Mini-course:
Structure of tempered homogeneous spaces

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The three lectures introduce recent theories of tempered spaces, and I plan to provide an overview of these topics, using plenty of elementary examples to make the basic concepts and key ideas more accessible.

Monday, February 17 - 14:00

Structure of tempered homogeneous spaces, I – Dynamical approach

In the first lecture, I will review basic concepts such as tempered unitary representations of real reductive groups, like $GL(n, \mathbb{R})$, as well as "tempered spaces" and "tempered subgroups". I will begin with some geometric observations of group actions, including the properness criterion for reductive homogeneous spaces. Subsequently, I will introduce a "quantification" of proper actions and incorporate a dynamical approach into analytic representation theory, including the temperedness criterion for homogeneous spaces, which was developed recently by Y. Benoist and the speaker, drawing on the Cowling-Haagerup-Howe theory and other related ideas.

Wednesday, February 19 - 14:00

Structure of tempered homogeneous spaces, II – Combinatorics approach

The criterion for tempered spaces, explained in the first lecture, is computable. In this lecture, I will explain how this criterion leads to the classification theory of non-tempered reductive homogeneous spaces by breaking it down into several steps. The technical methods used in the second lecture differ from the dynamical approach presented in the first lecture. Our approach relies on elementary results from finite-dimensional representations and some combinatorics of convex polyhedral cones.

Friday, February 21 - 14:00

Structure of tempered homogeneous spaces, III – Limit algebras

Recently, surprising and intriguing connections have been observed between the concept of "tempered spaces" for unitary representations and various other areas of mathematics. In this lecture, we will explore different aspects of tempered spaces from the perspectives of topology and geometry, including limit algebras (collapsing Lie algebras) and geometric quantization.