NCG and (Quantum) Group·oid·s

Talks by early career participants in the RTNCG trimester

Institut Henri Poincaré – Amphithéâtre Darboux
February 11 – 12, 2025

Tuesday, February 11

11:00 – 11:45 **Roberto Steven Vargas Peña** (Universidad de los Andes)
The noncommutative geometry of the space of positive elements in a C-algebra*

11:45 – 14:00 **Lunch break**

14:00 – 14:45 **Pegah Pournajafi** (Collège de France)
Quantum groups meet graphs

15:00 – 15:45 **Heon Lee** (Harbin Institute of Technology)
First-order differential calculi and Laplacians on q*-deformations of compact semisimple Lie groups*

The weekly AIM-RTNCG joint talk will take place at 16:00, independently from this mini-workshop.

Wednesday, February 12

9:30 – 10:30 **Max Carter** (Université Catholique de Louvain)
Unitary representations and spectral synthesis on contraction groups

10:30 – 11:00 **Coffe break**

11:00 – 12:00 **Paul Le Breton** (Université de Paris Cité)
Lie groupoids and differential operators

12:00 – 14:00 **Lunch break**

14:00 – 15:00 **Moudrik Chamoux** (Université Paris-Est Créteil)
Deformation to the Normal Cone, zoom action and distributions

15:00 – 15:30 **Coffe break**

15:30 – 16:30 **Lucas Lemoine** (Université Paris-Est Créteil)
Automorphisms of pseudodifferential operators on Lie groupoids

Abstracts

Roberto Steven Vargas Peña (Universidad de los Andes)

The noncommutative geometry of the space of positive elements in a C-algebra*

This talk delves into the fascinating differential geometry of positive elements of a C^*-algebra, highlighting its complexity, principal results, and depth. It also proposes an intriguing connection between this geometry and Connes's non-commutative approach, opening up new avenues for exploration in this field.

Pegah Pournajafi (Collège de France)

Quantum groups meet graphs

Quantum groups and graph theory may seem like distant fields, yet intriguing connections arise when they intersect. After a gentle introduction to the topic, we explore these interactions through the study of quantum automorphism groups of block graphs. We conclude with some open problems and future directions. These results are parts of a common project with Amaury Freslon and Paul Meunier.

Heon Lee (Harbin Institute of Technology)

First-order differential calculi and Laplacians on q*-deformations of compact semisimple Lie groups*

In this talk, we suggest a simple definition of Laplacian on a compact quantum group (CQG) associated with a first-order differential calculus (FODC) on it. Applied to the classical differential calculus on a compact Lie group, this definition yields classical Laplacians, as it should. Moreover, on the CQG K_q arising from the q-deformation of a compact semisimple Lie group K, we can find many interesting linear operators that satisfy this definition, which converge to a classical Laplacian on K as q tends to 1. In light of this, we call them q-Laplacians on K_q and investigate some of their operator theoretic properties. This work is based on the preprint arXiv:2410.00720.

Max Carter (Université Catholique de Louvain)

Unitary representations and spectral synthesis on contraction groups

The solution to Hilbert's fifth problem dictates that every connected locally compact group is an inverse limit of connected Lie groups, and thus Lie theoretic techniques can be used in the study of connected locally compact groups. In contrast, totally disconnected locally compact (tdlc) groups cannot be approximated by Lie groups or algebraic group over tdlc fields, and their structure is not well understood, generally speaking. Modern research in topological group theory is largely focussed on understanding the class of tdlc groups. Furthermore, our understanding of the (unitary) representation theory and harmonic analysis of tdlc groups lags far behind that of connected locally compact groups. In this talk, I will discuss recent progress on the harmonic analysis of (tdlc) contraction groups. Contraction groups can be viewed, in some sense, as analogues/generalisations of unipotent groups in the theory of tdlc groups. The focus will be on discussing recent progress on the unitary representation theory of these groups, and time pending, I will say a bit about spectral synthesis of certain convolution algebras on these groups.

Paul Le Breton (Université de Paris Cité)

Lie groupoids and differential operators

In numerous PDE problems, differential operators naturally appear with some geometrical constraints. For instance, if $(M, \mathcal{F})$ is a foliated manifold one may be interested in operators acting longitudinally on the leaves, if M has borders (or corners) one may ask operators to be well-behaved around the borders, on a manifold endowed with an action of a Lie group G one may consider G-equivariant operators...
The aim of this talk is to explain how Lie groupoids provide a unified framework to study the analytic behaviour of all of these operators. This point of view allows to adapt the notions of principal symbols and ellipticity to these contexts and to define a well behaved pseudodifferential calculus in order to build parametrix. One can then use these operators to build classes in the K-theory of the C^* algebras of the groupoids, as a first step towards Atiyah-Singer type theorems.

Moudrik Chamoux (Université Paris-Est Créteil)

Deformation to the Normal Cone, zoom action and distributions

The C^*-analysis of Lie groupoids provides powerful tools for understanding analysis on smooth manifolds and Lie groups. Deformations to the Normal Cone appear in the construction of fundamental instances of such Lie groupoids. In this talk we will discuss deformations to the normal cone, the zoom action and homogeneous distributions on a DNC. If time permits, we will also discuss applications to analysis on manifolds such as the pseudodifferential calculus *à la* Van Erp and Yuncken.

Lucas Lemoine (Université Paris-Est Créteil)

Automorphisms of pseudodifferential operators on Lie groupoids

The study of pseudodifferential operators on Lie groupoids offers a natural extension of classical analysis on smooth manifolds, encompassing singular spaces. Moreover, pseudodifferential operators can be generalized in the classical setting by Fourier integral operators. A key result in this context is Egorov's theorem: conjugation by a Fourier integral operator preserves pseudodifferential operators. Duistermaat and Singer later established a converse: any automorphism of the filtered algebra of classical pseudodifferential operators can be realized as conjugation by a Fourier integral operator. In this talk, I will discuss a generalization of this result in the setting of Lie groupoids. This involves a detailed examination of the symplectic structure of the cotangent groupoid, which is fundamental for constructing Fourier integral operators in this context.