20 janvier 2025 11:00 (50 minutes)*

The symmetry breaking of infinite-dimensional representations reveals several remarkable families of equivariant differential operators. We will demonstrate how their global properties can be captured using the concept of a generating operator and explore some applications to branching problems.

**Orateur:** PEVZNER, Michael

ID de Contribution: 3

Type: **Non spécifié**

## Cohomological insights into the Connes-Kasparov isomorphism

*lundi 20 janvier 2025 15:00 (50 minutes)*

A Riemann-Roch type formula serves as the the cornerstone in establishing the Atiyah-Singer index theory via the K-theory method. The classical deformation-to-the-normal-cone approach offers a perspective from noncommutative geometry on formulating the analytic index. In this work, we propose a topological method that combines a Riemann-Roch theorem with deformation-to-the-normal-cone techniques to provide a cohomological depiction of the Connes-Kasparov isomorphism. This is joint work with Paulo Carrillo Rouse and Zijing Wang.

**Orateur:** WANG, Hang (East China Nomal University)

ID de Contribution: 4

Type: **Non spécifié**

## **BGG complex and Poisson transforms as ingredients for a proof of the Baum-Connes conjecture with coefficients for real rank one Lie groups**

*lundi 20 janvier 2025 16:30 (50 minutes)*

We describe the construction of a Fredholm module adapted to the proof of the Baum-Connes (or Connes-Kasparov) conjecture with coefficients for real rank one simple Lie groups (e.g.  $Sp(n, 1)$ ). The main ingredients are a BGG complex on the flag manifold associated to the Borel subgroup, and a suitable Poisson transform from the above complex to the space of L2-harmonic forms on the associated symmetric space.

**Orateur:** JULG, Pierre (Université d'Orléans)

ID de Contribution: 5

Type: **Non spécifié**

## The Torasso representation

*mardi 21 janvier 2025 09:30 (50 minutes)*

For unitary representations of a reductive Lie group, in addition to finding all equivalence classes, there is the problem of describing concrete models of individual representations. This is important for several applications, note the example of the metaplectic representation for the double cover of the symplectic group. For the covering group of three by three real matrices of determinant one, there is an analogous representation studied by many authors, in particular P. Torasso who gave a concrete model. We shall describe other models using intertwining operators, both differential and integral operators. This talk is based on work with T. Kubo and a thesis by T. T. Dahl; also some open problems are stated for this representation.

**Orateur:** ØRSTED, Bent (Aarhus University, Denmark)

ID de Contribution: 6

Type: **Non spécifié**

## Tensor product decomposition for rank one spin groups

*mardi 21 janvier 2025 11:00 (50 minutes)*

Let  $G$  be a real reductive group. Let  $\pi_1$  and  $\pi_2$  be unitary irreducible representations of  $G$ . The decomposition of the tensor product  $\pi_1 \otimes \pi_2$  has been a long-standing problem in harmonic analysis. In this talk, we will discuss this problem for the case where  $G = Spin(n, 1)$ . It turns out that the decomposition of  $\pi_1 \otimes \pi_2$  in this case is closely related to the branching problem of unitary irreducible representations of  $G$  with respect to a minimal parabolic subgroup  $P$ . Especially, in the case where  $\pi_1$  is a unitary principal series (and  $\pi_2$  is an arbitrary unitary irreducible representation of  $G$ ), the tensor product  $\pi_1 \otimes \pi_2$  can be decomposed explicitly based on the knowledge of explicit branching laws with respect to  $P$  and other results and techniques in harmonic analysis and representation theory. If time permits, we will also discuss the case where  $\pi_1$  is a complementary series. This is ongoing joint work with S. Afentoulidis-Almpanis.

**Orateur:** LIU, Gang (IECL, université de Lorraine)

ID de Contribution: 7

Type: **Non spécifié**

## Differential symmetry breaking operators for the pair $(GL(n+1, \mathbb{R}), GL(n, \mathbb{R}))$

*mardi 21 janvier 2025 14:30 (40 minutes)*

In this talk, I will present a way to construct differential symmetry breaking operators between principal series representations induced from the minimal parabolic for the pair  $(GL(n+1, \mathbb{R}), GL(n, \mathbb{R}))$ . The construction, based on the so-called the source operator method, leads to DSBO for some “generic” parameters. I will also discuss the non-generic case based on the  $n=2$  example.

**Orateur:** LABRIET, Quentin (Université de Montréal)



ID de Contribution: 8

Type: **Non spécifié**

## On sporadic symmetry breaking operators of the pair $(SO_0(4, 1), SO_0(3, 1))$

*mardi 21 janvier 2025 15:15 (40 minutes)*

In this talk, we construct and classify all differential symmetry breaking operators between certain principal series representations of the groups  $SO_0(4, 1)$  and  $SO_0(3, 1)$ . Moreover, we prove that for these representations, we obtain a localness theorem, namely, we have that any symmetry breaking operator is given by a differential (local) symmetry breaking operator.

**Orateur:** PÉREZ-VALDÉS, Víctor (Ryukoku University)

ID de Contribution: 9

Type: **Non spécifié**

## Illustrating Delorme's intertwining conditions on $SL(2, \mathbb{R})$ and beyond

*mardi 21 janvier 2025 16:30 (40 minutes)*

The Paley-Wiener space for compactly supported smooth functions  $C_c^\infty(G)$  on a semisimple Lie group  $G$  is characterised by certain intertwining conditions, known as \textit{Delorme's intertwining conditions}, which are challenging to work with. Using the concept of Collingwood's boxes, we demonstrate how these relationships can be simplified and visualised in specific cases such as  $G = SL(2, \mathbb{R})$ , its finite products, and  $SL(2, \mathbb{C})$ . Additionally, we explore how this criterion for the Paley-Wiener space can be applied to analyse the solvability of invariant differential operators acting between sections of homogeneous vector bundles over the corresponding symmetric spaces.

**Orateur:** PALMIROTTA, Guendalina (Universität Paderborn)

ID de Contribution: 10

Type: Non spécifié

## Explicit Hilbert spaces for the unitary dual of rank one orthogonal groups

*mardi 21 janvier 2025 17:15 (40 minutes)*

We realize all irreducible unitary representations of the group  $\mathrm{SO}_0(n+1, 1)$  on explicit Hilbert spaces of vector-valued  $L^2$ -functions on  $\mathbb{R}^n \setminus \{0\}$ . The key ingredient in our construction is an explicit expression for the standard Knapp–Stein intertwining operators between arbitrary principal series representations in the so-called  $F$ -picture which is obtained from the non-compact picture on a maximal unipotent subgroup  $N \cong \mathbb{R}^n$  by applying the Euclidean Fourier transform. As an application, we describe the space of Whittaker vectors on all irreducible Casselman–Wallach representations. Moreover, the new realizations of the irreducible unitary representations immediately reveal their decomposition into irreducible representations of a parabolic subgroup, thus providing a simple proof of a recent result of Liu–Oshima–Yu. This is joint work with Frederik Bang-Jensen and Jan Frahm.

**Orateur:** ARENDS, Christian (Aarhus University)

ID de Contribution: 11

Type: **Non spécifié**

## Symmetry breaking for $PGL_2$ over a local fields

*mercredi 22 janvier 2025 09:00 (50 minutes)*

One major question in the representation theory of locally compact groups is how an irreducible representation of a group  $G$  decomposes if restricted to a subgroup  $H$ . For  $\pi$  and  $\tau$  irreducible representations of  $G$  and  $H$ , respectively, elements of  $\text{Hom}_H(\pi|_H, \tau)$  are referred to as symmetry breaking operators, a term coined by Kobayashi. In a recent joint paper with Jan Frahm we initiate the study of symmetry breaking operators over the  $p$ -adic fields. More precisely, we consider the pair  $(PGL_2(E), PGL_2(F))$ , when  $E$  is a quadratic field extension of an arbitrary  $p$ -adic field  $F$ , and explicitly construct and provide a classification of all symmetry breaking operators between principal series representations of  $PGL_2(E)$  and  $PGL_2(F)$ . Although our results are very similar to the Archimedean case, this talk will try to focus not only on the similarities but also on the differences between the Archimedean and non-Archimedean situations.

**Orateur:** CIOBOTARU, Corina (Aarhus University)

ID de Contribution: 12

Type: **Non spécifié**

## The character correspondence in the stable range over a p-adic field

*mercredi 22 janvier 2025 10:30 (50 minutes)*

Given a real irreducible dual pair there is an integral kernel operator which maps the distribution character of an irreducible admissible representation of the group with the smaller or equal rank to an invariant eigendistribution on the group with the larger or equal rank. If the pair is in the stable range and if the representation is unitary, then the resulting distribution is the character of the representation obtained via Howe's correspondence. This construction was transferred to the p-adic case and a conjecture was formulated. In this talk we verify a weaker version of this conjecture for dual pairs in the stable range over a p-adic field. This is a joint work with Hug Yean Loke, National University of Singapore.

**Orateur:** PRZEBINDA, Tomasz (University of Oklahoma)

ID de Contribution: 13

Type: **Non spécifié**

## Theta correspondence via $C^*$ -algebras

*mercredi 22 janvier 2025 11:30 (50 minutes)*

In this talk I will discuss how the well-known explicit construction of the local theta correspondence by J.S. Li has a simple interpretation in terms of induced representations group  $C^*$ -algebras in the sense of M.A.Rieffel. This picture allows us deduce that in the standard cases where Li's method works, local theta correspondence arises from a continuous functor. In special cases, the functor implements a continuous equivalence of representation categories called strong Morita equivalence. No background in  $C^*$ -algebras is required for this talk, as I will introduce the necessary concepts along the way. This is joint work with Magnus Goffeng (Lund) and Haluk Sengun (Sheffield).

**Orateur:** MESLAND, Bram (Universiteit Leiden)

ID de Contribution: 14

Type: **Non spécifié**

## Symmetry breaking for some representations of $GL(n, \mathbb{R})$

*jeudi 23 janvier 2025 09:30 (50 minutes)*

I will discuss the restriction of representations in the discrete spectrum of of the symmetric space  $GL(n, \mathbb{R})/GL(p, \mathbb{R})GL(n-p, \mathbb{R})$  to  $GL(n-1, \mathbb{R})$ .

**Orateur:** SPEH, Birgit (Cornell University)

ID de Contribution: 15

Type: **Non spécifié**

## **Cartan subalgebras in the branching problem and Hamiltonian action**

*jeudi 23 janvier 2025 11:00 (50 minutes)*

In this talk, I will discuss three Cartan subalgebras (or root systems) related to the branching problem of reductive Lie groups. One Cartan subalgebra describes complexity of an embedding of  $G$ -varieties. This is related to intertwining operators (symmetry breaking operators). The others are defined by the annihilators of  $\mathfrak{g}$ -modules or their non-zero vectors. They are related to the shape of the (continuous) spectrum. I will also discuss a relation between the Cartan subalgebras and the moment maps for Hamiltonian actions.

**Orateur:** KITAGAWA, Masatoshi (The University of Tokyo)



ID de Contribution: 16

Type: Non spécifié

## Intertwining operators for representations of covering groups of reductive $p$ -adic groups

*jeudi 23 janvier 2025 14:30 (40 minutes)*

In my talk, I will explain my PhD research project, which is about poles and zeros of the Harish-Chandra  $\mu$ -function. This function appears in the representation theory of  $p$ -adic groups, and is defined using intertwining operators between parabolically induced representations. It can be used to describe Bernstein blocks in the category of smooth representations of a reductive  $p$ -adic group. This work was done by my supervisor Maarten Solleveld, and the goal of my project is to generalize these results to covering groups of reductive  $p$ -adic groups. To do this, it is necessary to analyze the poles and zeros of the  $\mu$ -function, which can be seen as a complex rational function. For reductive groups, there is a formula for it given by Silberger, but it is not clear how his proof generalizes to covering groups. Therefore, my supervisor and I have been working on a different proof, which does work for covering groups of reductive  $p$ -adic groups. The proof uses techniques involving Hermitian and unitary representations, as well as  $C^*$ -algebras and operator theory. In my talk, I aim to provide the necessary background, before discussing the operator theoretical methods used to locate the poles and zeros of the  $\mu$ -function.

**Orateur:** FLIKKEMA, Janet (Radboud University)

ID de Contribution: 17

Type: **Non spécifié**

## Limit formulas for the trace of the functional calculus of quantum channels for $SU(2)$

*jeudi 23 janvier 2025 15:15 (40 minutes)*

Equivariant quantum channels are completely positive trace-preserving maps intertwining representations of a group  $G$ . Lieb and Solovej (2014) studied traces of the functional calculus of equivariant quantum channels for  $SU(2)$  to establish a Wehrl-type inequality for integrals of convex functions of matrix coefficients. In particular, they showed that coherent highest weight states minimize the Wehrl entropy, solving a long-standing conjecture. These quantum channels are defined by projecting onto the leading component in the decomposition of the tensor product of two irreducible representations of  $SU(2)$ . It is proved by Lieb and Solovej that the aforementioned integral of a convex function is the limit of the trace of the functional calculus of these equivariant quantum channels. I introduce new equivariant quantum channels for all the components in the tensor product and generalize their limit formula. I do this by realizing representations of  $SU(2)$  as reproducing kernel spaces and using explicit projections onto components of the tensor product. This allows me to pass from the trace to an integral to derive a limit formula and prove that the limit can be expressed using Berezin transforms, which are closely related to quantization on Kähler manifolds.

**Orateur:** VAN HAASTRECHT, Robin (Gothenburg University/Chalmers)

ID de Contribution: 18

Type: **Non spécifié**

## $(k, a)$ -generalized Fourier transform with negative $a$

*jeudi 23 janvier 2025 16:30 (40 minutes)*

The  $(k, a)$ -generalized Fourier transform  $F_{k,a}$  introduced by Ben Saïd–Kobayashi–Ørsted is a deformation family of the classical Fourier transform with a Dunkl parameter  $k$  and a parameter  $a > 0$  that interpolates minimal representations of two different simple Lie groups. In this session, we will talk about some new results when  $a$  is not positive. As a main result, we find a unitary transform that intertwines the known case  $a > 0$  and the new case  $a < 0$ .

**Orateur:** HIKAWA, Tatsuro (The University of Tokyo)

ID de Contribution: 19

Type: Non spécifié

## Deformation of heat kernels and Brownian motions from the viewpoint of Ben Saïd-Kobayashi-Ørsted $(k, a)$ -generalized Laguerre semigroup theory

*jeudi 23 janvier 2025 17:15 (40 minutes)*

In recent years, an analytical framework based on the “ $(k, a)$ -generalized Fourier analysis” introduced by Ben Saïd–Kobayashi–Ørsted has been actively studied. This is a novel branch of harmonic analysis that deforms the traditional Fourier analysis theory using two parameters,  $k$  and  $a$ , arising from Dunkl theory and the interpolation theory of minimal representations of Lie groups. In this talk, I will introduce what is currently known about how concepts related to harmonic analysis, such as the heat equation, heat kernel, and Brownian motion, are able to be generalized in this new framework and what fundamental properties they have.

**Orateur:** AOYAMA, Temma (The University of Tokyo)

ID de Contribution: 20

Type: **Non spécifié**

## On the multiplicity-freeness property of cohomology spaces and the visibility of group actions

*vendredi 24 janvier 2025 09:00 (50 minutes)*

With the aim of uniform treatment of multiplicity-free representations of Lie groups, T. Kobayashi introduced the notion of visible action for holomorphic actions of Lie groups on complex manifolds. His propagation theorem of the multiplicity-freeness property produces various kinds of multiplicity-free theorems for unitary representations realized in the space of holomorphic sections of an equivariant holomorphic vector bundle whose base space admits a visible action of a Lie group.

Kobayashi has indicated two directions of generalizations of his multiplicity-free theorem. One is a generalization to infinite dimensional manifolds and has been done by Miglioli and Neeb. The other is a generalization to cohomology spaces, which is the main concern of this talk.

I would like to talk about a cohomology version of Kobayashi's theorem and its application to multiplicity-free restrictions of Zuckerman derived functor modules to reductive subgroups.

**Orateur:** TANAKA, Yuichiro (The University of Tokyo)

ID de Contribution: 21

Type: **Non spécifié**

## Exceptional complex dual pair correspondences

*vendredi 24 janvier 2025 10:30 (50 minutes)*

Let  $E_n(\mathbb{C})$  denote the connected complex Lie group of type  $E_n$  for  $n = 6, 7$ . These two groups contain the following reductive pairs:

$\begin{aligned}$

$T_1(\mathbb{C}) \times \mathrm{Spin}(10, \mathbb{C}) \subset E_6(\mathbb{C}),$

$\backslash \mathrm{cr}$

$T_2(\mathbb{C}) \times \mathrm{Spin}(8, \mathbb{C}) \subset E_6(\mathbb{C}),$

$\backslash \mathrm{cr}$

$T_1(\mathbb{C}) \times E_6(\mathbb{C}) \subset E_7(\mathbb{C}),$

$\end{aligned}$  where  $T_1(\mathbb{C})$  and  $T_2(\mathbb{C})$  are complex tori of dimensions 1 and 2 respectively. In this

talk, I will describe the dual pair correspondences arising from the minimal representations of  $E_6(\mathbb{C})$  and  $E_7(\mathbb{C})$ . These are joint projects with Edmund Karasiewicz and Gordan Savin.

**Orateur:** LOKE, Hung Yean (National University of Singapore)

ID de Contribution: 22

Type: **Non spécifié**

## Intertwining operators in the $C^*$ -algebraic picture of the tempered dual

*vendredi 24 janvier 2025 11:30 (50 minutes)*

Intertwining operators between parabolically induced representations play a fundamental role in the study of the tempered dual of reductive groups. Therefore it is not surprising to see related objects, such as R-groups, appear in the description of the reduced  $C^*$ -algebra associated to these groups. The purpose of this talk will be to explain how various techniques of operator algebraic nature allow to study families of intertwining operators at the level of Hilbert modules, with the goal of analyzing the tempered dual as a noncommutative topological space. Most results presented are joint work with Tyrone Crisp and Nigel Higson.

**Orateur:** CLARE, Pierre (College of William & Mary)

ID de Contribution: 23

Type: **Non spécifié**

## **Opening by Dominique Mouhanna, IHP Director**

*lundi 20 janvier 2025 09:50 (10 minutes)*