Simplifying (super-)BMS algebras

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Bondi-van der Burg-Metzner-Sachs (BMS) symmetry

BMS, asymptotic symmetries of GR: found in the 60's in an asymptotic region of spacetime called *null infinity* (radiation).

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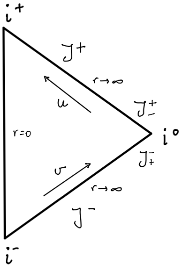

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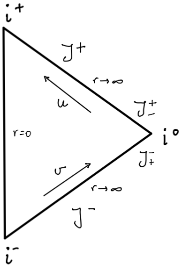

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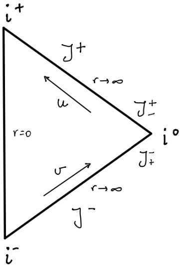

Infinite-dimensional extension of the Poincaré algebra by a set of angle-dependent translations: supertranslations (Abelian subgroup).

Connected to Weinberg's soft graviton theorems through Ward identities, leading to a deeper physical understanding of classical and quantum properties of gravity [Strominger's lectures: 1703.05448].

BMS, matching conditions and spatial infinity

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• Connecting i^0 with $\mathscr{I}^+_-$ and $\mathscr{I}^-_+$ is a non-trivial and subtle question. Evolution of reasonable Cauchy data makes null infinity not so smooth. Metric and Weyl tensor develop logarithmic singularities [Friedrich, Valiente-Kroon...].

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→ Symmetries are canonical: we can associate to any symmetry a charge-generator.

→ Strominger's matching conditions:

$$\Phi(\theta,\varphi)\Big|_{\mathscr{I}^+_-} = \Phi(\theta-\pi,\varphi+\pi)\Big|_{\mathscr{I}^-_+}$$

which lead to an infinity of conservation laws (energy and angular momentum at each angle on S^2), are really a consequence of the boundary (parity) conditions imposed at i^0 for having a well-defined action principle.

Logarithmic relaxation of the gravitational field

One could wonder whether it is possible to relax consistently the asymptotic behaviour of the gravitational field by log terms (finite action, finite/integrable canonical generators...):

$$g_{ij} = (g_{ij})_{\text{RT}} + U_{ij} \qquad U_{ij} = \Delta_{ij}^{\log} + \Delta_{ij}^{\text{diff}}$$
$$\pi^{ij} = (\pi^{ij})_{\text{RT}} + V^{ij} \qquad V^{ij} = \Gamma_{\log}^{ij} + \Gamma_{\text{diff}}^{ij}$$

Asymptotically:

$$g_{ij} = \delta_{ij} + \frac{\ln r}{r}\overline{\Delta}_{ij}^{\log} + \frac{\overline{h}_{ij}}{r} + o\left(r^{-1}\right) \qquad \pi^{ij} = \frac{\ln r}{r^2}\overline{\Gamma}_{\log}^{ij} + \frac{\overline{\pi}^{ij}}{r^2} + 0\left(r^{-2}\right)$$

where

$$\overline{h}_{ij} = (\overline{h}_{ij})^{\text{even}} + \overline{\Delta}_{ij}^{\text{odd}} \qquad \overline{\pi}^{ij} = (\overline{\pi}^{ij})_{\text{odd}} + \overline{\Gamma}_{\text{even}}^{ij}$$

$$\overline{\Delta}_{ij}^{\text{odd}} = r\left(\partial_i V_j + \partial_j V_i\right) \qquad \overline{\Delta}_{ij}^{\log} = r\left(\partial_i \tilde{V}_j + \partial_j \tilde{V}_i\right) = \text{even}$$
$$\overline{\Gamma}_{\text{even}}^{ij} = r^2\left(\partial^i\partial^j V - \delta^{ij}\triangle V\right) \qquad \overline{\Gamma}_{\log}^{ij} = r^2\left(\partial^i\partial^j \tilde{V} - \delta^{ij}\triangle \tilde{V}\right) = \text{odd}$$

Asymptotic conditions

The asymptotic behaviour of the gravitational field in spherical coordinates

$$g_{rr} = 1 + \frac{1}{r}\bar{h}_{rr} + \frac{1}{r^2}\left(\ln^2 r\, h_{rr}^{\log(2)} + \ln r\, h_{rr}^{\log(1)} + h_{rr}^{(2)}\right) + o(r^{-2})$$

$$g_{rA} = \bar{\lambda}_A + \frac{1}{r}\left(\ln^2 r\, h_{rA}^{\log(2)} + \ln r\, h_{rA}^{\log(1)} + h_{rA}^{(2)}\right) + o(r^{-1})$$

$$g_{AB} = r^2 \bar{g}_{AB} + r\left(\ln r\, \theta_{AB} + \bar{h}_{AB}\right) + \ln^2 r\, \theta_{AB}^{(2)} + \ln r\, \sigma_{AB} + h_{AB}^{(2)} + o(1)$$

and

$$\pi^{rr} = \ln r\, \pi^{rr}_{\log} + \bar{\pi}^{rr} + \frac{1}{r}\left(\ln^2 r \pi^{rr}_{\log(2)} + \ln r \pi^{rr}_{\log(1)} + \pi^{rr}_{(2)}\right) + o(r^{-1})$$

$$\pi^{rA} = \frac{\ln r}{r}\pi^{rA}_{\log} + \frac{1}{r}\bar{\pi}^{rA} + \frac{1}{r^2}\left(\ln^2 r \pi^{rA}_{\log(2)} + \ln r \pi^{rA}_{\log(1)} + \pi^{rA}_{(2)}\right) + o(r^{-2})$$

$$\pi^{AB} = \frac{\ln r}{r^2}\pi^{AB}_{\log} + \frac{1}{r^2}\bar{\pi}^{AB} + \frac{1}{r^3}\left(\ln^2 r \pi^{AB}_{\log(2)} + \ln r \pi^{AB}_{\log(1)} + \pi^{AB}_{(2)}\right) + o(r^{-3})$$

All the log subleading terms are required by preservation under Poincaré transformations (non-linearity of GR!). For details see [OF, Henneaux, Troessaert JHEP 2211.10941].

Logarithmic relaxation of the gravitational field

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• Boundary conditions invariant (besides the BMS supertranslations S_β) under a new kind of logarithmic supertranslations L^α.

• These logarithmic supertranslations are canonically conjugate to the pure supertranslations:

$$\{L^\alpha, S_\beta\} = \delta^\alpha_\beta$$

Decoupling of the pure supertranslations from Poincaré

- The presence of these central charges allows to decouple all pure supertranslations from the Poincaré algebra:

$$\text{Lorentz} \ltimes (\text{supertranslations} \times \text{log-supertranslations})$$

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• Other proposals to solve this "problem" in an independent form at null infinity by Yau et al [2102.03235, 2107.05316...], Porrati et al [1607.03120, 2202.03442...] and Compère et al [1912.03164, 2303.17124...].

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→ All these proposals are indeed equivalent! Nonetheless, analysis at i^0 is more complete concerning the nature of the redefinitions (Poisson brackets of all canonical variables...) [OF, Henneaux, Troessaert PRL 2305.05436].

The log-BMS algebra

$$\{M_a, M_b\} = f_{ab}^c M_c$$

$$\{M_a, T_i\} = R_{ai}^j T_j$$

$$\{M_a, S_\alpha\} = G_{a\alpha}^\beta S_\beta + G_{a\alpha}^i T_i$$

$$\{M_a, L^\alpha\} = -G_{a\beta}^\alpha L^\beta$$

$$\{L^\alpha, S_\beta\} = \delta_\beta^\alpha$$

Lorentz generators: M_a (spatial rotations and Lorentz boosts)

Translations generators : T_i (rigid)

Supertranslations generators: S_α (BMS supertranslations)
L^β (Log supertranslations)

Decoupling of the pure supertranslations from Lorentz

The searched-for redefinition ("nonlinear automorphism" of the Lorentz algebra) reads

$$\tilde{M}_a = M_a - G^i_{a\beta} T_i L^\beta - G^\alpha_{a\beta} S_\alpha L^\beta$$

The asymptotic symmetry algebra then becomes

$$\{\tilde{M}_a, \tilde{M}_b\} = f^c_{ab} M_c \qquad \{\tilde{M}_a, T_i\} = R^j_{ai} T_j$$

$$\{\tilde{M}_a, S_\alpha\} = \{\tilde{M}_a, L^\alpha\} = 0$$

$$\{L^\alpha, S_\beta\} = \delta^\alpha_\beta$$

This mechanism is implemented through suitable field-dependent diffeomorphisms [OF, Henneaux, Troessaert JHEP 2211.10941].

Decoupling of the pure supertranslations from Lorentz

- The charges L^α match the null infinity potential C: electric part of the Bondi shear or the Goldstone boson of spontaneously broken supertranslation invariance [OF, Henneaux, Troessaert PRL 2305.05436].

- We will extend this construction to

  - The higher-dimensional generalization of the BMS algebra (BMS_5).

  - The supersymmetric extension of BMS (super-BMS).

  Common feature: nonlinear algebras.

Structure of BMS_5 and super-BMS

BMS_5

$$[M_a, M_b] = f^c_{ab} M_c$$
$$[M_a, T_i] = R_{ai}{}^j T_j$$
$$\{M_a, S_\alpha\} = G_{a\alpha}{}^i T_i + G_{a\alpha}{}^\beta S_\beta + U_{a\alpha\beta\gamma} L^\beta L^\gamma$$
$$[M_a, L^\alpha] = -G_{a\beta}{}^\alpha L^\beta$$
$$[L^\alpha, S_\beta] = \delta^\alpha_\beta$$

Susy: Q_I (rigid)

Local susy: q_A (inf-dim)

Ferm. symmetry: s^B (inf-dim)

super-BMS

$$[M_a, M_b] = f^c_{ab} M_c$$
$$[M_a, T_i] = R^j_{ai} T_j$$
$$[M_a, S_\alpha] = G^i_{a\alpha} T_i + G^\beta_{a\alpha} S_\beta$$
$$[M_a, Q_I] = g^J_{aI} Q_J + V^i_{aIB} s^B T_i + V^\alpha_{aIB} s^B S_\alpha$$
$$[M_a, q_A] = h^B_{aA} q_B + U^i_{aAB} s^B T_i + U^\alpha_{aAB} s^B S_\alpha$$
$$[M_a, s^B] = -h^B_{aC} s^C$$
$$\{s^A, q_B\} = \delta^A_B$$
$$\{Q_I, q_A\} = d^i_{IA} T_i + d^\alpha_{IA} S_\alpha$$
$$\{q_A, q_B\} = d^i_{AB} T_i + d^\alpha_{AB} S_\alpha$$
$$\{Q_I, Q_J\} = d^i_{IJ} T_i$$

Canonical generator of the asymptotic symmetries

• Asymptotic symmetries: preservation of boundary conditions and action $\Leftrightarrow$ Canonical transformations (well-defined canonical generator).

• Canonical generator:

$$G_\xi = \int d^d x \xi^\alpha \mathcal{H}_\alpha + B_\xi \qquad B_\xi = \oint_{S_\infty^{d-1}} d^{d-1} y f$$

$\rightarrow$ B_ξ is necessary in order to satisfy $\iota_{X_\xi}\Omega = -d_V G_\xi$.

• Trivial asymptotic symmetries **(proper)** are those that decay fast enough so that $B_\xi = 0 \Rightarrow G_\xi \approx 0$. They form an ideal.

• Non-trivial or large asymptotic symmetries **(improper)** are diffeos that do not vanish at infinity, i.e., $B_\xi \neq 0 \Rightarrow G_\xi \neq 0$. These can change the physical state of the system.

[Benguria, Cordero and Teitelboim, Nucl. Phys. B 122 (1977), 61-99].

Canonical generator of the asymptotic symmetries

• Physically equivalent generators $G[\xi^\alpha]$ and $G[\xi'^\alpha]$ generate gauge transformation that coincide at infinity (G's differ by constraint terms).

• Asymptotic symmetries depend on the asymptotic values of the gauge parameters at infinity:

$$\xi^\alpha(r,y) \xrightarrow[r\to\infty]{} \overset{\circ}{\xi}{}^\alpha(r, y^A, U^s, T^a, \partial_A T^a, \cdots) + \text{“more”}$$

with

U^s: constant parameters (Poincaré transformations)

T^a: functions on S^{d-1}_∞ (supertranslations)

$\overset{\circ}{\xi}{}^\alpha$ could also depend on the asymptotic values of the fields.

Canonical generator of the asymptotic symmetries

- The charge-generator then takes the form

$$G[\xi^\alpha] = \int d^d x \xi^\alpha \mathcal{H}_\alpha + U^s \oint d^{d-1} y \mathcal{Q}_s + \oint d^{d-1} y T^a \mathcal{G}_a$$

We assume that U^s and T^a do not depend on the fields (G has well-defined functional derivatives).

- What if we make redefinitions involving the fields through the charges?... but why?...

Canonical generator of the asymptotic symmetries

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• What if we make redefinitions involving the fields through the charges?... but why?... The asymptotic charges are

$$Q_s = \oint d^{d-1} y \mathcal{Q}_s \qquad Q_a(y) = \mathcal{G}_a(y)$$

and their Poisson brackets

$$\{Q_s, Q_r\} \qquad \{Q_s, Q_a(y)\} \qquad \{Q_a(y), Q_b(y')\}$$

are (in general nonlinear) functions of the charges.

Canonical generator of the asymptotic symmetries

• Nonlinear algebras are indeed the rule rather than a fancy exception! (many examples in the literature –higher spin gravity, conformal gravity, extended supergravity in 3D, etc).

• The nonlinear functions of the charges that occur in the brackets and the redefinitions are still of the form

$$G[\xi^\alpha] = \int d^d x \xi^\alpha \mathcal{H}_\alpha + U^s \oint d^{d-1} y \mathcal{Q}_s + \oint d^{d-1} y T^a \mathcal{G}_a$$

with appropriate gauge parameters and boundary terms, and not by nonlocal expressions such as

$$\left(\int d^d x \xi^\alpha \mathcal{H}_\alpha + B_\xi \right)^2$$

Equations obeyed by the charges

- Let us consider the symplectic form

$$\Omega = \int d^d x \, d_V \pi_\Gamma \wedge d_V \phi^\Gamma$$

$(\phi^\Gamma, \pi_\Gamma)$: canonically conjugate fields.

- The condition $\iota_{X_M} \Omega = -d_V M$ for

$$M = \int d^d x \mathcal{M} + \oint d^{d-1} y \, m$$

implies that (with no boundary term)

$$d_V M = \int d^d x \Big(L_\Gamma d_V \phi^\Gamma + N^\Gamma d_V \pi_\Gamma \Big)$$

with

$$\frac{\delta M}{\delta \phi^\Gamma(x)} = L_\Gamma(x) \qquad \frac{\delta M}{\delta \pi_\Gamma(x)} = N^\Gamma(x)$$

Equivalent to the condition in [Regge, Teitelboim Annals. Phys. 1974]

Equations obeyed by the charges

- Let us now take the variation of the canonical generator $G[\xi^\alpha]$:

$$d_V G[\xi^\alpha] = \int d^dx (d_V \xi^\alpha)\mathcal{H}_\alpha + \int d^dx \xi^\alpha d_V \mathcal{H}_\alpha + U^s \oint d^{d-1}y d_V \mathcal{Q}_s + \oint d^{d-1}y T^a d_V \mathcal{G}_a$$

- The bulk terms can be written as

$$\int d^dx \xi^\alpha d_V \mathcal{H}_\alpha = \int d^dx A^\Gamma_\xi d_V \pi_\Gamma - \int d^dx d_V \phi^\Gamma B_{\xi,\Gamma} + \oint d^{d-1}y \mathcal{V}$$

$$\int d^dx (d_V \xi^\alpha)\mathcal{H}_\alpha = \int d^dx A'^\Gamma_\xi d_V \pi_\Gamma - \int d^dx d_V \phi^\Gamma B'_{\xi,\Gamma} + \oint d^{d-1}y \mathcal{V}'$$

After integration by parts, the above surface integrals become

$$\oint d^{d-1}y \mathcal{V} = U^s \oint d^{d-1}y k_s + \oint d^{d-1}y T^a s_a$$

$$\oint d^{d-1}y \mathcal{V}' = U^s \oint d^{d-1}y k'_s + \oint d^{d-1}y T^a s'_a$$

Equations obeyed by the charges

- $G[\xi^\alpha]$ must have well-defined functional derivatives ($d_V G[\xi^\alpha]$ must reduce to a bulk term). Then

$$U^s \oint d^{d-1}y(d_V \mathcal{Q}_s + k_s + k'_s) + \oint d^{d-1}y T^a (d_V \mathcal{G}_a + s_a + s'_a) = 0$$

which holds by providing a suitable set of boundary conditions!

- Since the $U^s s$ and the $T^a s$ *are arbitrary*, we get the equations to be obeyed by the charge-generators:

$$d_V Q_s + \oint d^{d-1}y(k_s + k'_s) = 0 \qquad d_V \mathcal{G}_a + s_a + s'_a = 0$$

- Given these conditions, what is the $\bar{\xi}^\alpha$ that must be included in

$$G[\bar{\xi}^\alpha] = \int d^d x \bar{\xi}^\alpha \mathcal{H}_\alpha + F[Q_s, Q_a(y)]$$

where $F[Q_s, Q_a(y)]$ is some given functional of the charges?

Nonlinear charges

The answer is a vector that behaves asymptotically as

$$\bar{\xi}^\alpha \quad \xrightarrow[r\to\infty]{} \quad \overset{\circ}{\xi}{}^\alpha(r, y^A, \overline{U}^s, \overline{T}^a, \partial_A \overline{T}^a, \cdots)$$

where

$$\overline{U}^s = \frac{\partial F}{\partial Q_s} \qquad \overline{T}^a(y) = \frac{\delta F}{\delta Q_a(y)}$$

The proof can be found in section 2.3 of [OF, Henneaux JHEP 2309.07600].

• No difficulty in handling non-linear expressions, charges take the standard form! (the asymptotic redefinition determines everything modulo physically irrelevant proper gauge symmetries).

• Integrability is manifest. The corresponding transformations are derived by taking the Poisson bracket.

Nonlinear BMS$_5$ algebra

Algebraic structure of BMS$_5$:

$$\begin{aligned}
[M_a, M_b] &= f^c_{ab} M_c \\
[M_a, T_i] &= R_{ai}{}^j T_j \\
\{M_a, S_\alpha\} &= G_{a\alpha}{}^i T_i + G_{a\alpha}{}^\beta S_\beta + U_{a\alpha\beta\gamma} L^\beta L^\gamma \\
[M_a, L^\alpha] &= -G_{a\beta}{}^\alpha L^\beta \\
[L^\alpha, S_\beta] &= \delta^\alpha_\beta
\end{aligned}$$

Supertranslations generators: S_α (BMS supertranslations)
L^β (subleading supertranslations)

→ The role of the log supertranslations is played by subleading supertranslations!
→ Structure constants are constrained by Jacobi identities.

[OF, Henneaux, Matulich, Troessaert PRL 2111.09664]
[OF, Henneaux, Matulich, Troessaert JHEP 2206.04972]

Nonlinear BMS_5 algebra

The searched-for redefinition of the Lorentz generators:

$$\tilde{M}_a = M_a - G_{a\beta}{}^{i} L^\beta T_i - G_{a\beta}{}^{\gamma} L^\beta S_\gamma - \frac{1}{3} U_{a\beta\gamma\delta} L^\beta L^\gamma L^\delta$$

The asymptotic symmetry algebra then takes the form

$$\begin{aligned}
[\tilde{M}_a, \tilde{M}_b] &= f^c_{ab} \tilde{M}_c \\
[\tilde{M}_a, T_i] &= R_{ai}{}^{j} T_j \\
[\tilde{M}_a, S_\alpha] &= 0 \\
[\tilde{M}_a, L^\alpha] &= 0 \\
[L^\alpha, S_\beta] &= \delta^\alpha_\beta
\end{aligned}$$

It explicitly exhibits the direct sum structure

Poincaré $\oplus$ Supertranslations

The exactly same structure found in the 4D case.

Nonlinear log-BMS$_4$ superalgebra

$$
\begin{aligned}
[M_a, M_b] &= f^c_{ab} M_c \\
[M_a, T_i] &= R^j_{ai} T_j \\
[M_a, S_\alpha] &= G^i_{a\alpha} T_i + G^\beta_{a\alpha} S_\beta \\
[M_a, L^\alpha] &= -G^\alpha_{a\beta} L^\beta \\
[L^\alpha, S_\beta] &= \delta^\alpha_\beta \\
[M_a, Q_I] &= g^J_{aI} Q_J + V^i_{aIB} s^B T_i + V^\alpha_{aIB} s^B S_\alpha \\
[M_a, q_A] &= h^B_{aA} q_B + U^i_{aAB} s^B T_i + U^\alpha_{aAB} s^B S_\alpha \\
[M_a, s^B] &= -h^B_{aC} s^C \\
\{s^A, q_B\} &= \delta^A_B \\
\{Q_I, q_A\} &= d^i_{IA} T_i + d^\alpha_{IA} S_\alpha \\
\{q_A, q_B\} &= d^i_{AB} T_i + d^\alpha_{AB} S_\alpha \\
\{Q_I, Q_J\} &= d^i_{IJ} T_i
\end{aligned}
$$

Jacobi identities (L^α, q_A, q_B) and (L^α, q_A, Q_I):

$$
[L^\alpha, q_A] = n^\alpha_{AB} s^B\,, \quad [L^\alpha, Q_I] = d^\alpha_{IB} s^B\,, \qquad d^\alpha_{AB} = n^\alpha_{AB} + n^\alpha_{BA}
$$

Nonlinear log-BMS$_4$ superalgebra

The algebraic decoupling is achieved by implementing the redefinitions:

$$
\begin{aligned}
\tilde{Q}_I &= Q_I - d^i_{IB} s^B T_i - d^\alpha_{IB} s^B S_\alpha \\
\tilde{q}_A &= q_A - \frac{1}{2} d^i_{AB} s^B T_i - \frac{1}{2} d^\alpha_{AB} s^B S_\alpha \\
\tilde{M}_a &= M_a - G^i_{a\beta} L^\beta T_i - G^\gamma_{a\beta} L^\beta S_\gamma + h^B_{aA} s^A q_B \\
&\quad + \frac{1}{2}\left(U^i_{aAB} - h^C_{aA} d^i_{BC} - n^\beta_{AB} G^i_{a\beta} \right) s^A s^B T_i - h^C_{aA} n^\alpha_{CB} s^A s^B S_\alpha \\
\tilde{L}^\alpha &= L^\alpha + \frac{1}{2} n^\alpha_{[AB]} s^A s^B
\end{aligned}
$$

Nonlinear log-BMS$_4$ superalgebra

The algebra then takes the direct sum structure

super-Poincaré ⊕ Heisenberg superalgebra

- Super-Poincaré:

$$[\tilde{M}_a, \tilde{M}_b] = f^c_{ab} \tilde{M}_c$$
$$[\tilde{M}_a, T_i] = R^j_{ai} T_j$$
$$[\tilde{M}_a, \tilde{Q}_I] = g^J_{aI} \tilde{Q}_J$$
$$\{\tilde{Q}_I, \tilde{Q}_J\} = d^i_{IJ} T_i$$

- Infinite-dimensional Heisenberg superalgebra:

$$[\tilde{L}^\alpha, S_\beta] = \delta^\alpha_\beta$$
$$\{s^A, \tilde{q}_B\} = \delta^A_B$$

No square roots of supertranslations!

Final remarks

• The method that simplifies the BMS algebra in 4D, can be extended to 5D and supersymmetric extensions, whose common feature is the nonlinearity.

• These simplifications were achieved by appropriate nonlinear redefinitions of the generators.

• Nonlinear redefinitions are implemented through the action of field-dependent gauge transformations, which always take to the canonical generators to the form

$$\int d^d x \xi^\alpha \mathcal{H}_\alpha + B_\xi$$

• **Key:** BMS supertranslations and BMS supersymmetries possess canonically conjugate charges.

• (super-)Poincaré generators free from supertranslations and angle-dependent supergauge ambiguities.