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Helical effects in fermionic plasma

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Abstract

A quantum fluid in thermal equilibrium can be described in the grand canonical ensemble using the density operator $\hat{\rho}$. At finite temperature and chemical potential, the expectation values of the energy-momentum tensor and the charge current reveal the well-known thermodynamics of the Fermi-Dirac fluid. When the system is rotating or immersed in a magnetic field, deviations from the Fermi-Dirac thermodynamics can be seen, a particular form of which gives rise to anomalous transport. Anomalous transport was originally uncovered at the level of the axial current: a rotating fluid exhibits a flow of chirality along the rotation vector (the chiral vortical effect). Similarly, Dirac fermions in a magnetic field exhibit the chiral separation effect, by which vector charge imbalance drives a flow of chirality. Conversely, chiral imbalance drives a flow of vector charge (the chiral magnetic effect). In this poster, we address similar effects at the level of the helicity current, describing the flow of helicity (as opposed to chirality) at finite rotation and in the presence of a magnetic field. Because the helicity has opposite charge conjugation parity compared to chirality, these transport laws complement each other. At high temperature and under rotation, the axial conductivity is dominant; while under a magnetic field, the helical conductivity becomes dominant.

[Based on Refs. [1, 2]]

1 The $V/A/H$ currents

- Free Dirac fermions are described by $\mathcal{L} = \frac{i}{2}\bar\psi \overleftrightarrow{\not\partial} \psi - m\bar\psi\psi$ and satisfy $(i\not\partial - m)\psi = 0$.
- Invariance under $U(1)_V$ transf. $\psi \to e^{i\alpha_V}\psi \Rightarrow$ conserved **vector current** $J_V^\mu = \bar\psi\gamma^\mu\psi$.
- For $m = 0$, $\mathcal{L}$ is invariant under the $U(1)_A$ symmetry, $\psi \to e^{i\alpha_A\gamma^5}\psi$. The conservation of the associated **axial current** $J_A^\mu = \bar\psi\gamma^\mu\gamma^5\psi$ is broken by $m \neq 0$ and by quantum effects:

$$\partial_\mu J_A^\mu = 2im\bar\psi\gamma^5\psi + \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}.$$

- A third conserved quantity is the **helicity current** $J_H^\mu = \bar\psi\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi$ [3, 1], where $h = \frac{1}{p}\mathbf{S}\cdot\mathbf{P}$ and $\mathbf{S}$ is the spin matrix, satisfying

$$\partial_\mu J_H^\mu = 0 + \text{quantum terms?}$$

In the non-interacting classical theory, J_H^μ is conserved even for $m \neq 0$!

2 Quantum theory with $V/A/H$ charges

	Q_V	Q_A	Q_H	$\mathbf{J}_V$	$\mathbf{J}_A$	$\mathbf{J}_H$
C	−	+	−	−	+	−
P	+	−	−	−	+	+
T	+	+	+	−	−	−

Caption. Behaviour of the vector (V), axial (A), and helical (H) charges (Q) and currents ($\mathbf{J}$) of a massless Dirac fermion under the C-, P-, and T-inversions. The signs $+/-$ indicate the even/odd nature of these quantities under the corresponding discrete transformations.

- The system of free, massless fermions supports the $V/A/H/$ conserved charges:

$$Q_V = \int d^3x\, \psi^\dagger\psi, \quad Q_A = \int d^3x\, \psi^\dagger\gamma^5\psi, \quad Q_H = 2\int d^3x\, \psi^\dagger h\psi.$$

- Taking $\hat\psi = \sum_j [U_j\hat{b}_j + V_j\hat{d}_j^\dagger]$ with U_j and $V_j = i\gamma^2 U_j^*$ as eigenfunctions of $\{H, h\}$,

$$HU_j = E_jU_j, \quad hU_j = \lambda_jU_j, \qquad HV_j = -E_jV_j, \quad hV_j = \lambda_jV_j,$$

we have : $\widehat{Q}_\ell := \sum_j [q^\ell_{+,\lambda}\hat{b}_j^\dagger\hat{b}_j + q^\ell_{-,\lambda}\hat{d}_j^\dagger\hat{d}_j]$, with charges

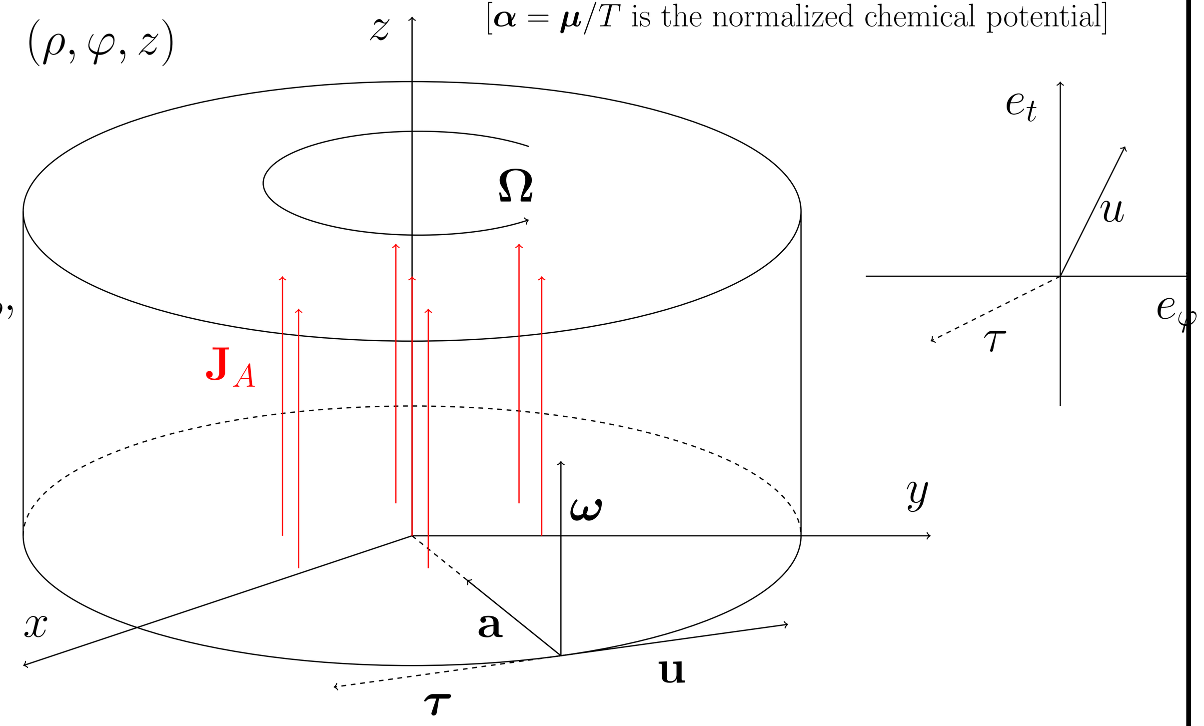
$$q^V_{\sigma,\lambda} = \sigma, \quad q^A_{\sigma,\lambda} = 2\lambda, \quad q^H_{\sigma,\lambda} = 2\lambda\sigma.$$

3 Vortical effects in $V/A/H$ plasmas

Finite temperature QFT under rigid rotation given by $\hat\rho = \exp[-\beta(\widehat{H} - \Omega\widehat{J}^z) + \boldsymbol{\alpha}\cdot\boldsymbol{Q}]$.

[$\boldsymbol{\alpha} = \boldsymbol{\mu}/T$ is the normalized chemical potential]

Velocity :
$$u^\alpha\partial_\alpha = \Gamma(\partial_t + \Omega\partial_\varphi),$$
Acceleration :
$$a^\alpha\partial_\alpha = (\nabla_u u)^\alpha\partial_\alpha = -\rho\Omega^2\Gamma^2\partial_\rho,$$
Vorticity :
$$\omega^\alpha\partial_\alpha = \tfrac{1}{2}\varepsilon^{\alpha\beta\gamma\sigma}u_\beta(\partial_\gamma u_\sigma)\partial_\alpha = \Gamma^2\Omega\partial_z,$$
Fourth vector :
$$\tau^\alpha\partial_\alpha = -\varepsilon^{\alpha\beta\gamma\sigma}\omega_\beta a_\gamma u_\sigma\partial_\alpha = -\Omega^3\Gamma^5(\rho^2\Omega\partial_t + \partial_\varphi).$$



The rotating fermionic plasma develops currents $J^\mu_\ell = Q_\ell u^\mu + \sigma^\tau_\ell\tau^\mu + \sigma^\omega_\ell\omega^\mu$, where the conductivities σ^ω_ℓ are given by [1, 3]:

- **Axial** vortical effect: $$\sigma^\omega_A = \frac{T^2}{6} + \frac{{\mu_V}^2 + \mu_A^2 + \mu_H^2}{2\pi^2}.$$
- **Helical** vortical effects: $$\sigma^\omega_V = -\frac{T^2}{2\pi^2}\sum_{\sigma,\lambda} q^H_{\sigma,\lambda}\mathrm{Li}_2(-e^{\mathbf{q}_{\sigma,\lambda}\cdot\boldsymbol{\alpha}_{\sigma,\lambda}}) \simeq \frac{2\mu_H T}{\pi^2}\ln 2,$$ $$\sigma^\omega_H = -\frac{T^2}{2\pi^2}\sum_{\sigma,\lambda} q^V_{\sigma,\lambda}\mathrm{Li}_2(-e^{\mathbf{q}_{\sigma,\lambda}\cdot\boldsymbol{\alpha}_{\sigma,\lambda}}) \simeq \frac{2\mu_V T}{\pi^2}\ln 2.$$
- σ^ω_A and σ^ω_H are non-vanishing even when $\mu_A = \mu_H = 0$!

4 Helicity in a constant magnetic field

- Under minimal coupling, $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$ and

$$\mathcal{L} = \frac{i}{2}\bar\psi\overleftrightarrow{\not\partial}\psi - q\bar\psi\not{A}\psi - m\bar\psi\psi \quad \Rightarrow \quad (i\not\partial - q\not{A} - m)\psi = 0. \tag{1}$$

- We take A^μ in the Coulomb gauge, $A^\mu = (0, \mathbf{A})$, with $\mathbf{A} \equiv \mathbf{A}(\mathbf{x})$, such that

$$\mathbf{E} = \partial_t\mathbf{A} - \nabla A^0 = 0, \quad \mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}, \quad F_{\mu\nu} = -B(g_{\mu x}g_{\nu y} - g_{\mu y}g_{\nu x}). \tag{2}$$

- In this case, the Dirac Hamiltonian satisfying $H\psi = i\partial_t\psi$ reads

$$H = m\gamma_0 + \gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\pi}, \qquad \boldsymbol{\pi} = -i\boldsymbol{\nabla} - q\mathbf{A}. \tag{3}$$

- We seek for h such that $[h, H] = 0$. Writing $H = m\gamma_0 + 2\gamma^5\mathbf{S}\cdot\boldsymbol{\pi}$, we see that

$$h = \frac{\mathbf{S}\cdot\boldsymbol{\pi}}{\sqrt{H^2 - m^2}}. \tag{4}$$

- The divergence of J^μ_H reads $\partial_\mu J^\mu_H = \bar\psi\not\partial h\psi + \overline{\not\partial\psi}h\psi + \text{h.c.}$
- Because $[H, h] = 0$, if $H\psi = i\partial_t\psi$ then $Hh\psi = i\partial_t(h\psi)$, such that

$$\not\partial h\psi = -i(q\not{A} + m)h\psi \quad \Rightarrow \quad \bar\psi\not\partial h\psi = -\overline{\not\partial\psi}h\psi \quad \Rightarrow \quad \partial_\mu J^\mu_H = 0. \tag{5}$$

- **J^μ_H is conserved in a background magnetic field, even at finite m!**

5 Landau levels

- The Dirac eq. can be solved with respect to eigenmodes U_j of H, P^y, P^z and h:

$$HU_j = E_jU_j, \quad P^yU_j = p^y_jU_j, \quad P^zU_j = p^z_jU_j, \quad hU_j = \lambda_jU_j. \tag{6}$$

- Transverse dynamics is quantized into Landau levels n_j,

$$E_j^2 = m^2 + (p^z_j)^2 + 2n_j|qB|. \tag{7}$$

- The lowest Landau level (LLL), $n_j = 0$, exists only when $2\sigma\lambda_j = \mathrm{sgn}(p^z_j)$.

6 Helical separation effect

- The t.e.v.s of the charge currents can be put in the form

$$J^\mu_\ell = Q_\ell u^\mu + \sigma^B_\ell B^\mu, \quad u^\mu = \delta^\mu_0, \quad B^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta} = B\delta^\mu_z, \tag{8}$$

- $Q_\ell u^\mu$ is the classical transport of the charge density Q_ℓ.
- The anomalous transport component $\sigma^B_\ell B^\mu$ is given only by the LLL:

Chiral magnetic effect: Chiral separation effect: Helical separation effect:

$$\sigma^B_V \simeq \frac{q\mu_A}{2\pi^2} + \frac{q\beta\mu_V\mu_H}{4\pi^2}, \quad \sigma^B_A \simeq \frac{q\mu_V}{2\pi^2} + \frac{q\beta\mu_A\mu_H}{4\pi^2}, \quad \sigma^B_H \simeq \frac{q}{\pi^2\beta}\ln 2 + \frac{q\beta\boldsymbol{\mu}^2}{8\pi^2}. \tag{9}$$

- σ^B_A and σ^B_H are non-vanishing even when $\mu_A = \mu_H = 0$!

References

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