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Acceleration as a circular motion along an imaginary circle

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Motivation

Accelerating states:

- Unruh effect [W. G. Unruh, PRD **14** (1976) 870]
- Global thermodynamic equilibrium state [F. Becattini, PRD **97** (2018) 085013]
- Black hole evaporation
- Eary universe

Why investigate?

- In rigid rotation, $\langle T^{\mu\nu} \rangle$ receives quantum corrections from $\boldsymbol{\omega}$ and $\boldsymbol{a}$.
- What is the effect of acceleration on strongly-interacting matter EoS?
- Lattice studies require imaginary time and suitable boundary conditions $\Rightarrow$ what is the KMS relation giving these bcs?

Global equilibrium with constant acceleration a

- In global equilibrium, the temperature four-vector $\beta^\mu = u^\mu/T$ satisfies the Killing equation:
$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0. \tag{1}$$
- For constant acceleration,
$$\beta^\mu \partial_\mu = \beta_T[(1+az)\partial_t + at\partial_z], \tag{2}$$
where $\beta_T = 1/T$ represents the (constant) inverse temperature,
- Imposing $u^2 = 1$, we have
$$T(x) = \frac{1}{\beta_T}[(1+az)^2 - (at)^2]^{-1/2}, \tag{3}$$
$$u^\mu \partial_\mu = \beta_T T(x)\left[(1+az)\partial_t + at\partial_z\right], \tag{4}$$
$$a^\mu \partial_\mu = a\beta_T^2 T^2(x)[at\partial_t + (1+az)\partial_z]\,. \tag{5}$$
- The state diverges at the Rindler horizon:
$$(1+az)^2 - (at)^2 = 0, \qquad z \geqslant -\frac{1}{a}\,. \tag{6}$$
- The proper thermal acceleration $\alpha^\mu = a^\mu/T(x)$ has constant magnitude: $\alpha = \sqrt{-\alpha^\mu \alpha_\mu} = a\beta_T$.

Density operator for global equilibrium

- The density operator reads

$$\hat{\rho} = \exp\left[-\int d\Sigma_\mu T^{\mu\nu}\beta_\nu\right] = e^{-b\cdot\hat{P}+\varpi:\hat{J}/2}, \tag{7}$$

  where $b^\mu = b^\mu = \beta_T \delta^\mu_0$ and $\varpi^\mu{}_\nu = \alpha(\delta^\mu_3 g_{0\nu} - \delta^\mu_0 g_{3\nu})$.
- Taking into account the property $\mathsf{T}(a)\Lambda\mathsf{T}(a)^{-1} = \mathsf{T}[(\mathsf{I}-\Lambda)(a)]\Lambda$, one can factorize $\hat{\rho}$ as

[Becattini et al, JHEP **02**(2021)101]

$$\hat{\rho} = e^{-\tilde{b}(\varpi)\cdot\hat{P}} e^{\varpi:\hat{J}/2}, \tag{8}$$

  where

$$\tilde{b}(\varpi)^\mu = \sum_{k=0}^\infty \frac{i^k}{(k+1)!}(\varpi^\mu{}_{\nu_1}\varpi^{\nu_1}{}_{\nu_2}\cdots\varpi^{\nu_{k-1}}{}_{\nu_k})b^{\nu_k} = B\delta^\mu_0 + A\delta^\mu_3,$$
$$B = \frac{\sin\alpha}{a}, \qquad A = \frac{i}{a}(1-\cos\alpha), \tag{9}$$

  such that $\hat{\rho} = e^{-B\hat{H}+A\hat{P}^z} e^{\alpha\hat{K}^z}$.

KMS relation for accelerating states

- Taking into account the properties

$$e^{i\tilde{b}\cdot\hat{P}}\hat{\Phi}(x)e^{-i\tilde{b}\cdot\hat{P}} = \hat{\Phi}(x+\tilde{b}), \qquad \hat{\Lambda}\hat{\Phi}(x)\hat{\Lambda}^{-1} = D[\Lambda^{-1}]\hat{\Phi}(\Lambda x), \tag{10}$$

we arrive at $\hat{\rho}\hat{\Phi}(t,z)\hat{\rho}^{-1} = e^{-\alpha S^{0z}}\hat{\Phi}(\tilde{t},\tilde{z})$, where

$$\begin{aligned} \tilde{t} &= \cos(\alpha)t + i\sin(\alpha)z + \frac{i}{a}\sin(\alpha), \\ \tilde{z} &= i\sin(\alpha)t + \cos(\alpha)z - \frac{1}{a}[1-\cos(\alpha)]. \end{aligned} \tag{11}$$

- The KMS are formulated at the level of the Wightman functions:

$$G^{+}(x,x') = \langle\hat{\Phi}(x)\hat{\overline{\Phi}}(x')\rangle, \qquad G^{-}(x,x') = (-1)^{2s}\langle\hat{\overline{\Phi}}(x')\hat{\Phi}(x)\rangle, \tag{12}$$

with $s=0$ for scalars and $s=1/2$ for Dirac fermions.

- At finite temperature and under acceleration, we derive the KMS relations:

$$G^{+}(x,x') = (-1)^{2s}e^{-\alpha S^{0z}}G^{-}(\tilde{t},\tilde{z};x'). \tag{13}$$

Euclidean two-point functions: rational acceleration

- The KMS relations transfer to the Euclidean two-point functions:

$$G_E(\tilde{\tau}, \tilde{z}; X') = (-1)^{2s} e^{-\alpha S^{0z}} G_E(\tau, z; X')\,, \tag{14}$$

which are solved formally by

$$G_E^{(\alpha)}(\tau, z; X') = \sum_{j=-\infty}^{\infty} (-1)^{2sj} e^{-j\alpha S^{0z}} G_E^{\rm vac}(\tau_{(j)}, z_{(j)}; X')\,, \tag{15a}$$

where $G_E^{\rm vac}(X, X')$ is the vacuum Euclidean propagator, while

$$\begin{aligned} \tau_{(j)} &= \tau \cos(j\alpha) - \frac{1}{a}(1 + az)\sin(j\alpha), \\ z_{(j)} &= \tau \sin(j\alpha) + \frac{1}{a}(1 + az)\cos(j\alpha) - \frac{1}{a}. \end{aligned} \tag{16}$$

- When $\alpha/2\pi = p/q \in \mathbb{Q}$, the sum over j in Eqs. (15) contains only q terms:

$$G_E^{(p,q)}(\tau, z; x') = \sum_{j=0}^{q-1} (-1)^{2sj} e^{-j\alpha S^{0z}} G_E^{\rm vac}(\tau_{(j)}, z_{(j)}; x'). \tag{17}$$

Energy density

- Using
$$G_E^{\rm vac}(X,X') = \frac{1}{4\pi^2 \Delta X^2}, \qquad S_E^{\rm vac}(X,X') = \gamma^E_\mu \partial_\mu G_E^{\rm vac} \tag{18}$$
we find (in agreement with [*Becattini*, JHEP, 202*])
$$\mathcal{E}^\xi_{\rm KG} = \frac{3[\alpha T(x)]^4}{16\pi^2}\left[G_4(\alpha) + 4\xi G_2(\alpha)\right], \quad \mathcal{E}_D = \frac{3[\alpha T(x)]^4}{4\pi^2} S_4(\alpha),$$
$$G_n(\alpha) = \sum_{j=1}^\infty \frac{1}{[\sin(j\alpha/2)]^n}, \qquad S_n(\alpha) = -\sum_{j=1}^\infty \frac{(-1)^j \cos(j\alpha/2)}{[\sin(j\alpha/2)]^n}.$$
- Since $e^{-q\alpha S^{0z}} = (-1)^p$, $S_E^{(p,q)}(\tau_{(q)}, z_{(q)}) = (-1)^{p+q} S_E^{p,q}(\tau, z)$, such that $S_E^{(p,q)} = 0$ when $p+q$ is odd.
- **For** $\alpha = p/q$**:** In this case, both G_n and S_n can be computed in closed form. We find:
$$\mathcal{E}_\xi^{(p,q)} = \frac{[\alpha T(x)]^4}{480\pi^2}(q^2-1)(q^2+11+60\xi),$$
$$\mathcal{E}_D^{(p,q)} = \frac{[\alpha T(x)]^4}{960\pi^2}(q^2-1)(7q^2+17)\frac{1+(-1)^{p+q}}{2}. \tag{19}$$