

Categorified Crystal Bases on Localized Quantum Coordinate Rings and Cellular Crystals

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For a monoidal category (τ, \circ) , if there exists a “real commuting family $(C_i, R_{C_i}, \phi_i)_{i \in I}$ ”, we can define a localization $\tilde{\tau}$ of τ by $(C_i, R_{C_i}, \phi_i)_{i \in I}$. Let $R = R(\mathfrak{g})$ be the quiver Hecke algebra (=KLR algebra) associated with a simple Lie algebra \mathfrak{g} and $R\text{-gmod}$ the category of finite-dimensional graded R -modules, which is a monoidal category with a real commuting family $(C_i, R_{C_i}, \phi_i)_{i \in I}$. Thus, we get its localization $\tilde{R}\text{-gmod}$. It has been shown that $R\text{-gmod}$ categorifies the unipotent quantum coordinate ring $A_q(\mathfrak{g})$, that is, the Grothendieck ring $K(R\text{-gmod})$ is isomorphic to $A_q(\mathfrak{g})$. For the localized category $\tilde{R}\text{-gmod}$, its Grothendieck ring $K(\tilde{R}\text{-gmod})$ defines the localized (unipotent) quantum coordinate ring $A_q(\mathfrak{g})$.

We shall give a certain crystal structure on the set of self-dual simple objects $\mathbb{B}(\tilde{R}\text{-gmod})$ in $\tilde{R}\text{-gmod}$. We also give the isomorphism of crystals from $\mathbb{B}(\tilde{R}\text{-gmod})$ to the cellular crystal $\mathbb{B}_{\mathbf{i}} = B_{i_1} \otimes \cdots \otimes B_{i_N}$ for an arbitrary reduced word $\mathbf{i} = i_1 \cdots i_N$ of the longest Weyl group element. This result can be seen as a localized version for the categorification of the crystal base $B(\infty)$ for the subalgebra $U_q^-(\mathfrak{g}) (\cong A_q(\mathfrak{g}))$ of the quantum algebra $U_q(\mathfrak{g})$, given by Lauda-Vazirani.

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