

# The Nullstellensatz and Positivstellensatz for Sparse Tropical Polynomial Systems, and Parametric Mean-Payoff Games

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Grigoriev and Podolskii (2018) have established a tropical analog of the effective Nullstellensatz, showing that a system of tropical polynomial equations is solvable if and only if a linearized system obtained from a truncated Macaulay matrix is solvable. They provided an upper bound of the minimal admissible truncation degree, as a function of the degrees of the tropical polynomials. We establish a tropical Nullstellensatz adapted to sparse tropical polynomial systems. Our approach is inspired by a polyhedral construction of Canny-Emiris (1993), refined by Sturmfels (1994). This leads to an improved bound of the truncation degree, which coincides with the classical Macaulay degree in the case of  $n+1$  equations in  $n$  unknowns. We also derive a tropical Positivstellensatz, allowing one to decide the inclusion of tropical basic semialgebraic sets. We finally show that solutions can be computed by a reduction to parametric mean-payoff games, providing a tropical analog of eigenvalue methods to solve polynomial systems. This is a joint work with Marianne Akian and Antoine Bérau, based in particular on the following article: *The Tropical Nullstellensatz and Positivstellensatz for Sparse Polynomial Systems*; In Proceedings of the 2023 International Symposium on Symbolic and Algebraic Computation. ACM, July 2023

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