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Helical effects in fermionic plasma

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Motivation

- **Helicity current:**
  - Fermion polarization can be described using helicity $\lambda = \pm 1/2$.
  - The helicity operator h commutes with the Hamiltonian.
  - The helicity current $J^\mu_H = \bar{\psi}\gamma^\mu h \psi + \mathrm{h.c.}$ is conserved for free fermions.
- **Helical vortical effect:**
  - Due to spin-orbit coupling leads to polarization under external Ω.
  - Well-known effect: Chiral vortical effect, involving $J^\mu_A \propto T^2, \mu_V^2$.
  - New helical vortical effect: $J^\mu_H \propto \mu_V T$.
- **Helical separation effect:**
  - In external $\mathbf{B}$, the transverse-plane dynamics is quantized $\Rightarrow$ Landau levels.
  - For $|qB| \gg T$, the transverse dynamics is suppressed $\Rightarrow$ **LLL** dominates.
  - LLL selects only one polarization $\Rightarrow$ **anomalous transport**.
  - CME and CSE are well-known and involve J^μ_A and μ_A.
  - **New conserved helicity operator** h in external $\mathbf{B}$.
  - $\langle J^\mu_H \rangle$ reveals leading-order **Helical Separation Effect** (**HSE**).

The $V/A/H$ currents

- Dirac fermions are described by $\mathcal{L} = \frac{i}{2}\bar{\psi}\overleftrightarrow{\not{\partial}}\psi - m\bar{\psi}\psi$.
- $J_V^\mu = \bar{\psi}\gamma^\mu\psi$ is conserved thanks to the $U(1)_V$ symmetry of $\mathcal{L}$.
- The conservation of $J_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ is broken by $m \neq 0$ and axial anomaly:

$$\partial_\mu J_A^\mu = 2im\bar{\psi}\gamma^5\psi + \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}. \tag{1}$$

- A third conserved current is the **helicity current**:

$$J_H^\mu = \bar{\psi}\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi, \quad h = \frac{1}{p}\mathbf{S}\cdot\mathbf{P}, \tag{2}$$

where $\mathbf{S}$ is the spin matrix, satisfying

$$\partial_\mu J_H^\mu = 0 + \text{quantum terms?} \tag{3}$$

- In the non-interacting classical theory, J_H^μ is conserved even for $m \neq 0$!

Quantum theory with $V/A/H$ charges

	Q_V	Q_A	Q_H	$\mathbf{J}_V$	$\mathbf{J}_A$	$\mathbf{J}_H$
C	$-$	$+$	$-$	$-$	$+$	$-$
P	$+$	$-$	$-$	$-$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$

Caption. Behaviour of the $V/A/H$ charges (Q) and currents ($\mathbf{J}$) under the C-, P-, and T-inversions.

- For free massless fermions, the $V/A/H/$ charges are conserved:

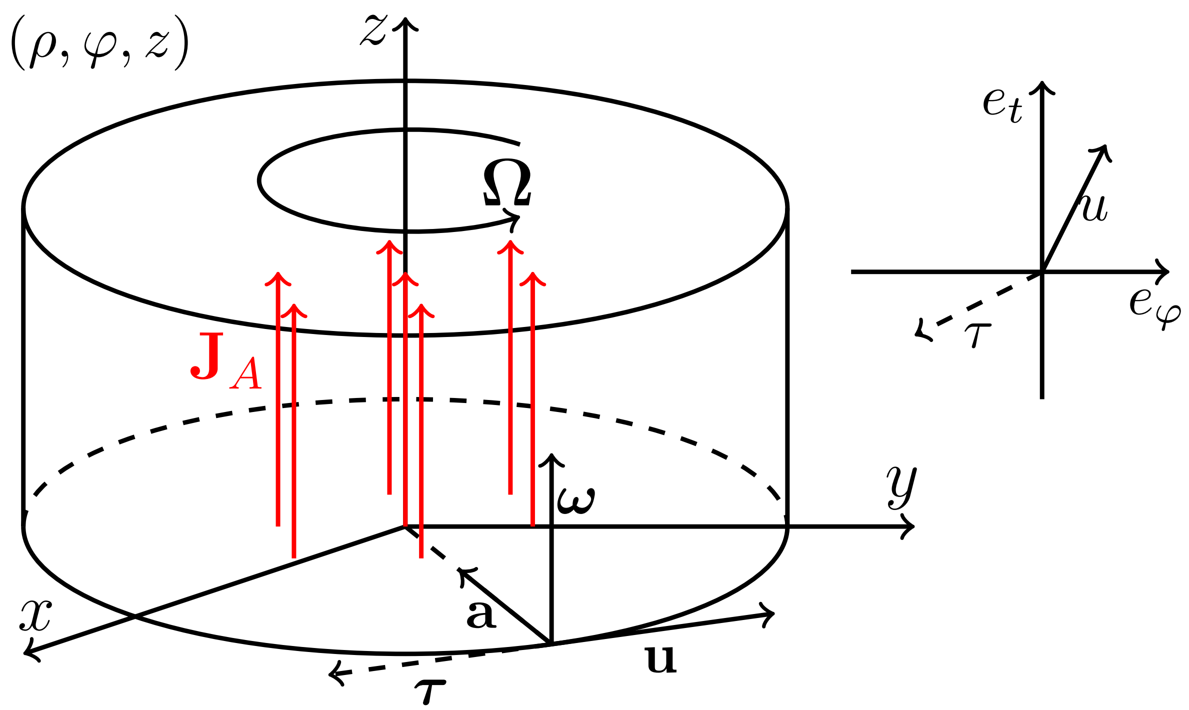
$$Q_V = \int d^3x\, \psi^\dagger \psi, \quad Q_A = \int d^3x\, \psi^\dagger \gamma^5 \psi, \quad Q_H = 2\int d^3x\, \psi^\dagger h \psi.$$

- In the quantum theory, $\hat{\psi} = \sum_j [U_j \hat{b}_j + V_j \hat{d}_j^\dagger]$ and

$$:\widehat{Q}_\ell := \sum_j [q^\ell_{+,\lambda} \hat{b}_j^\dagger \hat{b}_j + q^\ell_{-,\lambda} \hat{d}_j^\dagger \hat{d}_j], \tag{4}$$

with $(q^V_{\sigma,\lambda}, q^A_{\sigma,\lambda}, q^H_{\sigma,\lambda}) = (\sigma, 2\lambda, 2\lambda\sigma)$.

Vortical effects in $V/A/H$ plasmas



- The plasma develops $J^\mu_\ell = Q_\ell u^\mu + \sigma^\tau_\ell \tau^\mu + \sigma^\omega_\ell \omega^\mu$, where
- **Axial** vortical effect: $\sigma^\omega_A = \frac{T^2}{6} + \frac{{\mu_V}^2 + \mu_A^2 + \mu_H^2}{2\pi^2}$.
- **Helical** vortical effects: $\sigma^\omega_V = \frac{2\mu_H T}{\pi^2} \ln 2, \qquad \sigma^\omega_H = \frac{2\mu_V T}{\pi^2} \ln 2$.
- σ^ω_A and σ^ω_H are non-vanishing even when $\mu_A = \mu_H = 0$!

Helicity in a constant magnetic field

- Under minimal coupling, $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$ and

$$\mathcal{L} = \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial\!\!\!/}\psi - q\bar{\psi}A\!\!\!/\psi - m\bar{\psi}\psi \quad \Rightarrow \quad (i\partial\!\!\!/ - qA\!\!\!/ - m)\psi = 0. \tag{5}$$

- We take A^μ in the Coulomb gauge, $A^\mu = (0, \mathbf{A})$, with $\mathbf{A} \equiv \mathbf{A}(\mathbf{x})$, such that

$$\mathbf{E} = \partial_t \mathbf{A} - \nabla A^0 = 0, \quad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}, \quad F_{\mu\nu} = -B(g_{\mu x} g_{\nu y} - g_{\mu y} g_{\nu x}). \tag{6}$$

- In this case, the Dirac Hamiltonian satisfying $H\psi = i\partial_t\psi$ reads

$$H = m\gamma_0 + \gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{\pi}, \qquad \boldsymbol{\pi} = -i\boldsymbol{\nabla} - q\mathbf{A}. \tag{7}$$

- We seek for h such that $[h, H] = 0$. Writing $H = m\gamma_0 + 2\gamma^5 \mathbf{S} \cdot \boldsymbol{\pi}$, we see that

$$h = \frac{\mathbf{S} \cdot \boldsymbol{\pi}}{\sqrt{H^2 - m^2}}. \tag{8}$$

- The divergence of J_H^μ reads $\partial_\mu J_H^\mu = \bar{\psi}\partial\!\!\!/ h\psi + \overline{\partial\!\!\!/ \psi} h\psi + \text{h.c.}$
- Because $[H, h] = 0$, if $H\psi = i\partial_t\psi$ then $Hh\psi = i\partial_t(h\psi)$, such that

$$\partial\!\!\!/ h\psi = -i(qA\!\!\!/ + m)h\psi \quad \Rightarrow \quad \bar{\psi}\partial\!\!\!/ h\psi = -\overline{\partial\!\!\!/ \psi} h\psi \quad \Rightarrow \quad \partial_\mu J_H^\mu = 0. \tag{9}$$

- **J_H^μ is conserved in a background magnetic field, even at finite m!**

Landau levels & Helical separation effect

- Transverse dynamics is quantized into Landau levels n_j,

$$E_j^2 = m^2 + (p_j^z)^2 + 2n_j|qB|. \tag{10}$$

- The lowest Landau level (LLL), $n_j = 0$, exists only when $2\sigma\lambda_j = \text{sgn}(p_j^z)$.
- The t.e.v.s of the charge currents can be put in the form

$$J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^B B^\mu, \quad u^\mu = \delta_0^\mu, \quad B^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta} = B\delta_z^\mu, \tag{11}$$

- $Q_\ell u^\mu$ is the classical transport of the charge density Q_ℓ.
- The anomalous transport component $\sigma_\ell^B B^\mu$ is given only by the LLL:

Chiral magnetic effect: Chiral separation effect: Helical separation effect:

$$\sigma_V^B \simeq \frac{q\mu_A}{2\pi^2} + \frac{q\beta\mu_V\mu_H}{4\pi^2}, \quad \sigma_A^B \simeq \frac{q\mu_V}{2\pi^2} + \frac{q\beta\mu_A\mu_H}{4\pi^2}, \quad \sigma_H^B \simeq \frac{q}{\pi^2\beta}\ln 2 + \frac{q\beta\boldsymbol{\mu}^2}{8\pi^2}. \tag{12}$$

- σ_A^B and σ_H^B are non-vanishing even when $\mu_A = \mu_H = 0$!