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Helicity relaxation time in an interacting fermionic plasma

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Abstract

The polarization of free Dirac fermions can be described by helicity, which represents the projection of the spin along the direction of motion. The helicity operator commutes with the Hamiltonian and therefore helicity is a good" quantum number, even in the case of massive fermions. This opens the possibility of defining a helicity current", J^μ_H, which is conserved for free fermions. In the case of massless fermions, J^μ_H transforms covariantly under Lorentz transformations. Integrating its zeroth component over the spatial volume gives the helicity charge, Q_H.

Consider now an ensemble of interacting fermions with a slight helicity imbalance. Due to the helicity-violating pair annihilation (HVPA) processes, the helical imbalance will dissipate in time. This poster addresses the calculation of the typical timescale of the helicity relaxation time in the high-temperature, deconfined phase of the quark-gluon plasma, by employing the Boltzmann collision integral for the HVPA processes.

[Based on Refs. [1, 2]]

1 Helicity current

- Helicity $h = \mathbf{S} \cdot \mathbf{P}/p$ represents the projection of spin onto the direction of movement.
- Working with momentum particle $[U_{p,\lambda}(x)]$ and anti-particle $[V_{p,\lambda}(x) = i\gamma^2 U^*_{p,\lambda}(x)]$, we have

$$\begin{pmatrix}1\\ \mathbf{P}\\ h\end{pmatrix}\hat{\psi}(x) = \sum_{\lambda=\pm\frac{1}{2}} \int d^3p \left[\begin{pmatrix}1\\ \mathbf{p}\\ \lambda\end{pmatrix} U_{\lambda,p}(x)\hat{b}_\lambda(p) + \begin{pmatrix}1\\ -\mathbf{p}\\ \lambda\end{pmatrix} V_{\lambda,p}(x)\hat{d}^\dagger_\lambda(p)\right]. \quad (1)$$

- For massless particles, $H = i\partial_t = -i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} = 2p\gamma^5 h$ and $U_{p,\lambda}$ $(V_{p,\lambda})$ become simultaneous eigenvectors of H, h and γ^5:

$$\begin{pmatrix}H\\ h\\ \gamma^5\end{pmatrix} U_{p,\lambda} = \begin{pmatrix}E_{\mathbf{p}}\\ \lambda\\ 2\lambda\end{pmatrix}, \qquad \begin{pmatrix}H\\ h\\ \gamma^5\end{pmatrix} V_{p,\lambda} = \begin{pmatrix}-E_{\mathbf{p}}\\ \lambda\\ -2\lambda\end{pmatrix}. \quad (2)$$

- The helicity current $J^\mu_H = \bar{\psi}\gamma^\mu h\psi + (\overline{h\psi})\gamma^\mu\psi$ is conserved for on-shell fermions, $\partial_\mu J^\mu_H = 0$, allowing the conserved helical charge to be introduced:

$$Q_H = \int d^3x J^0_H \quad \to \quad : \widehat{Q}_H := \sum_{\lambda=\pm 1/2} 2\lambda \int d^3p [\hat{b}^\dagger_\lambda(p)\hat{b}_\lambda(p) - \hat{d}^\dagger_\lambda(p)\hat{d}_\lambda(p)]. \quad (3)$$

2 Lorentz covariance of the helicity current

- The helicity of a massive Dirac particle depends on reference frame.
- In contrast, the helicity of a massless particle is Lorentz-invariant.
- **Question:** How does J^μ_H transform under Lorentz transformation?
- Consider the Lorentz transformation Λ:

$$\psi(x) \to \psi^\Lambda(x) = D(\Lambda)\psi(\Lambda^{-1}x), \qquad D(\Lambda) = e^{-iS_{\alpha\beta}\omega^{\alpha\beta}}, \qquad S^{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta]. \quad (4)$$

- For $U_{p,\lambda}(x)$, one can show that

$$U_{p,\lambda}(x) \to U^\Lambda_{p,\lambda}(x) = D(\Lambda)U_{p,\lambda}(\Lambda^{-1}x) = \sqrt{\frac{(\Lambda p)^0}{p^0}} e^{i\lambda\theta(\Lambda,p)} U_{\Lambda p,\lambda}(x), \quad (5)$$

where θ is a real number defined by $\quad [\zeta = \frac{1}{2}(\alpha^2 + \beta^2)]$

$$W(\Lambda,p) \equiv L^{-1}(\Lambda p)\Lambda L(p) \equiv S[\alpha(\Lambda,p), \beta(\Lambda,p)]R_3[\theta(\Lambda,p)],$$

$$[S(\alpha,\beta)]^\mu{}_\nu = \begin{pmatrix}1+\zeta & \alpha & \beta & -\zeta\\ \alpha & 1 & 0 & -\alpha\\ \beta & 0 & 1 & -\beta\\ \zeta & \alpha & \beta & 1-\zeta\end{pmatrix}, \qquad [R_3(\theta)]^\mu{}_\nu = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{pmatrix}. \quad (6)$$

- Consider now $J^{\Lambda;\mu}_H(x) = \overline{\psi^\Lambda(x)}\gamma^\mu h_x\psi^\Lambda(x) + (\overline{h_x\psi^\Lambda(x)})\gamma^\mu\psi^\Lambda(x)$, which reads

$$J^{\Lambda;\mu}_H(x) = \Lambda^\mu{}_\nu\bar{\psi}(\Lambda^{-1}x)\gamma^\nu\overline{D(\Lambda)}h_x D(\Lambda)\psi(\Lambda^{-1}x) + \text{h.c.}, \quad (7)$$

where we used $\overline{D(\Lambda)}\gamma^\mu D(\Lambda) = \Lambda^\mu{}_\nu\gamma^\nu$.
- $\overline{D(\Lambda)}h_x D(\Lambda)\psi(\Lambda^{-1}x)$ can be evaluated at the level of the expansion (1), by considering:

$$\overline{D(\Lambda)}h_x D(\Lambda)U_{p,\lambda}(\Lambda^{-1}x) = \lambda\sqrt{\frac{(\Lambda p)^0}{p^0}} e^{i\lambda\theta(\Lambda,p)}\overline{D(\Lambda)}U_{\Lambda p,\lambda}(x), \quad (8)$$

where we used $h_x U_{\Lambda p,\lambda}(x) = U_{\Lambda p,\lambda}(x)\lambda$. Noting that $\overline{D(\Lambda)} = D(\Lambda^{-1})$, Eq. (5) implies

$$\overline{D(\Lambda)}U_{\Lambda p,\lambda}(x) = \sqrt{\frac{p^0}{(\Lambda p)^0}} e^{i\lambda\theta(\Lambda^{-1},\Lambda p)}U_{p,\lambda}(\Lambda^{-1}x). \quad (9)$$

- It can be shown that $\theta(\Lambda^{-1}, \Lambda p) = -\theta(\Lambda, p)$, and thus $\overline{D(\Lambda)}h_x D(\Lambda)U_{p,\lambda}(\Lambda^{-1}x) = U_{p,\lambda}(\Lambda^{-1}x)\lambda$. Noting that a similar relation holds also for $V_{p,\lambda}(\Lambda^{-1}x)$, we find $\overline{D(\Lambda)}h_x D(\Lambda)\psi(\Lambda^{-1}x) = h_{\Lambda^{-1}x}\psi(\Lambda^{-1}x)$.
- We thus conclude that $J^\mu_H(x)$ transforms covariantly:

$$J^\mu_H(x) \to J^{\Lambda,\mu}_H(x) = \Lambda^\mu{}_\nu J^\nu_H(\Lambda^{-1}x). \quad (10)$$

3 Kinetic model: Helicity imbalance

- A fluid of spin 1/2 (anti-)particles can be described using four distribution functions,

$$f^{\text{eq};\sigma}_{\mathbf{p},\lambda} = \left[\exp\left(\frac{p\cdot u - \mathbf{q}_{\sigma,\lambda}\cdot\boldsymbol{\mu}}{T}\right) + 1\right]^{-1}, \quad (11)$$

where $p^\mu \equiv$ four-momentum ($p^0 = |\mathbf{p}|$), $\sigma = \pm 1$ for particles/anti-particles, $\lambda = \pm 1/2 \equiv$ polarization and u^μ is the local velocity of the fluid.
- $\boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H) \equiv$ charge imbalance and $\mathbf{q}_{\sigma,\lambda} = (\sigma, 2\lambda, 2\sigma\lambda)$ correspond to : $\widehat{Q}_{V/A/H}$:.
- We impose the Boltzmann equation, $p^\mu\partial_\mu f_{\mathbf{p},\lambda} = C[f]$ and assume spatial homogeneity.
- Considering $2 \to 2$ vector interactions, $C[f]$ conserves Q_V and Q_A. Q_H broken by helicity-violating pair annihilation (HVPA) processes of the form $e^+_R e^-_L \to e^+_L e^-_R$, with cross section:

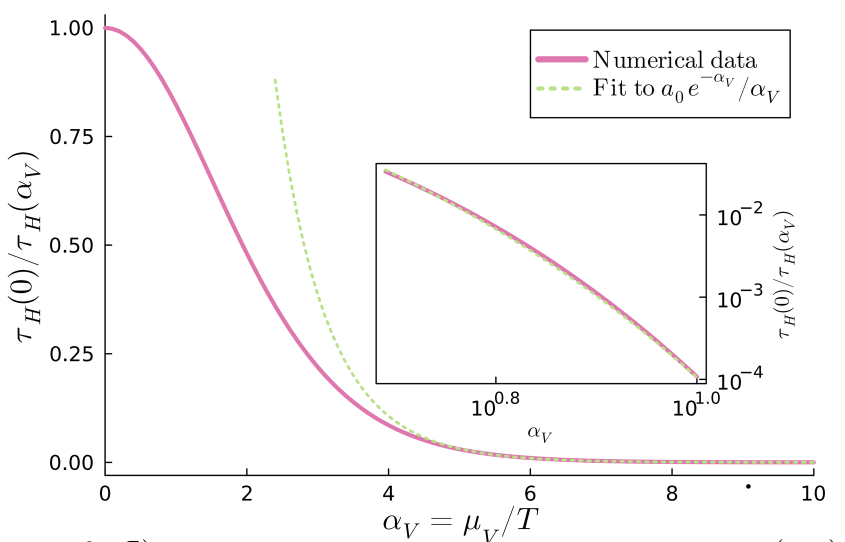
$$\frac{d\sigma}{d\Omega}(q^i_R\bar{q}^j_L \to q^{i'}_L\bar{q}^{j'}_R) = \frac{\alpha^2_{QCD}}{4E^2_{\text{cm}}}(1 - \cos\theta_{cm})^2\sum_{a,b} t^a_{ji}t^a_{j'i'}t^b_{ij}t^b_{i'j'}, \qquad \text{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}. \quad (12)$$

- The charge currents can be computed via $\quad (dP = d^3p/[(2\pi)^3 E_{\mathbf{p}}];\ g \equiv$ no. of dofs$)$

$$J^\mu_\ell = g\sum_{\lambda,\sigma} q^\ell_{\sigma,\lambda}\int dP\, p^\mu f^{\text{eq};\sigma}_{\mathbf{p},\lambda} \qquad \Rightarrow \qquad \frac{dQ_H}{dt} = g\sum_{\lambda,\sigma} 2\lambda\sigma\int dP\, C_{\text{HVPA}}[f],$$

$$C_{\text{HVPA}}[f] = \int dK dP' dK' \delta^4(p + k - p' - k')s(2\pi)^6$$
$$\times [f^\sigma_{\mathbf{p}',-\lambda}f^{-\sigma}_{\mathbf{k}',\lambda}\tilde{f}^\sigma_{\mathbf{p},\lambda}\tilde{f}^{-\sigma}_{\mathbf{k},-\lambda} - f^\sigma_{\mathbf{p},\lambda}f^{-\sigma}_{\mathbf{k},-\lambda}\tilde{f}^\sigma_{\mathbf{p}',-\lambda}\tilde{f}^{-\sigma}_{\mathbf{k}',\lambda}]N_f\sum_{i',j,j'}\frac{d\sigma}{d\Omega}(q^{\sigma,i}_{\mathbf{p},\lambda}q^{-\sigma,j}_{\mathbf{k},-\lambda} \to q^{\sigma,i'}_{\mathbf{p}',-\lambda}q^{-\sigma,j'}_{\mathbf{k}',\lambda}). \quad (13)$$

- We seek to compute the relaxation time $\tau_H \equiv \tau_H(T, \mu_V)$, defined by $dQ_H/dt \simeq -Q_H/\tau_H$, for small μ_H (but arbitrary μ_V), when $f^{\text{eq};\sigma}_{\mathbf{p},\lambda} \simeq f^\sigma_{0\mathbf{p}} + 2\lambda\sigma\beta\mu_H f^\sigma_{0\mathbf{p}}\tilde{f}^\sigma_{0\mathbf{p}}$ and $\quad [f^\sigma_{0\mathbf{p}} = [e^{\beta E_{\mathbf{p}} - \sigma\alpha_V} + 1]^{-1},\ \tilde{f}^\sigma_{0\mathbf{p}} = 1 - f^\sigma_{0\mathbf{p}}]$



$$\tau_H^{-1} = \frac{8}{3}(2\pi)^6 g\alpha^2_{\text{QCD}}\beta^3\int dP dK dP' dK'$$
$$\times (1 - \cos\theta_{\text{cm}})^2\delta^4(p + k - p' - k')$$
$$\times \frac{1}{4}\sum_{\sigma=\pm 1} f^\sigma_{0\mathbf{p}}f^{-\sigma}_{0\mathbf{k}}\tilde{f}^\sigma_{0\mathbf{p}'}\tilde{f}^{-\sigma}_{0\mathbf{k}'}(\tilde{f}^\sigma_{0\mathbf{p}} + \tilde{f}^{-\sigma}_{0\mathbf{k}} + f^\sigma_{0\mathbf{p}'} + f^{-\sigma}_{0\mathbf{k}'}). \quad (14)$$

- Eventually, $\tau_H = 6\pi^3\beta/(g\alpha^2_{\text{QCD}}\mathcal{I})$, with [2]

$$\mathcal{I} = \int_0^\infty dz\, z^3\int_{-1}^1 dx\int_{-1}^1 d\delta\int_{|\delta|}^1 d\xi\left[\frac{3 - x^2}{2}\xi - 2x\delta + \frac{3x^2 - 1}{2\xi}\delta^2\right]$$
$$\times \frac{1}{2}\sum_{\sigma=\pm 1}[e^{\frac{z}{2}(1+\delta) - \sigma\alpha_V} + 1]^{-1}[e^{\frac{z}{2}(1-\delta) + \sigma\alpha_V} + 1]^{-1}[e^{-\frac{z}{2}(1+x\xi) + \sigma\alpha_V} + 1]^{-1}[e^{-\frac{z}{2}(1-x\xi) - \sigma\alpha_V} + 1]^{-1}. \quad (15)$$

- When $\mu_V = 0$, we find

$$\tau_H = 0.196 \times \frac{6\pi^3\beta}{g\alpha^2_{\text{QCD}}} \simeq \left(\frac{250\ \text{MeV}}{k_B T}\right)\left(\frac{1}{\alpha_{\text{QCD}}}\right)^2\left(\frac{2}{N_f}\right) \times 4.804\ \text{fm}/c. \quad (16)$$

References

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[2] S. Morales-Tejera, V. E. Ambruș, M. N. Chernodub, arXiv:2403.19756.

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