Operator Algebras, Index Theory and Geometry

Thematic day in celebration of Pierre Julg's career

Institut Henri Poincaré
February 5, 2025

8:45 – 9:45 **Michael Cowling** (University of New South Wales)
Admissibility of irreducible representations

9:45 – 10:15 **Coffee break**

10:15 – 11:15 **Georges Skandalis** (Université de Paris Cité)
Groupoids as cyclic sets

11:15 – 12:15 **Alain Valette** (Université de Neuchâtel)
Reciprocal hyperbolic elements in $\mathrm{PSL}_2(\mathbb{Z})$

12:15 – 13:45 **Lunch break**

13:45 – 14:45 **Nigel Higson** (Pennsylvania State University)
*Tempiric representations, Connes-Kasparov, and pseudodifferential operators
on symmetric spaces*

14:45 – 15:45 **Robert Yuncken** (Université de Lorraine)
On pseudodifferential calculi for subelliptic operators

15:45 – 16:00 **Short break**

16:00 – 17:00 **Gennadi Kasparov** (Vanderbilt University)
Index theory on manifolds with a tangent Lie structure

17:00 – 18:00 **Alain Connes** (Institut des Hautes Études Scientifiques)
From class field theory to zeta spectral triples

The talks will take place in Amphithéâtre Darboux.

Abstracts

Michael Cowling (University of New South Wales)

Admissibility of irreducible representations

This is an account of joint work with Francesca Astengo (Genova) and Bianca Di Blasio (Milano Bicocca).

Harish-Chandra showed that irreducible unitary representations of semisimple Lie groups are admissible; these are easier to understand than general representations. But, as Pierre Julg and others have shown, there are good reasons to look at representations that are not unitary, so it is natural to ask whether "all" irreducible representations on (say) Banach spaces are admissible. We show that the answer to this question is closely related to a classical question in functional analysis, the invariant subspace problem.

Georges Skandalis (Université de Paris Cité)

Groupoids as cyclic sets

Associated to a groupoid is a natural cyclic set. Actually, one can give several characterisations of groupoids based on this set. It also gives a nice interpretation of well known notions of on groupoids: symplectic groupoids, duals of VB groupoids and in particular the Weinstein groupoid... Joint work with Claire Debord.

Alain Valette (Université de Neuchâtel)

Reciprocal hyperbolic elements in $\mathrm{PSL}_2(\mathbb{Z})$

An element A in $\mathrm{PSL}_2(\mathbb{Z})$ is hyperbolic if $|\operatorname{Tr}(A)| > 2$. The maximal virtually abelian subgroup of $\mathrm{PSL}_2(\mathbb{Z})$ containing A is either infinite cyclic or infinite dihedral; say that A is reciprocal if the second case happens (A is then conjugate to its inverse). We give a characterization of reciprocal hyperbolic elements in $\mathrm{PSL}_2(\mathbb{Z})$ in terms of the continued fractions of their fixed points in $\mathrm{P}^1(\mathbb{R})$ (those are quadratic surds). Doing so we revisit results of P. Sarnak (2007) and C.-L. Simon (2022), themselves rooted in classical work by Gauss and Fricke & Klein.

Nigel Higson (Penn State University)

Tempiric representations, Connes-Kasparov, and pseudodifferential operators on symmetric spaces

Let G be a real reductive group, connected for simplicity, with maximal compact subgroup K. The Connes-Kasparov isomorphism attaches a single parameter to most, but not all, of the components of the tempered dual of a real reductive group. That parameter is a shifted version of a highest weight for K, and every such parameter is attached to a unique component in the tempered dual. On the other hand, Vogan's theory of minimal K-types associates to every component of the tempered dual

of G a finite collection of highest weights for K. These collections are disjoint, and every highest weight appears in one of them. Most of Vogan's collections are singletons, but not all of them. The Connes-Kasparov and Vogan correspondences seem to be more similar than they are different, and in the online precursor to the current thematic program at IHP, Vogan asked whether they can be reconciled by adjusting the definition of the reduced C*-algebra of G? I shall discuss one answer, involving pseudodifferential operators, and some of the new questions that arise from that answer. This is joint work with Peter Debello.

Robert Yuncken (Université de Lorraine)

On pseudodifferential calculi for subelliptic operators

Many constructions in representation theory and index theory are based upon elliptic differential operators. The work of Julg and Kasparov shows that subelliptic differential operators, such as the Berstein-Gelfand-Gelfand complex, are often required to make certain key constructions. In this talk, we will describe the tangent groupoid associated a very general class of such operators, namely operators of Helffer-Nourrigat type. Generalising ideas of Connes and Debord-Skandalis then allows us to prove the analytic properties of such operators. Joint work with Iakovos Androulidakis, Omar Mohsen and Erik Van Erp.

Gennadi Kasparov (Vanderbilt University)

Index theory on manifolds with a tangent Lie structure

In recent years there was a significant progress in the theory of pseudo-differential operators on filtered manifolds. I will introduce in my talk a wider class of manifolds which I call manifolds with a tangent Lie structure. I will explain a coarse approach to pseudo-differential theory which gives a simplified pseudo-differential calculus containing only operators of order 0 and negative order. This calculus easily leads to the Atiyah-Singer type index theorem for operators of order 0 on manifolds with a tangent Lie structure. For filtered manifolds this calculus agrees with the known Hörmander and van Erp - Yuncken calculi, which allows to extend the index theorem to operators of any order.

Alain Connes (IHES)

From class field theory to zeta spectral triples

J'exposerai les résultats récents (< 1 an) de ma collaboration avec C. Consani et H. Moscovici sur ia compréhension conceptuelle de l'espace des classes d'adèles comme extension de la théorie du corps de classe et la construction de triplets spectraux donnant les zéros de zêta.