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Helicity relaxation time in an interacting fermionic plasma

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Motivation

- Helicity $\equiv$ projection of spin along the direction of motion.
- h commutes with $H \Rightarrow$ helicity is a good" quantum number (even when $m \neq 0$).
- Helicity current J_H^μ is conserved for free fermions.
- In the case of massless fermions, J_H^μ transforms covariantly under Lorentz transformations and Q_H is well-defined.
- Due to the helicity-violating pair annihilation (HVPA) processes, the helical imbalance dissipates in time.
- Aim: calculate the relaxation time τ_H due to HVPA processes.

Helicity current

- Helicity $h = \mathbf{S} \cdot \mathbf{P}/p \equiv$ the projection of spin on the direction of movement.
- Working with momentum particle $[U_{p,\lambda}(x)]$ and anti-particle $[V_{p,\lambda}(x) = i\gamma^2 U^*_{p,\lambda}(x)]$, we have

$$\begin{pmatrix} 1 \\ \mathbf{P} \\ h \end{pmatrix} \hat{\psi}(x) = \sum_{\lambda = \pm\frac{1}{2}} \int d^3p \left[\begin{pmatrix} 1 \\ \mathbf{p} \\ \lambda \end{pmatrix} U_{\lambda,p}(x) \hat{b}_\lambda(p) + \begin{pmatrix} 1 \\ -\mathbf{p} \\ \lambda \end{pmatrix} V_{\lambda,p}(x) \hat{d}^\dagger_\lambda(p) \right]. \tag{1}$$

- For massless particles, $H = i\partial_t = -i\gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} = 2p\gamma^5 h$ and $U_{p,\lambda}$ ($V_{p,\lambda}$) become simultaneous eigenvectors of H, h and γ^5:

$$\begin{pmatrix} H \\ h \\ \gamma^5 \end{pmatrix} U_{p,\lambda} = \begin{pmatrix} E_\mathbf{p} \\ \lambda \\ 2\lambda \end{pmatrix}, \qquad \begin{pmatrix} H \\ h \\ \gamma^5 \end{pmatrix} V_{p,\lambda} = \begin{pmatrix} -E_\mathbf{p} \\ \lambda \\ -2\lambda \end{pmatrix}. \tag{2}$$

- The helicity current $J^\mu_H = \bar{\psi}\gamma^\mu h \psi + (\overline{h\psi})\gamma^\mu \psi$ is conserved for on-shell fermions, $\partial_\mu J^\mu_H = 0$, allowing the conserved helical charge to be introduced:

$$Q_H = \int d^3x J^0_H \quad \to \quad : \widehat{Q}_H := \sum_{\lambda=\pm 1/2} 2\lambda \int d^3p [\hat{b}^\dagger_\lambda(p) \hat{b}_\lambda(p) - \hat{d}^\dagger_\lambda(p) \hat{d}_\lambda(p)].$$

Lorentz covariance of the helicity current

- The helicity of a massive Dirac particle depends on reference frame.
- In contrast, the helicity of a massless particle is Lorentz-invariant.
- **Question:** How does J_H^μ transform under Lorentz transformation?
- Consider the Lorentz transformation Λ:

$$\psi(x) \to \psi^\Lambda(x) = D(\Lambda)\psi(\Lambda^{-1}x), \quad D(\Lambda) = e^{-iS_{\alpha\beta}\omega^{\alpha\beta}}, \quad S^{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta].$$

- For $U_{p,\lambda}(x)$, one can show that

$$U_{p,\lambda}(x) \to U^\Lambda_{p,\lambda}(x) = D(\Lambda)U_{p,\lambda}(\Lambda^{-1}x) = \sqrt{\frac{(\Lambda p)^0}{p^0}} e^{i\lambda\theta(\Lambda,p)} U_{\Lambda p,\lambda}(x),$$

  where θ is a real number related to $W(\Lambda, p) \equiv L^{-1}(\Lambda p)\Lambda L(p)$.
- Consider now $J_H^{\Lambda;\mu}(x) = \overline{\psi^\Lambda(x)}\gamma^\mu h_x \psi^\Lambda(x) +$ h.c.. Using $\overline{D(\Lambda)}\gamma^\mu D(\Lambda) = \Lambda^\mu{}_\nu\gamma^\nu$:

$$J_H^{\Lambda;\mu}(x) = \Lambda^\mu{}_\nu \bar{\psi}(\Lambda^{-1}x)\gamma^\nu \overline{D(\Lambda)} h_x D(\Lambda)\psi(\Lambda^{-1}x) + \text{h.c.}, \tag{3}$$

Lorentz covariance of the helicity current

- $\overline{D(\Lambda)} h_x D(\Lambda) \psi(\Lambda^{-1}x)$ can be evaluated at the level of the expansion (1), by considering:

$$\overline{D(\Lambda)} h_x D(\Lambda) U_{p,\lambda}(\Lambda^{-1}x) = \lambda \sqrt{\frac{(\Lambda p)^0}{p^0}} e^{i\lambda\theta(\Lambda,p)} \overline{D(\Lambda)} U_{\Lambda p,\lambda}(x), \tag{4}$$

where we used $h_x U_{\Lambda p,\lambda}(x) = U_{\Lambda p,\lambda}(x)\lambda$. Noting that $\overline{D(\Lambda)} = D(\Lambda^{-1})$, Eq. (**??**) implies

$$\overline{D(\Lambda)} U_{\Lambda p,\lambda}(x) = \sqrt{\frac{p^0}{(\Lambda p)^0}} e^{i\lambda\theta(\Lambda^{-1},\Lambda p)} U_{p,\lambda}(\Lambda^{-1}x). \tag{5}$$

- It can be shown that $\theta(\Lambda^{-1}, \Lambda p) = -\theta(\Lambda, p)$, and thus $\overline{D(\Lambda)} h_x D(\Lambda) U_{p,\lambda}(\Lambda^{-1}x) = U_{p,\lambda}(\Lambda^{-1}x)\lambda$. Noting that a similar relation holds also for $V_{p,\lambda}(\Lambda^{-1}x)$, we find $\overline{D(\Lambda)} h_x D(\Lambda)\psi(\Lambda^{-1}x) = h_{\Lambda^{-1}x}\psi(\Lambda^{-1}x)$.
- We thus conclude that $J^\mu_H(x)$ transforms covariantly:

$$J^\mu_H(x) \to J^{\Lambda,\mu}_H(x) = \Lambda^\mu{}_\nu J^\nu_H(\Lambda^{-1}x). \tag{6}$$

Kinetic model: Helicity imbalance

- A fluid of spin $1/2$ (anti-)particles can be described using four distribution functions,

$$f^{\text{eq};\sigma}_{\mathbf{p},\lambda} = \left[\exp\left(\frac{p\cdot u - \mathbf{q}_{\sigma,\lambda}\cdot\boldsymbol{\mu}}{T}\right) + 1\right]^{-1}, \tag{7}$$

- $\boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H) \equiv$ charge imbalance and $\mathbf{q}_{\sigma,\lambda} = (\sigma, 2\lambda, 2\sigma\lambda)$.
- We impose $p^\mu \partial_\mu f_{\mathbf{p},\lambda} = C[f]$ and assume spatial homogeneity.
- Considering $2 \to 2$ vector interactions, $C[f]$ conserves Q_V and Q_A.
- Q_H broken by helicity-violating pair annihilation (HVPA) processes of the form $e^+_R e^-_L \to e^+_L e^-_R$, with cross section:

$$\frac{d\sigma}{d\Omega}(q^i_R \bar{q}^j_L \to q^{i'}_L \bar{q}^{j'}_R) = \frac{\alpha^2_{QCD}}{4E^2_{\text{cm}}}(1-\cos\theta_{cm})^2 \sum_{a,b} t^a_{ji} t^a_{j'i'} t^b_{ij} t^b_{i'j'},$$

where $\text{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$.

Kinetic model: Helicity imbalance

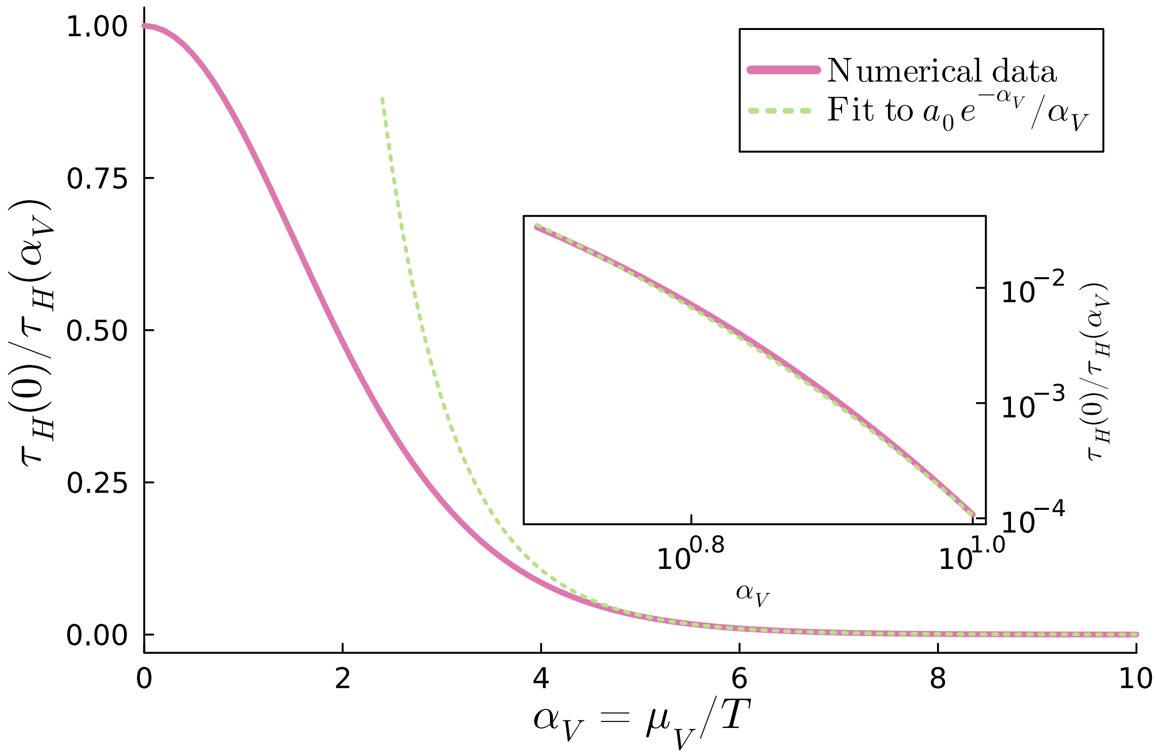
- The charge currents can be computed via ($dP = d^3p/[(2\pi)^3 E_{\mathbf{p}}]$; $g \equiv$ no. of dofs)

$$J^\mu_\ell = g \sum_{\lambda,\sigma} q^\ell_{\sigma,\lambda} \int dP\, p^\mu f^{\text{eq};\sigma}_{\mathbf{p},\lambda} \quad \Rightarrow \quad \frac{dQ_H}{dt} = g \sum_{\lambda,\sigma} 2\lambda\sigma \int dP\, C_{\text{HVPA}}[f],$$

$$C_{\text{HVPA}}[f] = \int dK dP' dK' \delta^4(p + k - p' - k') s (2\pi)^6$$
$$\times [f^\sigma_{\mathbf{p}',-\lambda} f^{-\sigma}_{\mathbf{k}',\lambda} \tilde{f}^\sigma_{\mathbf{p},\lambda} \tilde{f}^{-\sigma}_{\mathbf{k},-\lambda} - f^\sigma_{\mathbf{p},\lambda} f^{-\sigma}_{\mathbf{k},-\lambda} \tilde{f}^\sigma_{\mathbf{p}',-\lambda} \tilde{f}^{-\sigma}_{\mathbf{k}',\lambda}]$$
$$\times N_f \sum_{i',j,j'} \frac{d\sigma}{d\Omega} (q^{\sigma,i}_{\mathbf{p},\lambda} q^{-\sigma,j}_{\mathbf{k},-\lambda} \to q^{\sigma,i'}_{\mathbf{p}',-\lambda} q^{-\sigma,j'}_{\mathbf{k}',\lambda}). \quad (8)$$

- We seek to compute the relaxation time $\tau_H \equiv \tau_H(T, \mu_V)$, defined by $dQ_H/dt \simeq -Q_H/\tau_H$, for small μ_H (but arbitrary μ_V), when $f^{\text{eq};\sigma}_{\mathbf{p},\lambda} \simeq f^\sigma_{0\mathbf{p}} + 2\lambda\sigma\beta\mu_H f^\sigma_{0\mathbf{p}} \tilde{f}^\sigma_{0\mathbf{p}}$ and $[f^\sigma_{0\mathbf{p}} = [e^{\beta E_{\mathbf{p}} - \sigma\alpha_V} + 1]^{-1}$, $\tilde{f}^\sigma_{0\mathbf{p}} = 1 - f^\sigma_{0\mathbf{p}}]$

$$\tau_H^{-1} = \frac{8}{3}(2\pi)^6 g \alpha^2_{\text{QCD}} \beta^3 \int dP dK dP' dK' (1 - \cos\theta_{\text{cm}})^2 \delta^4(p + k - p' - k')$$
$$\times \frac{1}{4} \sum_{\sigma=\pm 1} f^\sigma_{0\mathbf{p}} f^{-\sigma}_{0\mathbf{k}} \tilde{f}^\sigma_{0\mathbf{p}'} \tilde{f}^{-\sigma}_{0\mathbf{k}'} (\tilde{f}^\sigma_{0\mathbf{p}} + \tilde{f}^{-\sigma}_{0\mathbf{k}} + f^\sigma_{0\mathbf{p}'} + f^{-\sigma}_{0\mathbf{k}'}). \quad (9)$$



- Eventually, $\tau_H = 6\pi^3\beta/(g\alpha_{\rm QCD}^2\mathcal{I})$, with $\mathcal{I} \equiv \mathcal{I}(T,\mu)$.
- When $\mu_V = 0$, we find

$$\tau_H = 0.196 \times \frac{6\pi^3\beta}{g\alpha_{\rm QCD}^2} \simeq \left(\frac{250\ {\rm MeV}}{k_B T}\right)\left(\frac{1}{\alpha_{\rm QCD}}\right)^2\left(\frac{2}{N_f}\right) \times 4.804\ {\rm fm}/c. \quad (10)$$